An EOQ Model with Three-Parameter Weibull Deterioration, Trended Demand and Time Varying Holding Cost with Salvage

N. K. Sahoo[1], P. K. Tripathy[2]

[1]*Department of Mathematics, S.S.S. Mahavidyalaya, Gaudakateni, Dhalpur, Dhenkanal-759001, India*
[2]*P.G. Department of Statistics, Utkal University, Vani Vihar, Bhubaneswar-751004, India*

***Abstract** - An EOQ model is developed for deteriorating items with three parameter Weibull distribution deterioration and linear declined demand rate. The holding cost is considered as time dependent quadratic function. The model is introduced with salvage value. The sensitivity analysis is carried out to study the effect of salvage value and other parameters with a numerical example.*

Keywords- *Weibull distribution deterioration, linear demand, Salvage value, EOQ*

1. Introductions

Deterioration is defined as decay or damage such that the item cannot be used for its original purpose. Most of the physical goods undergo decay or deterioration over time. Commodities such as fruits, vegetables, foodstuffs, etc., suffer from depletion by direct spoilage while kept in store. Highly volatile liquids such as gasoline, alcohol, turpentine, etc., undergo physical depletion over time through the process of evaporation. Electronic goods, radioactive substances, photographic film, grain, etc. deteriorate through a gradual loss of potential or utility with the passage of time. Deterioration is a big critical problem for business world and unavoidable loss to production and inventory system. Hence young researchers have key attention on deteriorating inventory model. Here we have taken three parameter Weibull distribution deterioration rate, demand rate and holding cost as time depended to make model more realistic which is our main objective.

Now we will provide a brief literature review for deteriorating inventory model. Ghare and Schrader [6] developed inventory model for deteriorating items. Covert and Philip [8] proposed an inventory model with two parameter weibull distribution deterioration. An EOQ model with three-parameter Weibull deterioration and linear demand rate is considered by Chakrabarty, Giri and Chaudhuri [1]. S. K. Goyal and B. C. Giri. [10] suggested deteriorating inventory model. An inventory model with two-parameter Weibull deterioration and quadratic demand rate is proposed by Ghosh and Chaudhuri [9]. Poonam Mishra and Nita H. Saha [7] formulated an inventory model with time dependent deteriorating and salvage value. N. K. Sahoo, C. K. Sahoo and S. K. Sahoo [3] discussed about an EOQ model with ramp type demand rate, linear deterioration rate, unit production cost with shortage and backlogging. N. K. Sahoo et al. [4] recommended an EOQ model for deteriorating items with time-dependent deterioration rate, quadratic demand rate without shortage. P. K. Tripathy and S. Pradhan [5] suggested an inventory model having Weibull demand and time depended deterioration rate. Smaila S. Sanni and Walford I. E. Chukwu [11] adopted an inventory model with three-parameter Weibull deterioration, quadratic demand rate and shortages. N. K. Sahoo, Bhabani S. Mohanty and P. K. Tripathy [2] proposed an EOQ model with exponential demand and time varying holding cost.

2. Assumptions and notation

The model is developed with following assumptions and notations

- Demand $D(t)=a(1-bt)$ is assumed to be a decreasing function of time i.e. $a>0$ is fixed demand and $b(0<b<<1)$ is the rate of change of demand.
- The replenishment rate is infinite.
- The lead time is zero.
- Shortages are not allowed.
- A is the ordering cost per order.
- B is the purchase cost per units.
- Holding cost $h(t)$ per item per unit time is time dependent and is assumed as $h(t)=h_1+h_2t^2,\ h_1>0,\ h_2>0$.

- $\theta(t)=\alpha\,\beta(t-\gamma)^{\beta-1}$, where $0<\alpha<1$ is scale parameter, $\beta>0$ is shape parameter and $0<\gamma<1$ is location parameter represents the three-parameter Weibull distribution deterioration rate at any time $t>\gamma$.
- The salvage value λB $(0\le\lambda<1)$ is associated to deteriorated units during the cycle time.
- The deteriorated units cannot be repaired or replaced during the period under review.
- $I_{TC}(T)$ is the total inventory cost.

3. Mathematical Model

Let $I(t)$ be the inventory level at any instant of time t $(0\le t\le T)$. Using assumptions and notations the inventory system depicted in Fig.1 and the inventory system with respect to time t can be depicted by the adopting differential equation.

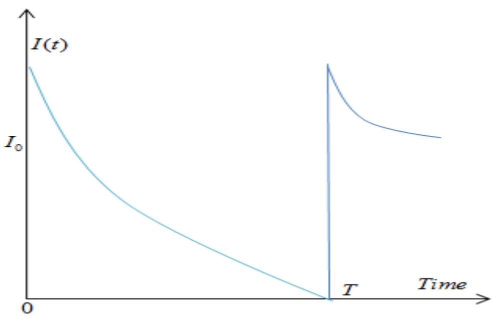

Fig. 1. Graphical representation of the inventory system

$$\frac{dI(t)}{dt}+\theta(t)I(t)=-D(t),0\le t\le T \quad (3.1)$$

With initial condition $I(0)=I_0$ and boundary condition $I(T)=0$.

Using $D(t)=a(1-bt)$ and $\theta(t)=\alpha\,\beta(t-\gamma)^{\beta-1}$, we get

$$\frac{dI(t)}{dt}+\alpha\,\beta(t-\gamma)^{\beta-1}I(t)=-a(1-bt),0\le t\le T \quad (3.2)$$

With $I(0)=I_0$ and $I(T)=0$.

So solution of equation (3.2) is

$$I(t)=\begin{bmatrix} a(T-t)-\frac{1}{2}ab(T^2-t^2) \\ +\ \frac{a\alpha}{\beta+1}((T-\gamma)^{\beta+1}-(t-\gamma)^{\beta+1}) \\ -a\alpha(T-t)(t-\gamma)^{\beta} \end{bmatrix} \quad (3.3)$$

Neglecting higher power of α.

Using $I(0)=I_0$, we get

$$I_0=aT-\frac{1}{2}abT^2-a\alpha T(-\gamma)^{\beta}$$
$$+\ \frac{a\alpha}{\beta+1}((T-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}) \quad (3.4)$$

The total cost per time unit I_{TC} is given by

$$I_{TC}(T)=\frac{1}{T}(I_{HC}+I_{OC}+I_{CD}-SV) \quad (3.5)$$

Where

(i) $I_{HC}=Inventory\ holding\ \cos t$

$$=\int_0^T h(t)\,I(t)\,dt$$

$$=h_1\left[\frac{aT^2}{2}-\frac{abT^3}{3}+\frac{a\alpha}{\beta+1}\begin{pmatrix} T(T-\gamma)^{\beta+1}-\frac{2(T-\gamma)^{\beta+2}}{(\beta+2)} \\ +\frac{2(-\gamma)^{\beta+2}}{(\beta+2)}+T(-\gamma)^{\beta+1} \end{pmatrix}\right]$$

$$+h_2\left[\frac{aT^4}{12}-\frac{abT^5}{15}+\frac{a\alpha}{\beta+1}\begin{pmatrix} \frac{T^3(T-\gamma)^{\beta+1}}{3}-\frac{2T^2(T-\gamma)^{\beta+2}}{\beta+2} \\ +\frac{6T(T-\gamma)^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{8(T-\gamma)^{\beta+4}}{(\beta+2)(\beta+3)(\beta+4)} \\ +\frac{2T(-\gamma)^{\beta+3}}{(\beta+2)(\beta+3)}+\frac{8(-\gamma)^{\beta+4}}{(\beta+2)(\beta+3)(\beta+4)} \end{pmatrix}\right] \quad (3.6)$$

(ii) $I_{OC}=Ordering\ \ Cost=A$

(iii) $I_{CD}=Cost\ due\ to\ Deterioration$

$$=B[I_0-\int_0^T D(t)dt]$$

$$=B\left(\frac{a\alpha}{\beta+1}((T-\gamma)^{\beta+1}-(-\gamma)^{\beta+1})-a\alpha T(-\gamma)^{\beta}\right) \quad (3.7)$$

(iv) $SV=Salvage\ value\ of\ Deterioration\ items$

$$=\lambda B[I_0-\int_0^T D(t)dt]$$

$$=\lambda B\left(\frac{a\alpha}{\beta+1}((T-\gamma)^{\beta+1}-(-\gamma)^{\beta+1})-a\alpha T(-\gamma)^{\beta}\right)$$

Thus,

$$I_{TC}(T)=\left[\begin{array}{l} A+h_1\left(\frac{aT^2}{2}-\frac{abT^3}{3}+\frac{a\alpha}{\beta+1}\left(\begin{array}{l}T(T-\gamma)^{\beta+1}-\frac{2(T-\gamma)^{\beta+2}}{(\beta+2)}\\ +\frac{2(-\gamma)^{\beta+2}}{(\beta+2)}+T(-\gamma)^{\beta+1}\end{array}\right)\right)\\ +h_2\left(\frac{aT^4}{12}-\frac{abT^5}{15}+\frac{a\alpha}{\beta+1}\left(\begin{array}{l}\frac{T^3(T-\gamma)^{\beta+1}}{3}-\frac{2T^2(T-\gamma)^{\beta+2}}{\beta+2}\\ +\frac{6T(T-\gamma)^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{8(T-\gamma)^{\beta+4}}{(\beta+2)(\beta+3)(\beta+4)}\\ +\frac{2T(-\gamma)^{\beta+3}}{(\beta+2)(\beta+3)}+\frac{8(-\gamma)^{\beta+4}}{(\beta+2)(\beta+3)(\beta+4)}\end{array}\right)\right)\\ +B\left(\frac{a\alpha}{\beta+1}((T-\gamma)^{\beta+1}-(-\gamma)^{\beta+1})-a\alpha T(-\gamma)^{\beta}\right)\\ -\lambda B\left(\frac{a\alpha}{\beta+1}((T-\gamma)^{\beta+1}-(-\gamma)^{\beta+1})-a\alpha T(-\gamma)^{\beta}\right)\end{array}\right] \tag{3.8}$$

The necessary condition for $I_{TC}(T)$ to be minimum is $\frac{dI_{TC}(T)}{dT}=0$ and solving it for T by Mathematica-5.1 software. For obtained T, $I_{TC}(T)$ is minimum only if $\frac{d^2I_{TC}(T)}{dT^2}>0$.

Thus $\frac{dI_{TC}(T)}{dT}=0$,

$$\Rightarrow\left[\begin{array}{l} h_1\left(aT-abT^2+\frac{a\alpha}{\beta+1}\left(\begin{array}{l}(\beta+1)T(T-\gamma)^{\beta}\\ -(T-\gamma)^{\beta+1}+(-\gamma)^{\beta+1}\end{array}\right)\right)\\ +h_2\left(\begin{array}{l}\frac{aT^3}{3}-\frac{abT^4}{3}\\ +\frac{a\alpha}{\beta+1}\left(\begin{array}{l}\frac{(\beta+1)T^3(T-\gamma)^{\beta}}{3}-T^2(T-\gamma)^{\beta+1}\\ +\frac{2T(T-\gamma)^{\beta+2}}{(\beta+2)}-\frac{2(T-\gamma)^{\beta+3}}{(\beta+2)(\beta+3)}+\frac{2(-\gamma)^{\beta+3}}{(\beta+2)(\beta+3)}\end{array}\right)\end{array}\right)\\ +B\left(a\alpha(T-\gamma)^{\beta}-a\alpha(-\gamma)^{\beta}\right)\\ -\lambda B\left(a\alpha(T-\gamma)^{\beta}-a\alpha(-\gamma)^{\beta}\right)\end{array}\right]=0 \tag{3.9}$$

And

$$\frac{d^2I_{TC}(T)}{dT^2}=\left[\begin{array}{l} h_1\left(a-2abT+\frac{a\alpha}{\beta+1}\left((\beta+1)\beta T(T-\gamma)^{\beta-1}\right)\right)\\ +h_2\left(aT^2-\frac{4abT^3}{3}+\frac{a\alpha}{\beta+1}\left(\frac{(\beta+1)\beta T^3(T-\gamma)^{\beta-1}}{3}\right)\right)\\ +B\,a\,\alpha\beta(T-\gamma)^{\beta-1}-\lambda B\,a\,\alpha\beta(T-\gamma)^{\beta-1}\end{array}\right]$$

4. Numerical Example

Example-1-Let $A=10000$, $a=800$, $b=0.6$, $B=40$, $h_1=2$, $h_2=0.2$, $\alpha=0.4$, $\beta=2$, $\gamma=0.2$ and $\lambda=0.1$ in appropriate units. By applying Mathematica 9.1, we obtain the optimum solution for T of equations (3.9) as $T=0.284685$. Substituting T in equation (3.8), we obtain the optimum average cost as $I_{TC}(T)=9959.11$.

5. Sensitivity Analysis

Table-1 (Effects of changes in the system parameters)

Parameters	*% change in parameters*	T^*	$I^*_{TC}(T)$	*Percentage change in* $I^*_{TC}(T)$
A	+50	0.284685	14959.1	50.20519
	+25	0.284685	12459.1	25.10254
	-25	0.284685	7459.11	-25.1026
	-50	0.284685	4959.11	-50.2053
a	+50	0.284685	9938.67	-0.20524
	+25	0.284685	9948.89	-0.10262
	-25	0.284685	9969.34	0.10272
	-50	0.284685	9979.56	0.20534
b	+50	0.298092	9955.15	-0.03976
	+25	0.291235	9957.2	-0.01918
	-25	0.278421	9960.9	0.017973
	-50	0.272425	9962.58	0.034842
B	+50	0.324999	9906.06	-0.53268
	+25	0.309124	9933.39	-0.25826
	-25	0.242098	9981.74	0.227229
	-50	0.148444	9997.4	0.384472
h_1	+50	0.219944	9982.06	0.230442
	+25	0.253047	9971.97	0.129128
	-25	0.315016	9943.34	-0.15835
	-50	0.344177	9924.53	-0.34722
h_2	+50	0.284517	9959.15	0.000402
	+25	0.284601	9959.13	0.000201
	-25	0.284769	9959.1	-0.0001
	-50	0.284853	9959.08	-0.0003
λ	+50	0.277344	9964.49	0.054021
	+25	0.281128	9961.82	0.027211
	-25	0.288033	9956.37	-0.02751
	-50	0.291192	9953.59	-0.05543

α	+50	0.324893	9905.92	-0.53408
	+25	0.309102	9933.32	-0.25896
	-25	0.241905	9981.8	0.227832
	-50	0.147447	9997.44	0.384874
γ	+50	0.501096	9775.3	-1.84565
	+25	0.393053	9891.97	-0.67416
	-25	0.175966	9990.38	0.313984
	-50	0.0668696	9999.47	0.405257

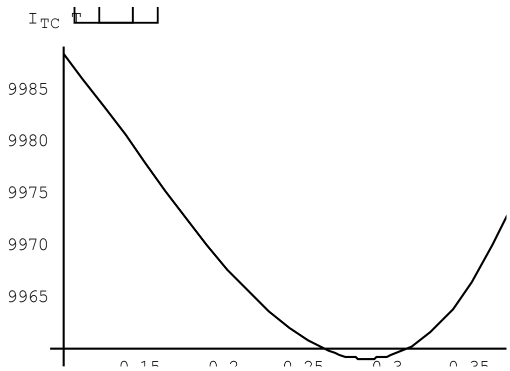

***Fig.2. Concavity of** $I_{TC}(T)$ w.r.t. T*

We now study the effects of changes in the system of parameters A, a, b, B, h_1, h_2, λ, α, β, γ on the optimal cost derived by the proposed method. The sensitivity analysis is performed by changing (increasing or decreasing) of parameters by 25% and 50% and taking one parameter at a time and keeping the remaining parameter at their fixed value.

The analysis is based on the example-1 and the results are shown in table-1. The following points are observed

(i) The optimum time T^* remain same while the optimum cost $I^*_{TC}(T)$ increase (decrease) with increase (decrease) of parameter A. The model is highly sensitive to the parameter A.

(ii) The optimum time T^* also remain same while the optimum cost $I^*_{TC}(T)$ decrease (increase) with increase (decrease) of parameter a. The model is moderately sensitive to the parameter a.

(iii) The optimum time T^* increase (decrease) while the optimum cost $I^*_{TC}(T)$ decrease (increase) with increase (decrease) of parameter b and B. The model is insensitive to the parameter b and B.

(iv) The optimum time T^* decrease (increase) while the optimum cost $I^*_{TC}(T)$ increase (decrease) with increase (decrease) of parameter h_1. The model is highly sensitive to the parameter h_1.

(v) The optimum time T^* slowly changes while the optimum cost $I^*_{TC}(T)$ is almost same with increase (decrease) of parameter h_2. The model is almost insensitive to the parameter h_2.

(vi) The optimum time T^* decrease (increase) while the optimum cost $I^*_{TC}(T)$ increase (decrease) with increase (decrease) of parameter λ. The model is insensitive to the parameter λ.

(vii) The optimum time T^* increase (decrease) while the optimum cost $I^*_{TC}(T)$ decrease (increase) with increase (decrease) of parameter α and γ. The model is insensitive to the parameter α and γ.

6. Conclusion:

The current article describes a deteriorating model with three parameter Weibull deterioration, linear declined demand rate and time-varying holding cost. The salvage value is analyzed in this paper. We also studied effect of change of different parameters for this model. The model is solved for minimizing inventory total cost with numerical example.

The above model can be improved into more realistic assumptions such as stochastic demand with shortages.

7. References

[1] B.C. Chakrabarty; B.C. Giri and K.S. Chaudhuri. (1998). An EOQ model for items with Weibull distributio deterioration,shortages and trended demand: An extention of Philip's model. *Computers & Operations Research.* **25**, 649-657.

[2] N. K. Sahoo; Bhabani S. Mohanty and P.K. Tripathy. (2016). Fuzzy inventiry model with exponential demand and time-varying deterioration. *Global Journal of Pure and Applied Mathematics.* **12**(3), 2573-2589.

[3] N. K. Sahoo; C. K. Sahoo and S. K. Sahoo. (2009). An EOQ model model with ramp type demand rate, linear deterioration rate, unit production cost with shortages and backlogging. *Journal of Rajasthan Academy of Physical Sciences.* **8**(3), 319-332.

[4] N.K.Sahoo; C.K.Sahoo and S.K.Sahoo. (2011). An EOQ model for deteriorating items with time-dependent deterioration rate, quadratic demand rate, unit production

cost and without shortage. *The Journal of the Indian Academy of Mathematics.* **33**(2), 583-594.

[5] P. K. Tripathy and S. Pradhan. (2011). An integrated partial backlogging inventory model having Weibull demand and variable deterioration rate with the effect of trade credit. *International Journal of Scientific & Engineering Research*. **2**(4), 1-4.

[6] P.M. Ghare and G.F. Schrader. (1963). An inventory model for exponentially deteriorating items. *Journal of Industrial Engineering, 14*, 238-243.

[7] Poonam Mishra and Nita H. Shah. (2008). Inventory management of time dependent deteriorating items with salvage value. *Applied Mathematical Sciences, 2*(16), 793-798.

[8] R. P. Covert and G. C. Philip. (1973). An EOQ model items with Weibull distribution deterioration. *AIIE Transactions.* **5**, 323-326.

[9] S. K. Ghosh and K. S. Chaudhuri. (2004). An order-level inventory model for deteriorating item with Weibull distribution deterioration, time-quadratic demand and shortages. *Advanced Modelling and Optimization.* **6**(1), 21-35.

[10] S. K. Goyal and B. C. Giri. (2001). Recent Trends in Modeling of Deteriorating Inventory. *European Journal of Operational Research*. **134**, 1-16.

[11] Smaila S. Sanni and Walford I. E. Chukwu. (2016). An inventory model with three-parameter Weibull deterioration, quadratic demand rate and shortages. *American Journal of Mathematical and Management Sciences.* **35**(2), 159-170.