On Strongly πgr-Irresolute Functions

C.Janaki
Assistant Professor, Department of Mathematics
L.R.G. Govt. Arts College for Women,Tirupur-4

V.Jeyanthi
Assistant Professor, Department of Mathematics
Sree Narayana Guru College,Coimbatore-105

Abstract:

The purpose of this paper is to introduce strongly πgr-irresolute functions, strongly regular πgr-irresolute functions and strongly β- πgr-irresolute functions and study some of their basic properties. Also, some new forms of homeomorphism are defined and obtained their characterizations.

Keywords: strongly πgr-irresolute, strongly regular πgr-irresolute, strongly β- πgr-irresolute, strongly πgr-homeomorphism, Strongly regular πgr-homeomorphism.

Mathematics Subject Classification: 54C10,54C08,54C05.

1.Introduction:

Levine [10]introduced the concept of generalized closed sets in topological spaces and a class of topological space called $T_{1/2}$-space. The concept of π-closed sets in topological spaces was initiated by Zaitsev[21] and the concept of πg-closed set was introduced by Noiri and Dontchev[6]. N.Palaniappan[18] studied and introduced regular closed sets in topological spaces. The notion of homeomorphism has been studied by many topologists[13,16].Maki et al [13]introduced β-homeomorphisms. The strong forms of continuous map have been discussed by Noiri[17], Levine[11], Arya and Gupta[2], Reily, Vamanamurthy[19] and Zorlutuna et.al[22], Munshi and Basan[15]. Strongly πgα-irresolute functions and its properties were studied by Janaki.C[7].

In this paper, we introduce strongly πgα-irresolute function, strongly regular πgα-irresolute functions and obtained their characterizations. Throughout this paper (X,τ),(Y,σ),(Z,η) or simply X,Y,Z represent the topological spaces on which no separation axioms are assumed unless otherwise mentioned.

2.Preliminaries:

Let (X,τ) or simply X be a topological space and A be a subset of X. The closure and interior of A are denoted by Cl(A) and Int(A) respectively.

Definition: 2.1
A subset A of X is called is called
(i) Pre-open[14] if $A \subseteq \text{Int}(\text{Cl}(A))$.
(ii) Regular open [18] if $\text{Int}(\text{Cl}(A)) = A$. 
(iii) β-open [1] if $A \subseteq \text{Cl}(\text{Int}(\text{Cl}(A)))$.

Finite union of regular open set is π-open[21] and its complement is π-closed.

Definition: 2.2
A subset A of X is called πgα-closed [8] if rcl(A) ⊆U whenever A ⊆U and U is π-open in X. Let πGRO(X) denote the collection of πgα-open set of X and πGRC(X) denote the collection of πgα-closed set of X.

Definition: 2.3
A function $f: X \rightarrow Y$ is called
(i) Continuous [11] if $f^{-1}(V)$ is closed in X for every closed set V of Y.
(ii) g-continuous [11] if $f^{-1}(V)$ is g-closed in X for every closed set V of Y.
(iii) r-continuous[18] if $f^{-1}(V)$ is regular closed in X for every closed set V of Y.
(iv) π-irresolute[6,7] if $f^{-1}(V)$ is π-closed in X for every π-closed set V of Y.
(v) an R-map [3] if $f^{-1}(V)$ is regular closed in X for every regular closed set V of Y.
(vi) πg-continuous[9] if $f^{-1}(V)$ is πg-closed in X for every closed set V of Y.
(vii) πg-irresolute[9] if $f^{-1}(V)$ is πg-closed in X for every πg-closed set V of Y.
(viii) β-irresolute[12] if $f^{-1}(V)$ is β-open in X for every β-open set V of Y.

Definition: 2.4
A topological space X is called
Thus, \( f(x) \) is \( \text{regular} \) in \( X \).

**Theorem 3.3**

If \( f: X \rightarrow Y \) is a \( \text{regular} \) \( Y \)-irresolute, then \( f \) is \( \text{r-irresolute} \).

**Proof:**

Let \( V \) be a regular open set in \( Y \) and hence \( V \) is \( \text{r-open} \) in \( Y \). Since \( f \) is \( \text{regular} \) \( Y \)-irresolute, \( f^{-1}(V) \) is open in \( X \). Therefore \( f^{-1}(V) \) is open in \( X \) for every \( \text{regular open set V} \) in \( Y \). Hence \( f \) is \( \text{r-irresolute} \).

**Theorem 3.4**

If \( f: X \rightarrow Y \) is a \( \text{continuous} \) and \( Y \) is a \( \text{r-T}_{1/2} \)-space, then \( f \) is \( \text{r-irresolute} \).

**Proof:**

Let \( V \) be a \( \text{regular} \) open set in \( Y \). Since \( Y \) is a \( \text{r-T}_{1/2} \)-space, \( V \) is open in \( Y \). Since \( f \) is \( \text{continuous} \), \( f^{-1}(V) \) is open in \( X \). Thus, \( f^{-1}(V) \) is open in \( X \) for every \( \text{regular open set V} \) in \( Y \). Hence \( f \) is \( \text{r-irresolute} \).

**Theorem 3.5**

If \( f: X \rightarrow Y \) is a \( \text{r-irresolute} \), \( X \) is a \( \text{r-T}_{1/2} \)-space, then \( f \) is \( \text{r-irresolute} \).

**Proof:**

Let \( V \) be a \( \text{regular} \) open set in \( Y \). Since \( f \) is \( \text{r-irresolute} \), \( f^{-1}(V) \) is \( \text{regular} \) open in \( X \). Since \( X \) is a \( \text{r-T}_{1/2} \)-space, \( f^{-1}(V) \) is \( \text{regular} \) open in \( X \) and hence \( f^{-1}(V) \) is \( \text{regular} \) open in \( X \). Thus, \( f^{-1}(V) \) is open in \( X \) for every \( \text{regular open set V} \) in \( Y \). Hence \( f \) is \( \text{r-irresolute} \).

**Theorem 3.6**

Let \( f: X \rightarrow Y \) and \( g: Y \rightarrow Z \) be any functions. Then

(i) \( g \circ f: X \rightarrow Z \) is \( \text{r-irresolute} \) if \( f \) is \( \text{continuous} \) and \( g \) is \( \text{r-irresolute} \).

(ii) \( g \circ f: X \rightarrow Z \) is \( \text{r-irresolute} \) if \( f \) is \( \text{r-irresolute} \) and \( g \) is \( \text{r-irresolute} \).

**Proof:**

(i) Let \( V \) be a \( \text{regular} \) open set in \( Z \). Since \( g \) is \( \text{r-irresolute} \), \( g^{-1}(V) \) is open in \( Y \). Since \( f \) is \( \text{continuous} \), \( f^{-1}(g^{-1}(V)) \) is \( \text{regular} \) open in \( X \).

\[ \Rightarrow (g \circ f)^{-1}(V) \text{ is \( \text{regular} \) open in \( X \) for every \( \text{regular open set V} \) in \( Z \).} \]

\[ \Rightarrow (g \circ f)^{-1}(V) \text{ is \( \text{r-irresolute} \).} \]

(ii) Let \( V \) be a \( \text{regular} \) open set in \( Z \). Since \( g \) is \( \text{r-irresolute} \), \( g^{-1}(V) \) is \( \text{regular} \) open in \( Y \). Since \( f \) is \( \text{r-irresolute} \), \( f^{-1}(g^{-1}(V)) \) is open in \( X \).
The following are equivalent for a function $f: X \to Y$:

(i) $f$ is strongly $\pi gr$-irresolute.
(ii) For each $x \in X$ and each $\pi gr$-open set $V$ of $Y$ containing $f(x)$, there exists an open set $U$ in $X$ containing $x$ such that $f(U) \subseteq V$.
(iii) $f^{-1}(V) \subseteq \text{int}(f^{-1}(V))$ for each $\pi gr$-open set $V$ of $Y$.
(iv) $f^{-1}(F)$ is closed in $X$ for every $\pi gr$-closed set $F$ of $Y$.

**Proof:** (i) $\Rightarrow$ (ii):

Let $x \in X$ and $V$ be a $\pi gr$-open set in $Y$ containing $f(x)$. By hypothesis, $f^{-1}(V)$ is open in $X$ and contains $x$.

Set $U = f^{-1}(V)$. Then $U$ is open in $X$ and $f(U) \subseteq V$.

(ii) $\Rightarrow$ (iii):

Let $V$ be a $\pi gr$-open set in $Y$ and $x \in f^{-1}(V)$.

By assumption, there exists an open set $U$ in $X$ containing $x$ such that $f(U) \subseteq V$.

Then $x \in U \subseteq \text{int}(U) \subseteq \text{int}(f^{-1}(V))$.

Then $f^{-1}(V) \subseteq \text{int}(f^{-1}(V))$.

(iii) $\Rightarrow$ (iv):

Let $F$ be a $\pi gr$-closed set in $Y$. Set $V = Y - F$. Then $V$ is $\pi gr$-open in $Y$.

By (iii), $f^{-1}(V) \subseteq \text{int}(f^{-1}(V))$.

Hence $f^{-1}(F)$ is closed in $X$.

(iv) $\Rightarrow$ (i):

Let $V$ be $\pi gr$-open in $Y$. Let $F = Y - V$. That is $F$ is $\pi gr$-closed set in $Y$. Then $f^{-1}(F)$ is closed in $X$ (by (iv)). Then $f^{-1}(V)$ is open in $X$. Hence $f$ is strongly $\pi gr$-irresolute.

**Theorem 3.8**

A function $f: X \to Y$ is strongly $\pi gr$-irresolute if $A$ is open in $X$, then $f/A: A \to Y$ is strongly $\pi gr$-irresolute.

**Proof:** Let $V$ be a $\pi gr$-open set in $Y$. By hypothesis, $f^{-1}(V)$ is open in $X$. But $(f/A)^{-1}(V) = A \cap f^{-1}(V)$ is open in $A$ and hence $f/A$ is strongly $\pi gr$-irresolute.

**Theorem 3.9**

Let $f: X \to Y$ be a function and $\{A_i: i \in \Lambda\}$ be a cover of $X$ by open sets of $(X, \tau)$. Then $f$ is strongly $\pi gr$-irresolute if $f/A_i : (A_i, \tau/A_i) \to (Y, \sigma)$ is strongly $\pi gr$-irresolute for each $i \in \Lambda$.

**Proof:** Let $V$ be a $\pi gr$-open set in $Y$. By hypothesis, $(f/A_i)^{-1}(V)$ is open in $A_i$. Since $A_i$ is open in $X$, $(f/A_i)^{-1}(V)$ is open in $X$ for every $i \in \Lambda$.

$$f^{-1}(V) = X \cap f^{-1}(V) = \bigcup \{A_i \cap f^{-1}(V): i \in \Lambda\} = \bigcup \{(f/A_i)^{-1}(V): i \in \Lambda\}$$

Hence $f$ is strongly $\pi gr$-irresolute.

**Theorem 3.10**

Let $f: X \to Y$ be a strongly $\pi gr$-irresolute surjective function. If $X$ is compact, then $Y$ is $\pi$GRO-compact.

**Proof:** Let $\{A_i: i \in \Lambda\}$ be a cover of $\pi gr$-open sets of $Y$. Since $f$ is strongly $\pi gr$-irresolute and $X$ is compact, we get $X \subseteq \bigcup \{f^{-1}(A_i): i \in \Lambda\}$. Since $f$ is surjective, $Y = f(X) \subseteq \bigcup \{A_i: i \in \Lambda\}$. Hence $Y$ is $\pi$GRO-compact.

**Theorem 3.11**

If $f: X \to Y$ is strongly $\pi gr$-irresolute and a subset $B$ of $X$ is compact relative to $X$, then $f(B)$ is $\pi$GRO-compact relative to $Y$.

**Proof:** Obvious.

**Definition 3.12**

A function $f: X \to Y$ is said to be

(i) a strongly regular $\pi gr$-irresolute function if $f^{-1}(V)$ is regular open in $X$ for every $\pi gr$-open set $V$ in $Y$. 

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Theorem: 3.13
(i) If \( f: X \rightarrow Y \) is strongly regular \( \pi gr \)-irresolute, then \( f \) is strongly \( \pi gr \)-irresolute.

(ii) If \( f: X \rightarrow Y \) is strongly regular \( \pi gr \)-irresolute, then \( f \) is strongly \( \beta \)-\( \pi gr \)-irresolute.

Proof: (i) Let \( f \) be a strongly regular \( \pi gr \)-irresolute function and let \( V \) be a \( \pi gr \)-open set in \( Y \). Then \( f^{-1}(V) \) is regular open in \( X \) and hence open in \( X \).

\[ \Rightarrow f^{-1}(V) \text{ is open in } X \text{ for every } \pi gr \text{-open set } V \text{ in } Y. \]

Hence \( f \) is strongly regular \( \pi gr \)-irresolute.

(ii) Let \( f \) be a strongly regular \( \pi gr \)-irresolute function and let \( V \) be a \( \pi gr \)-open set in \( Y \). Then

\[ f^{-1}(V) \text{ is regular open in } X \text{ and hence open in } X. \]

\[ \Rightarrow f^{-1}(V) \text{ is open in } X \text{ for every } \pi gr \text{-open set } V \text{ in } Y. \]

Hence \( f \) is strongly \( \beta \)- \( \pi gr \)-irresolute.

Remark: 3.14
Converse of the above need not be true as seen in the following examples.

Example: 3.15
(i) Let \( X = \{a,b,c\} \), \( \tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}\} \) and \( \sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a,b\}\} \).

Let \( f: X \rightarrow Y \) be an identity map. Here for every \( \pi gr \)-open set \( V \) in \( Y \), \( f^{-1}(V) \) is open and \( \beta \)-open in \( X \). Hence \( f \) is strongly \( \pi gr \)-irresolute and strongly \( \beta \)-\( \pi gr \)-irresolute.

But for every \( \pi gr \)-open set \( V \) in \( Y \), \( f^{-1}(V) \) is not regular open in \( X \). Thus, \( f \) is not strongly regular \( \pi gr \)-irresolute. Hence strongly \( \pi gr \)-irresolute function need not be strongly regular \( \pi gr \)-irresolute function and strongly \( \beta \)-\( \pi gr \)-irresolute function.

Theorem: 3.16
If \( f: X \rightarrow Y \) and \( g: Y \rightarrow Z \), then \( g \circ f: X \rightarrow Z \) is

(i) strongly \( \pi gr \)-irresolute if \( f \) is strongly regular \( \pi gr \)-irresolute and \( g \) is \( \pi gr \)-irresolute.

Proof: Let \( V \) be an \( \pi gr \)-open set in \( Z \). Since \( g \) is \( \pi gr \)-irresolute, \( g^{-1}(V) \) is \( \pi gr \)-open in \( Y \). Since \( f \) is strongly regular \( \pi gr \)-irresolute, \( f^{-1}(g^{-1}(V)) \) is regular open in \( X \).

\[ \Rightarrow (g \circ f)^{-1}(V) \text{ is regular open in } X \text{ and hence open in } X. \]

Hence \((g \circ f)^{-1}(V) \text{ is } \beta \text{- open in } X \) for every \( \pi gr \)-open set \( V \) in \( Z \).

Therefore \((g \circ f)^{-1}(V) \text{ is } \beta \text{- open in } X \) for every \( \pi gr \)-open set \( V \) in \( Z \).

Hence \((g \circ f)^{-1}(V) \text{ is } \beta \text{- open in } X \) for every \( \pi gr \)-open set \( V \) in \( Z \).

(ii) Let \( V \) be an \( \pi gr \)-open set in \( Z \). Since \( g \) is strongly \( \pi gr \)-irresolute, \( g^{-1}(V) \) is regular open in \( Y \). Since \( f \) is regular irresolute, \( f^{-1}(g^{-1}(V)) \) is regular open in \( X \).

\[ \Rightarrow (g \circ f)^{-1}(V) \text{ is regular open in } X \text{ and hence open in } X. \]

Hence \((g \circ f)^{-1}(V) \text{ is } \beta \text{- open in } X \) for every \( \pi gr \)-open set \( V \) in \( Z \).

Therefore \((g \circ f)^{-1}(V) \text{ is } \beta \text{- open in } X \) for every \( \pi gr \)-open set \( V \) in \( Z \).

(ii) Let \( V \) be an \( \pi gr \)-open set in \( Z \). Since \( g \) is strongly \( \pi gr \)-irresolute, \( g^{-1}(V) \) is regular open in \( Y \). Since \( f \) is continuous, \( f^{-1}(g^{-1}(V)) \) is regular open in \( X \).

Since \( f \) is continuous, \( f^{-1}(g^{-1}(V)) \) is regular open in \( X \).

Hence \((g \circ f)^{-1}(V) \text{ is } \beta \text{- open in } X \) for every \( \pi gr \)-open set \( V \) in \( Z \).

Therefore \((g \circ f)^{-1}(V) \text{ is } \beta \text{- open in } X \) for every \( \pi gr \)-open set \( V \) in \( Z \).
\( (g \circ f)^{-1}(V) \) is open in X and hence \( \beta \)-open in X. Hence \( (g \circ f) \) is strongly \( \beta \)-\pi-gr-irresolute.

**Theorem 3.18**

The following are equivalent for a function \( f: X \rightarrow Y \):

(i) \( f \) is strongly \( \beta \)-\pi-gr-irresolute.

(ii) For each \( x \in X \) and each \( \pi-gr \)-open set \( V \) of \( Y \) containing \( f(x) \), there exists a \( \beta \)-open set \( U \) in \( X \) containing \( x \) such that \( f(U) \subset V \).

(iii) \( f^{-1}(V) \subset Cl(\text{Int}(f^{-1}(V))) \) for each \( \pi-gr \)-open set \( V \) of \( Y \).

(iv) \( f^{-1}(F) \) is \( \beta \)-closed in \( X \) for every \( \pi-gr \)-closed set \( F \) of \( Y \).

**Proof:** Similar to that of Theorem 3.7

**Theorem 3.19**

The following are equivalent for a function \( f: X \rightarrow Y \):

(i) \( f \) is strongly \( \beta \)-\pi-gr-irresolute.

(ii) For each \( x \in X \) and each \( \pi-gr \)-open set \( V \) of \( Y \) containing \( f(x) \), there exists a \( \beta \)-open set \( U \) in \( X \) containing \( x \) such that \( f(U) \subset V \).

(iii) \( f^{-1}(V) \subset Cl(\text{Int}(f^{-1}(V))) \) for each \( \pi-gr \)-open set \( V \) of \( Y \).

(iv) \( f^{-1}(F) \) is \( \beta \)-closed in \( X \) for every \( \pi-gr \)-closed set \( F \) of \( Y \).

**Proof:** Similar to that of Theorem 3.7.

**Lemma 3.20**

If \( f: X \rightarrow Y \) is strongly regular \( \pi-gr \)-irresolute and \( A \) is a regular open subset of \( X \), then \( f/A : A \rightarrow Y \) is strongly regular \( \pi-gr \)-irresolute.

**Proof:** Let \( f \) be a \( \pi-gr \)-open set in \( Y \). By hypothesis, \( f^{-1}(V) \) is \( \beta \)-open in \( X \). Let \( V \) be any \( \pi-gr \)-open set of \( Y \) such that \( f^{-1}(V) \) is \( \beta \)-open in \( X \). Hence \( f/A \) is strongly \( \beta \)-\pi-gr-irresolute.

**Theorem 3.21**

Let \( f: X \rightarrow Y \) and \( \{A_{\lambda} : \lambda \in \Lambda \} \) be a cover of \( X \) by \( \pi-gr \)-open set of \( (X, t) \). Then \( f \) is a strongly regular \( \pi-gr \)-irresolute function if \( f/A_{\lambda} : A_{\lambda} \rightarrow Y \) is strongly regular \( \pi-gr \)-irresolute for each \( \lambda \in \Lambda \).

Let \( f: X \rightarrow Y \) and \( \{A_{\lambda} : \lambda \in \Lambda \} \) be a cover of \( X \) by \( \pi-gr \)-open set of \( (X, t) \). Then \( f \) is a strongly regular \( \pi-gr \)-irresolute function if \( f/A_{\lambda} : A_{\lambda} \rightarrow Y \) is strongly regular \( \pi-gr \)-irresolute for each \( \lambda \in \Lambda \).

**Proof:** Let \( V \) be any \( \pi-gr \)-open set in \( Y \). By hypothesis, \( (f/A_{\lambda})^{-1}(V) \) is regular open in \( A_{\lambda} \). Since \( A_{\lambda} \) is regular open in \( X \), it follows that \( (f/A_{\lambda})^{-1}(V) \) is \( \pi-gr \)-open in \( X \) for each \( \lambda \in \Lambda \).

Let \( V \) be any \( \pi-gr \)-open set in \( Y \). By hypothesis, \( f^{-1}(V) \) is \( \beta \)-open in \( X \). But \( (f/A_{\lambda})^{-1}(V) = A_{\lambda} \cap f^{-1}(V) \) is \( \beta \)-open in \( A_{\lambda} \). Hence \( f/A \) is strongly \( \beta \)-\pi-gr-irresolute.

**Theorem 3.22**

If \( f: X \rightarrow Y \) is strongly \( \beta \)-\pi-gr-irresolute and \( A \) is a regular-open subset of \( X \), then \( f/A : A \rightarrow Y \) is strongly \( \beta \)-\pi-gr-irresolute.

**Proof:** Let \( V \) be a \( \pi-gr \)-open set in \( Y \). By hypothesis, \( f^{-1}(V) \) is \( \beta \)-open in \( X \). But \( (f/A_{\lambda})^{-1}(V) = A_{\lambda} \cap f^{-1}(V) \) is \( \beta \)-open in \( A_{\lambda} \). Hence \( f/A \) is strongly \( \beta \)-\pi-gr-irresolute.

**Theorem 3.23**

If a function \( f: X \rightarrow Y \) is strongly \( \beta \)-\pi-gr-irresolute, then \( f^{-1}(B) \) is \( \beta \)-closed in \( X \) for any nowhere dense set \( B \) of \( Y \).

**Proof:** Let \( B \) be any nowhere dense subset of \( Y \). Then \( Y-B \) is regular in \( Y \) and hence \( \pi-gr \)-open in \( Y \). By hypothesis, \( f^{-1}(Y-B) \) is \( \beta \)-open in \( X \). Hence \( f^{-1}(B) \) is \( \beta \)-closed in \( X \).

**Theorem 3.24**

If a function \( f: X \rightarrow Y \) is strongly \( \beta \)-\pi-gr-irresolute, then \( f^{-1}(B) \) is \( \beta \)-closed in \( X \) for any nowhere dense set \( B \) of \( Y \).

**Proof:** Let \( B \) be any nowhere dense subset of \( Y \). Then \( Y-B \) is regular in \( Y \) and hence \( \pi-gr \)-open in \( Y \). By hypothesis, \( f^{-1}(Y-B) \) is \( \beta \)-open in \( X \). Hence \( f^{-1}(B) \) is \( \beta \)-closed in \( X \).

**Theorem 3.25**

If a function \( f: X \rightarrow Y \) and \( g: Y \rightarrow Z \), then \( g \circ f : X \rightarrow Z \) is strongly \( \beta \)-\pi-gr-irresolute if

a) \( f \) is strongly \( \beta \)-\pi-gr-irresolute and \( g \) is \( \pi-gr \)-irresolute.
b) \( f \) is an R-map and \( g \) is strongly regular πgr-irresolute.

c) \( f \) is β-irresolute and \( g \) is strongly β-πgr-irresolute.

d) \( f \) is β-irresolute and \( g \) is strongly πgr-irresolute.

e) \( f \) is β-continuous and \( g \) is strongly πgr-irresolute.

f) \( f \) is β-irresolute and \( g \) is strongly regular πgr-irresolute.

**Proof:** Follows from the definitions.

**Theorem 3.26**

Let \( X \) be a sub maximal and extremally disconnected space. Then the following are equivalent for a function \( f: X \to Y \).

Then the following are equivalent:

a) \( f \) is strongly regular πgr-irresolute.

b) \( f \) is strongly πgr-irresolute.

c) \( f \) is strongly β-πgr-irresolute.

**Proof:**

If \( X \) is sub maximal and extremally disconnected, then \( \tau = RO(X) = \beta O(X) \) and hence the result follows.

**Definition 3.27**

A bijection \( f: X \to Y \) is

(i) a πgr-homeomorphism if both \( f \) and \( f^{-1} \) are πgr-continuous.

(ii) a πgrc-homeomorphism if both \( f \) and \( f^{-1} \) are πgr-irresolute.

(iii) a strongly πgrc-homeomorphism if both \( f \) and \( f^{-1} \) are strongly πgr-irresolute.

(iv) a strongly regular πgrc-homeomorphism if both \( f \) and \( f^{-1} \) are strongly regular πgr-irresolute.

(v) a strongly β-πgrc-homeomorphism if both \( f \) and \( f^{-1} \) are strongly β-πgr-irresolute.

**Theorem 3.28**

If a bijective function \( f: X \to Y \) is strongly regular πgrc-homeomorphism, then

1) \( f \) is πgrc-homeomorphism.

2) \( f \) is strongly πgrc-homeomorphism.

**Proof:** (1) Since a bijection \( f \) is strongly regular πgrc-homeomorphism, \( f \) and \( f^{-1} \) are strongly regular πgr-irresolute. Every strongly regular πgr-irresolute function is πgr-irresolute. Since every regular open set is πgr-open. Therefore, both \( f \) and \( f^{-1} \) are πgr-irresolute functions and hence \( f \) is a πgrc-homeomorphism.

(2) Since every strongly regular πgr-irresolute function is strongly πgr-irresolute and hence the result follows.

**Proposition 3.29**

(i) Every strongly regular πgrc-homeomorphism is a strongly πgrc-homeomorphism and a strongly β-πgrc-homeomorphism.

(ii) Every strongly regular πgrc-homeomorphism is a strongly β-πgrc-homeomorphism.

**Proof:** Follows from the definitions.

**Remark 3.30**

The family of all strongly πgrc-homeomorphism from \((X, \tau)\) onto itself is denoted by \( \pi r c h(X, \tau) \).

**Theorem 3.31**

If \( f: X \to Y \) and \( g: Y \to Z \) are strongly regular πgrc-homeomorphisms, then \( g \circ f: X \to Z \) is a strongly πgrc-homeomorphism.

**Proof:** Let \( V \) be a πgr-open set in \( Z \). Then \((g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V)) = f^{-1}(U), \) where \( U = g^{-1}(V) \). Since \( g \) is strongly regular πgrc-homeomorphism, \( g \) is strongly regular πgr-irresolute and \( g^{-1}(V) \) is regular open in \( Y \) for every πgr-open set \( V \) in \( Z \). Hence \( U = g^{-1}(V) \) is πgr-open in \( Y \). Since every regular open set is πgr-open. Also, since \( f \) is strongly regular πgr-irresolute, \( f^{-1}(U) \) is regular open in \( X \) and hence open in \( X \). Therefore, \((g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V)) \) is open in \( X \). Hence \((g \circ f)\) is strongly πgr-irresolute. Now, \((g \circ f)(A) = g(f(A)) = g(B), \) where \( B = f(A) \). Since \( f \) is strongly regular πgrc-homeomorphism, \( f(A) \) is regular open in \( Y \) and hence πgr-open in \( Y \). Now \( g \) is strongly regular πgrc-homeomorphism implies \( g(B) \) is regular open in \( Z \) and hence open in \( Z \). Hence \((g \circ f)^{-1} \) is strongly πgr-irresolute.

\( \Rightarrow (g \circ f) \) is a strongly πgrc-homeomorphism.

**Bibliography**


