Stolarsky-3 Mean Labeling on Triangular Snake Graphs

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Abstract - Let G = (V, E) be a graph with p vertices and q edges. G is said to be Stolarsky-3 Mean graph if each vertex x∈V is assigned distinct labels f(x) from 1,2,…,q+1 and each edge e = uv is assigned the distinct labels

\[ f(e = uv) = \sqrt{\frac{f(u)^2 + f(v)^2 + f(u)+ f(v)^2}{3}} \] (or) \[ \sqrt{\frac{f(u)^2 + f(v)^2 + f(u)+ f(v)^2}{3}} \]

then the resulting edge labels are distinct. In this case f is called a Stolarsky-3 Mean labeling of G and G is called a Stolarsky-3 Mean graph. In this paper we investigate the Stolarsky-3 Mean labeling for Double Triangular Snake, Triple Triangular Snake, Four Triangular Snake and Alternative Triple Triangular snake graphs.

Keywords - Graph Labeling, Stolarsky-3 Mean Labeling, Triangular Snake Graphs, Double Triangular Snake Graph, Triple Triangular Snake graph, Four Triangular Snake graph.

I. Introduction

All graphs in this paper are finite, simple and undirected graphs. We follow Gallian[1] for all detailed survey of graph labeling and we refer Harary[2] for all other standard terminologies and notations. The concept of “Mean Labeling” has been introduced by S.Somasundaram and R.Ponraj in 2004[3] and S.Somasundaram, R.Ponrajand S.S. Sandhya introduced the concept of “Harmonic Mean Labeling of graphs” in[4]. “Stolarsky-3 mean labeling” was introduced by S.S. Sandhya, E.Ebin Raja Merly and S.Kavitha in [5].

The following definitions are used here for our present study.

Definition 1.1: A graph G with p vertices and q edges is said to be Stolarsky-3 Mean graph if each vertex x∈V is assigned distinct labels f(x) from 1,2,…,q+1 and each edge e = uv is assigned the distinct labels

\[ f(e = uv) = \sqrt{\frac{f(u)^2 + f(v)^2 + f(u)+ f(v)^2}{3}} \] (or) \[ \sqrt{\frac{f(u)^2 + f(v)^2 + f(u)+ f(v)^2}{3}} \]

then the resulting edge labels are distinct. In this case f is called a Stolarsky-3 Mean labeling of G.

Definition 1.2: A Triangular Snake T_n is obtained from a path u_1, u_2, ..., u_n by joining u_i and u_i+1 to a new vertex v_i for 1≤ i ≤ n-1. That is, every edge of a path is replaced by a triangle C_3.

Definition 1.3: Double Triangular Snake D (T_n) consists of two Triangular snakes that have a common path.

Definition 1.4: Triple Triangular Snake T (T_n) consists of three Triangular snakes that have a common path.

Definition 1.5: An Alternate Triangular Snake A (T_n) is obtained from a path u_1, u_2, ..., u_n by joining u_i and u_i+1 alternatively to a new vertex v_i. That is every alternate edge of a path is replaced by C_3.

Definition 1.6: An Alternate Triple Triangular Snake A (T(T_n)) consists of three Alternate Triangular snakes that have a common path.

Definition 1.7: Four Triangular Snake F (T_n) consists of four Triangular snakes that have a common path.

Now we shall use the frequent reference to the following theorems.
**Theorem 1.8 [3]:** Triangular Snake graph is a Mean graph (S. Somasundaram & R. Ponraj).

**Theorem 1.9 [4]:** Triangular Snake graph is Harmonic Mean graph (S. Somasundaram, R. Ponraj & S.S. Sandhya).

**Theorem 1.10 [6]:** Triangular Snake graph is Stolarsky-3 Mean graph (S.S. Sandhya, E.Ebin Raja Merly and S.Kavitha).

II. Main Results

**Theorem 2.1:** Double Triangular Snake graph $D(T_n)$ is Stolarsky-3 Mean graph.

**Proof:** Let $P_n$ be the path $u_1, u_2, ..., u_n$.

To construct $D(T_n)$, join $u_i$ and $u_{i+1}$ to two new vertices $v_i, w_i$, $1 \leq i \leq n-1$.

Define a function $f: V(D(T_n)) \rightarrow \{1, 2, ..., q+1\}$ by

$f(u_i) = 5i - 4, 1 \leq i \leq n$.

$f(v_i) = 5i - 3, 1 \leq i \leq n - 1$.

$f(w_i) = 5i - 2, 1 \leq i \leq n - 1$.

Then the edges are labeled with

$f(u_iu_{i+1}) = 5i - 2, 1 \leq i \leq n - 1$.

$f(u_i v_i) = 5i - 4, 1 \leq i \leq n - 1$.

$f(u_i w_i) = 5i - 3, 1 \leq i \leq n - 1$.

$f(v_i u_{i+1}) = 5i - 1, 1 \leq i \leq n - 1$.

$f(w_i u_{i+1}) = 5i, 1 \leq i \leq n - 1$.

Then the edge labels are distinct.

Hence $D(T_n)$ is Stolarsky-3 Mean graph.

**Example 2.2:** The Stolarsky-3 Mean labeling of $D(T_4)$ is given below.

![Figure 1](image-url)
Theorem 2.3: Triple Triangular Snake graph $T(T_n)$ is Stolarsky-3 Mean graph.

Proof: Let $P_n$ be the path $u_1, u_2, \ldots, u_n$.

To construct $T(T_n)$, Join $u_i$ and $u_{i+1}$ to three new vertices $v_i, w_i$ and $x_i$, $1 \leq i \leq n-1$.

Define a function $f : V(T(T_n)) \rightarrow \{1, 2, \ldots, q+1\}$ by

$f(u_i) = 7i - 6$, $1 \leq i \leq n$.

$f(v_i) = 7i - 5$, $1 \leq i \leq n - 1$.

$f(w_i) = 7i - 4$, $1 \leq i \leq n - 1$.

$f(x_i) = 7i - 3$, $1 \leq i \leq n - 1$.

Then the edges are labeled with

$f(u_i u_{i+1}) = 7i - 3$, $1 \leq i \leq n - 1$.

$f(u_i v_i) = 7i - 6$, $1 \leq i \leq n - 1$.

$f(u_i w_i) = 7i - 5$, $1 \leq i \leq n - 1$.

$f(u_i x_i) = 7i - 4$, $1 \leq i \leq n - 1$.

$f(v_i v_{i+1}) = 7i - 2$, $1 \leq i \leq n - 1$.

$f(w_i w_{i+1}) = 7i - 1$, $1 \leq i \leq n - 1$.

$f(x_i u_{i+1}) = 7i$, $1 \leq i \leq n - 1$.

Then the edge labels are distinct. Hence $T(T_n)$ is Stolarsky-3 Mean graph.

Example 2.4: The Stolarsky-3 Mean labeling of $T(T_4)$ is given below.

\[\text{Figure: 2}\]

Theorem 2.5: Alternate Triple Triangular snake $A(T(T_n))$ is Stolarsky-3 Mean graph.

Proof: Let $G$ be the graph $A(T(T_n))$.

Consider a path $u_1, u_2, \ldots, u_n$.

To construct $G$, join $u_i$ and $u_{i+1}$ (alternatively) with three new vertices $v_i, w_i$ and $x_i$, $1 \leq i \leq n - 1$. 
There are two different cases to be considered.

Case(1) If G is starts from \(u_1\), we consider two sub cases.

Sub case (1) (a) \(n\) is odd

Define a function \(f : V(G) \rightarrow \{1, 2, \ldots, q+1\}\) by

\[
\begin{align*}
    f(u_2i) &= 8i-7, \quad 1 \leq i \leq \frac{n-1}{2} + 1, \\
    f(u_2i) &= 8i, \quad 2 \leq i \leq \frac{n-1}{2}, \\
    f(v_i) &= 8i-6, \quad 1 \leq i \leq \frac{n-1}{2}, \\
    f(w_i) &= 8i-5, \quad 1 \leq i \leq \frac{n-1}{2}, \\
    f(x_i) &= 8i-4, \quad 1 \leq i \leq \frac{n-1}{2}.
\end{align*}
\]

Then the edges are labeled as

\[
\begin{align*}
    f(u_iu_{i+1}) &= 4i, \quad 1 \leq i \leq n - 1, \\
    f(u_{2i-1}v_i) &= 8i - 7, \quad 1 \leq i \leq \frac{n-1}{2}, \\
    f(u_{2i-1}w_i) &= 8i - 6, \quad 1 \leq i \leq \frac{n-1}{2}, \\
    f(u_{2i-1}x_i) &= 8i - 5, \quad 1 \leq i \leq \frac{n-1}{2}, \\
    f(v_iu_{2i}) &= 8i - 3, \quad 1 \leq i \leq \frac{n-1}{2}, \\
    f(w_iu_{2i}) &= 8i - 2, \quad 1 \leq i \leq \frac{n-1}{2}, \\
    f(x_iu_{2i}) &= 8i - 1, \quad 1 \leq i \leq \frac{n-1}{2}.
\end{align*}
\]

Thus we get distinct edge labels.

The labeling pattern is shown below.

\[\text{Figure: 3}\]

Sub case (1) (b) \(n\) is even
Define a function \( f : V(\mathcal{G}) \rightarrow \{1,2,.., q+1\} \) by

\[
\begin{align*}
  f(u_{2i-1}) &= 8i-7, \ 1 \leq i \leq \frac{n}{2} \\
  f(u_{2i}) &= 8i, \ 1 \leq i \leq \frac{n}{2} \\
  f(v_i) &= 8i - 6, \ 1 \leq i \leq \frac{n}{2} \\
  f(w_i) &= 8i - 5, \ 1 \leq i \leq \frac{n}{2} \\
  f(x_i) &= 8i - 4, \ 1 \leq i \leq \frac{n}{2}
\end{align*}
\]

Then the edges are labeled as

\[
\begin{align*}
  f(u_i u_{i+1}) &= 4i, \ 1 \leq i \leq n - 1. \\
  f(u_{2i-1} v_i) &= 8i - 7, \ 1 \leq i \leq \frac{n}{2}. \\
  f(u_{2i-1} w_i) &= 8i - 6, \ 1 \leq i \leq \frac{n}{2}. \\
  f(u_{2i-1} x_i) &= 8i - 5, \ 1 \leq i \leq \frac{n}{2}. \\
  f(v_i u_{2i}) &= 8i - 3, \ 1 \leq i \leq \frac{n}{2}. \\
  f(w_i u_{2i}) &= 8i - 2, \ 1 \leq i \leq \frac{n}{2}. \\
  f(x_i u_{2i}) &= 8i - 1, \ 1 \leq i \leq \frac{n}{2}
\end{align*}
\]

Thus we get distinct edge labels.

The labeling pattern is shown below.

![Diagram](image)

**Figure: 4**

**Case(2) If** \( G \) **is starts from** \( u_2 \) **then we consider two sub cases.**

**Sub case (2) (a) n is odd**

Define a function \( f : V(\mathcal{G}) \rightarrow \{1,2,.., q+1\} \) by

\[
\begin{align*}
  f(u_{2i-1}) &= 8i-7, \ 1 \leq i \leq \frac{n-1}{2} + 1. \\
  f(u_{2i}) &= 8i-6, \ 1 \leq i \leq \frac{n-1}{2}
\end{align*}
\]
\( f(v_i) = 8i - 2, \ 1 \leq i \leq \frac{n-1}{2}. \)
\( f(w_i) = 8i - 1, \ 1 \leq i \leq \frac{n-1}{2}. \)
\( f(x_i) = 8i - 5, \ 1 \leq i \leq \frac{n-1}{2}. \)

Then the edges are labeled as
\( f(u_i, u_{i+1}) = 4i - 3, \ 1 \leq i \leq n - 1. \)
\( f(u_2, v_i) = 8i - 5, \ 1 \leq i \leq \frac{n-1}{2}. \)
\( f(u_2, w_i) = 8i - 4, \ 1 \leq i \leq \frac{n-1}{2}. \)
\( f(u_2, x_i) = 8i - 6, \ 1 \leq i \leq \frac{n-1}{2}. \)
\( f(v_i, u_{2i+1}) = 8i - 1, \ 1 \leq i \leq \frac{n-1}{2}. \)
\( f(w_i, u_{2i+1}) = 8i, \ 1 \leq i \leq \frac{n-1}{2}. \)
\( f(x_i, u_{2i+1}) = 8i - 2, \ 1 \leq i \leq \frac{n-1}{2}. \)

Thus we get distinct edge labels.

The labeling pattern is shown below.

![Figure: 5](image-url)

**Sub case (2) (b) n is even**

Define a function \( f: V(G) \rightarrow \{1, 2, ..., q+1\} \) by
\( f(u_{2i-1}) = 8i - 7, \ 1 \leq i \leq \frac{n}{2}. \)
\( f(u_{2i}) = 8i - 6, \ 1 \leq i \leq \frac{n}{2}. \)
\( f(v_i) = 8i - 2, \ 1 \leq i \leq \frac{n}{2}. \)
\( f(w_i) = 8i - 1, \ 1 \leq i \leq \frac{n}{2}. \)
\( f(x_i) = 8i - 5, \ 1 \leq i \leq \frac{n}{2}. \)

Then the edges are labeled as
\( f(u_i, u_{i+1}) = 4i - 3, \ 1 \leq i \leq n - 1. \)
\( f(u_2v_i) = 8i - 5 \), \( 1 \leq i \leq \frac{n-2}{2} \).

\( f(u_2w_i) = 8i - 4 \), \( 1 \leq i \leq \frac{n-2}{2} \).

\( f(u_2x_i) = 8i - 6 \), \( 1 \leq i \leq \frac{n-2}{2} \).

\( f(v_1u_{2i+1}) = 8i - 1 \), \( 1 \leq i \leq \frac{n-2}{2} \).

\( f(w_1u_{2i+1}) = 8i \), \( 1 \leq i \leq \frac{n-2}{2} \).

\( f(x_1u_{2i+1}) = 8i - 2 \), \( 1 \leq i \leq \frac{n-2}{2} \).

Thus we get distinct edge labels.

The labeling pattern is shown below.

![Figure 6](image-url)

From all the above cases, we can conclude that Alternate Triple Triangular snake \( A(T(T_n)) \) is Stolarsky-3 mean graph.

**Theorem 2.6:** Four Triangular Snake graph \( F(T_n) \) is Stolarsky-3 Mean graph.

**Proof:** Let \( P_n \) be the path \( u_1, u_2, \ldots, u_n \).

To construct \( F(T_n) \), join \( u_i \) and \( u_{i+1} \) with four new vertices \( v_i, w_i, v'_i \) and \( w'_i \), \( 1 \leq i \leq n-1 \).

Define a function \( f : V(F(T_n)) \rightarrow \{1, 2, \ldots, q+1\} \) by

\( f(u_1) = 1 \).

\( f(v_1) = 6 \).

\( f(w_1) = 2 \).

\( f(v'_1) = 8 \).

\( f(w'_1) = 4 \).

\( f(u_i) = 9(i-1), 2 \leq i \leq n \).

\( f(v_i) = 9i - 4 \), \( 2 \leq i \leq n - 1 \).

\( f(w_i) = 9i - 8 \), \( 2 \leq i \leq n - 1 \).
\[ f(v_i') = 9i \cdot 2, \ 2 \leq i \leq n - 1. \]
\[ f(w_i') = 9i \cdot 6, \ 2 \leq i \leq n - 1. \]

Then the edges are labeled with
\[ f(u_i u_{i+1}) = 9i \cdot 4, \ 1 \leq i \leq n - 1. \]
\[ f(u_i v_i) = 9i \cdot 6, \ 1 \leq i \leq n - 1. \]
\[ f(u_i w_i) = 9i \cdot 5, \ 1 \leq i \leq n - 1. \]
\[ f(u_i v_i) = 9i \cdot 8, \ 1 \leq i \leq n - 1. \]
\[ f(u_i w_i) = 9i \cdot 6, \ 1 \leq i \leq n - 1. \]
\[ f(v_i u_{i+1}) = 9i \cdot 8, \ 1 \leq i \leq n - 1. \]
\[ f(w_i u_{i+1}) = 9i \cdot 3, \ 1 \leq i \leq n - 1. \]
\[ f(v_i u_{i+1}) = 9i, \ 1 \leq i \leq n - 1. \]
\[ f(w_i u_{i+1}) = 9i \cdot 2, \ 1 \leq i \leq n - 1. \]

Then the edge labels are distinct.

Hence \( F(T_n) \) is Stolarsky-3 Mean graph.

**Example 2.7:** The Stolarsky-3 Mean labeling of \( F(T_4) \) is given below.

![Figure 7](image_url)

**REFERENCES**


