Super Geometric Mean Labeling On Double Triangular Snakes

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ABSTRACT

Let f: V(G) → {1,2,…,p+q} be an injective function. For a vertex labeling “f” the induced edge labeling f* (e=uv) is defined by,

f* (e) = ⌈√(f(u)f(v))⌉ or ⌊√(f(u)f(v))⌋.

Then f is called a Super Geometric mean labeling if

\{f(V(G)) \cup \{f(e): e \in E(G)\}\} = \{1,2,...,p + q\}, A graph which admits Super Geometric mean labeling is called Super Geometric mean graph.

In this paper, we prove that Double Triangular Snakes and Alternate Double Triangular Snake graphs and Super Geometric mean graphs.

Key words: Graph, Geometric mean graph, Super Geometric mean graph, Double Triangular snake, Alternate Double Triangular snake.

1. Introduction

All graphs in this paper are finite, simple and undirected graph G = (V,E) with p vertices and q edges. For all detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notations we follow Harary [2]. The concept of Geometric mean labeling has been introduced by S. Somasundaram, R. Ponraj and P. Vidhyarani in [5]. We will provide a brief summary of definitions and other informations which are necessary for our present investigation.

Definition 1.1

A graph G = (V,E) with p vertices and q edges is called a Geometric mean graph if it is possible to label vertices x ∈ V with distinct label f(x) from 1,2,...,q+1 in such a way that when each edge e=uv is labeled with, f(e=uv) = [√(f(u)f(v))] or [√(f(u)f(v))] then the edge labels are distinct. In this case “f” is called Geometric mean labeling of G.
Definition 1.2:
Let \( f: V(G) \rightarrow \{1,2,...,p+q\} \) be an injective function. For a vertex labeling “f”, the induced edge labeling \( f^*(e=uv) \) is defined by,
\[
f^*(e) = \left\lfloor \sqrt{f(u)f(v)} \right\rfloor \text{ or } \left\lceil \sqrt{f(u)f(v)} \right\rceil \text{ then “f” is called a Super Geometric mean labeling if } \{f(V(G)) \cup \{ f(e): e \in E(G) \} = \{1,2,...,p+q \} \}. \text{ A graph which admits Super Geometric mean labeling is called Super Geometric mean graph.}
\]

Definition 1.3:
A Triangular Snake \( T_n \) is obtained from a Path \( u_1u_2,...,u_n \) by joining \( u_i \) and \( u_{i+1} \) to a new vertex \( v_i, 1\leq i \leq n-1 \). That is every edge of a Path is replaced by a triangle \( C_3 \).

Definition 1.4:
An Alternate Triangular snake \( A(T_n) \) is obtained from a Path \( u_1u_2...u_n \) by joining \( u_i \) and \( u_{i+1} \) alternatively to a new vertex \( v_i \). That is every alternate edge of a Path is replaced by \( C_3 \).

Definition 1.5:
A Double Triangular snake \( D(T_n) \) consists of two Triangular snakes that have a common Path.

Definition 1.6:
An Alternate Double Triangular snake \( A(D(T_n)) \) consists of two Alternate Triangular snakes that have a common Path.

Now we shall use frequent reference to the following theorems.

Theorem 1.7 [5]: Triangular snakes and Alternate Triangular snakes are Geometric mean graphs.

Theorem 1.8: Double Triangular and Alternate Double Triangular snakes are Geometric mean graphs.

2. Main Results
Theorem 2.1:
A Double Triangular snake \( D(T_n) \) is a Super Geometric mean graph.

Proof
Let \( D(T_n) \) be the Double Triangular snake.
Consider a Path \( u_1u_2...u_n \).
Join \( u_i u_{i+1} \) with two new vertices \( v_i \) and \( w_i, 1\leq i \leq n-1 \). The graph is displaced below.
Define a function $f: V(D(T_n)) \rightarrow \{1,2,\ldots,p+q\}$ by,

- $f(u_1) = 4$
- $f(u_i) = 8i-7$, $2 \leq i \leq n$
- $f(v_1) = 1$
- $f(v_i) = 8i-4$, $2 \leq i \leq n-1$
- $f(w_i) = 8i-1$, $1 \leq i \leq n-1$

Edges are labeled with,

- $f(u_1u_2) = 6$
- $f(u_iu_{i+1}) = 8i-3$, $2 \leq i \leq n-1$
- $f(u_iw_i) = 8i-6$, $1 \leq i \leq n-1$
- $f(v_1u_2) = 3$
- $f(v_iw_{i+1}) = 8i-2$, $2 \leq i \leq n-1$
- $f(u_1w_1) = 5$
- $f(u_iw_i) = 8i-5$, $2 \leq i \leq n-1$
- $f(w_iw_{i+1}) = 8i$, $1 \leq i \leq n-1$

Thus we get distinct edge labels.

Hence $D(T_n)$ is a Super Geometric mean graph.

**Example 2.2**

A Super Geometric mean labeling of $D(T_5)$ is given below.
Theorem 2.3

Alternate Double Triangular snake $A(D(T_n))$ is a Super Geometric mean graph.

Proof:

Let $G$ be the graph $A(D(T_n))$

Consider the Path $u_1u_2...u_n$

To construct $G$, join $u_i$ and $u_{i+1}$ (alternatively) with two new vertices $v_i$ and $w_i$

There are two different cases to be considered.

Case 1: If $A(D(T_n))$ starts from $u_1$, we need two subcases.

Subcase 1(a) If $n$ is odd, then

Define a function $f: V(G) \rightarrow \{1,2,...,p+q\}$ by,

$f(u_1) = 4$

$f(u_{2i-1}) = 10i-9, 2 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor + 1$

$f(u_{2i}) = 10i-1, 1 \leq i \leq \frac{n-1}{2}$

$f(v_1) = 1$

$f(v_i) = 10i-6, 2 \leq i \leq \frac{n-1}{2}$

$f(w_i) = 10i-3, 1 \leq i \leq \frac{n-1}{2}$

Edges are labeled with,

$f(u_1u_2) = 6$

$f(u_iu_{i+1}) = 5i, 2 \leq i \leq n-1$
\( f(u_{2i-1}v_i) = 10i-8, \ 1 \leq i \leq \frac{n-1}{2} \)

\( f(v_1u_2) = 3 \)

\( f(v_1u_{2i}) = 10i-4, \ 2 \leq i \leq \frac{n-1}{2} \)

\( f(u_1w_1) = 5 \)

\( f(u_{2i-1}w_i) = 10i-7, \ 2 \leq i \leq \frac{n-1}{2} \)

\( f(w_iu_{2i}) = 10i-2, \ 1 \leq i \leq \frac{n-1}{2} \)

Thus we get distinct edge labels.

The labeling pattern of \( A (D(T_4)) \) is shown below

![Figure: 3](image)

**Subcase 1(b):** If \( n \) is even, then

Define a function \( f: V(G) \rightarrow \{1,2,\ldots,p+q\} \) by,

\( f(u_1) = 4 \)

\( f(u_{2i-1}) = 10i-9, \ 2 \leq i \leq \frac{n}{2} \)

\( f(u_{2i}) = 10i-1, \ 1 \leq i \leq \frac{n}{2} \)

\( f(v_1) = 1 \)

\( f(v_i) = 10i-6, \ 2 \leq i \leq \frac{n}{2} \)

\( f(w_1) = 10i-3, \ 1 \leq i \leq \frac{n}{2} \)

Edges are labeled with

\( f(u_1u_2) = 6 \)

\( f(u_iu_{i+1}) = 5i, \ 2 \leq i \leq n-1 \)

\( f(u_{2i-1}v_i) = 10i-8, \ 1 \leq i \leq \frac{n}{2} \)

\( f(v_1u_2) = 3 \)
\[ f(v_i u_{2i}) = 10i-4, \ 2 \leq i \leq \frac{n}{2} \]

\[ f(u_{1} w_{1}) = 5 \]

\[ f(u_{2i-1} w_{i}) = 10i-7, \ 2 \leq i \leq \frac{n}{2} \]

\[ f(w_{i} u_{2i}) = 10i-2, \ 1 \leq i \leq \frac{n}{2} \]

Therefore the edge labels are distinct.

The labeling pattern of \( A(D(T_4)) \) is displaced below.

![Figure: 4]

In this case \( f \) provides a Super Geometric mean labeling of \( G \).

**Case 2:** If \( A(D(T_n)) \) Starts from \( u_2 \), we have to consider two subcases.

**Subcase 2(a):** If \( n \) is odd then,

Define a function \( f: V(G) \rightarrow \{1,2,\ldots,p+q\} \) by,

\[ f(u_{2i-1}) = 10i-9, \ 1 \leq i \leq \left(\frac{n-1}{2}\right)+1 \]

\[ f(u_{2i}) = 10i-7, \ 1 \leq i \leq \frac{n-1}{2} \]

\[ f(v_i) = 10i-3, \ 1 \leq i \leq \frac{n-1}{2} \]

\[ f(w_i) = 10i-1, \ 1 \leq i \leq \frac{n-1}{2} \]

Edges are labeled with,

\[ f(u_{2i-1} u_{2i}) = 10i-8, \ 1 \leq i \leq \frac{n-1}{2} \]

\[ f(u_{2i} u_{2i+1}) = 10i-4, \ 1 \leq i \leq \frac{n-1}{2} \]

\[ f(v_i u_{2i}) = 10i-6, \ 1 \leq i \leq \frac{n-1}{2} \]
f(v_i u_{2i+1}) = 10i-2, \quad 1 \leq i \leq \frac{n-1}{2}

f(u_{2i} w_i) = 10i-5, \quad 1 \leq i \leq \frac{n-1}{2}

f(w_i u_{2i+1}) = 10i, \quad 1 \leq i \leq \frac{n-1}{2}

Thus we get distinct edge labels.

The labeling pattern of A(D(T_4)) is given below.

Figure: 5

Subcase 2(b): If n is even, then

Define a function, f: V(G) \rightarrow \{1,2,\ldots,p+q\} by,

f(u_{2i-1}) = 10i-9, \quad 1 \leq i \leq \frac{n}{2}

f(u_{2i}) = 10i-7, \quad 1 \leq i \leq \frac{n}{2}

f(v_i) = 10i-3, \quad 1 \leq i \leq \frac{n-2}{2}

f(w_i) = 10i-1, \quad 1 \leq i \leq \frac{n-2}{2}

Edges are labeled with

f(u_{2i-1} u_{2i}) = 10i-8, \quad 1 \leq i \leq \frac{n}{2}

f(u_{2i} u_{2i+1}) = 10i-4, \quad 1 \leq i \leq \frac{n-2}{2}

f(v_i u_{2i}) = 10i-6, \quad 1 \leq i \leq \frac{n-2}{2}

f(v_i u_{2i+1}) = 10i-2, \quad 1 \leq i \leq \frac{n-2}{2}

f(u_{2i} w_i) = 10i-5, \quad 1 \leq i \leq \frac{n-2}{2}

f(w_i u_{2i+1}) = 10i \quad 1 \leq i \leq \frac{n-2}{2}

\therefore The edge labels are distinct

\therefore The labeling pattern of A (D(T_3)) is shown below.
In this case, $f$ provides a Super Geometric mean labeling of $G$.

$\therefore$ From all the above cases, we conclude that Alternate Double Triangular snake $A(D(T_n))$ is a Super Geometric mean graph.

References:


