A New Approach on Anti-L-Fuzzy Soft Subhemiring of a Hemiring

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Abstract
The notion of Anti-L-Fuzzy soft subhemiring in a Hemiring is introduced and its basic properties are investigated. The purpose of this study is to implement the concept of anti-L-fuzzy soft subhemiring of a hemiring in L-fuzzy soft subhemiring of a hemiring.


Keywords

INTRODUCTION
Hemirings appear in a natural manner in some applications to the automata, the theory of formal languages, graph theory, design theory and combinatorial geometry. Hemirings which are regarded as a generalization of rings have been found useful in solving problems in different areas of applied Mathematics and Computer sciences. So many researchers have studied different aspects of hemirings. Zhan et al gave the concept of h-hemirregularity of hemirings and investigated some properties in terms of fuzzy theory. The concept of fuzzy subset was introduced by L.A Zadeh [16], Fuzzy set theory has been developed in many directions by many scholars and has evoked great interest among Mathematicians working in different field of Mathematics. A Semiring R is said to be a hemiring if it is additively commutative with zero. The notion of anti-fuzzy left h-ideals in Hemirings was introduced by Akram. M and K.H.Dar [1]. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by M.Palaniappan and K.Arjunan [10]. In this paper we introduce the some theorems in anti-L-fuzzy soft subhemirings of a hemiring.

II. PERLIMINARIES
In this section we have some basic definitions which we need for our further studies.

1.1 DEFINITION
A pair (F,E) is called a soft set (over U) iff F is a mapping E into the set of all subsets of the set U.

1.2 DEFINITION
Let (U,E) be a soft universe and A ≤ E. Let f(U) be the set of all fuzzy subsets in U. A pair (F,A) is a called a fuzzy soft set over U, where F is a mapping given by F:A → f(U).

1.3 DEFINITION
Let X be a non-empty set and L=(L,≤ ) be a lattice with least element 0 and greatest element 1.

1.4 DEFINITION
Let X be a non-empty set. A L-fuzzy subset A of X is function A: X → L.

1.5 DEFINITION
Let (R,+,:) be a hemiring. A L-fuzzy subset (F,A) of R is said to be an anti-L-fuzzy soft subhemiring (ALFSSSH) of R if it satisfies the following conditions.

1.6 DEFINITION
Let (R,+,:) be a hemiring. An anti-L-fuzzy soft subhemiring (F,A) of R is said to be an anti-L-fuzzy soft normal subhemiring (ALFSNSHR) of R if for all x(F,A) and y(F,A) in R.

1.7 DEFINITION
Let (F,A) and (G,B) be L-fuzzy soft subsets of sets G and H, respectively. The anti-product of (F,A) and (G,B) denoted by (F,A)×(G,B) is defined as

\[ \{(x(F,A),y(G,B)), \mu(F,A)×(G,B)(x(F,A),y(G,B)) \} \text{ for all } x(F,A) \text{ in } G \text{ and } y(G,B) \text{ in } H \}

Where

\[ \mu(F,A)×(G,B)(x(F,A),y(G,B)) = \max \{ \mu(F,A)(x(F,A)), \mu(G,B)(y(G,B)) \} \]
Let \((R,+,\cdot)\) and \((R',+,\cdot)\) be any two hemirings. Let \(f:R\rightarrow R'\) be any function and \((F,A)\) be an anti-L-fuzzy soft subhemiring in \(R\). \((G,V)\) be an anti-L-fuzzy soft subhemiring in \(f(R) = R'\) defined by
\[
\mu_{(G,V)}(y_{(G,V)}) = \inf_{x\in f^{-1}(y)}\mu_{(F,A)}(x_{(F,A)}) \quad \text{for all } x_{(F,A)} \text{ and } y_{(G,V)} \in R'.
\]
Then \((F,A)\) is called pre image of \((G,V)\) under \(f\) and is denoted by \(f^{-1}(V)\).

### 1.9 Definition
Let \((F,A)\) be an anti-L-fuzzy soft subhemiring of a hemiring \((R,+,\cdot)\) and \(a\) in \(R\). Then the pseudo anti-L-fuzzy soft coset \(a(F,A)^p\) is defined by
\[
(a\mu_{(F,A)})_{(x_{(F,A)})} = P(a)\mu_{(F,A)}(x_{(F,A)}) \quad \text{for every } x_{(F,A)} \text{ in } R \text{ and for some } p \text{ in } P.
\]

### 1.10 Definition
Let \((R,+,\cdot)\) and \((R',+,\cdot)\) be any two hemirings then the function \(f:R\rightarrow R'\) is called a hemiring anti-homomorphism. If it satisfies the following axioms.
(i) \(f(x + y) = f(x) + f(y)\)
(ii) \(f(xy) = f(y)f(x)\) for all \(x\) and \(y\) in \(R\).

### 1.11 Definition
Let \((R,+,\cdot)\) and \((R',+,\cdot)\) be any two hemirings. Then the function \(f:R\rightarrow R'\) is called a hemiring isomorphism. If it is one-to-one and on to, then \(f\) is called a hemiring isomorphism.

### 1.12 Definition
Let \((R,+,\cdot)\) and \((R',+,\cdot)\) be any two hemirings. Then the function \(f:R\rightarrow R'\) is called a hemiring anti-homomorphism. If it is one-to-one and onto, then \(f\) is called a hemiring anti-isomorphism.

### III. Properties of Anti-L-Fuzzy Soft Subhemirings of a Hemiring \(R\)
In this section, we investigate some properties of Anti-L-fuzzy soft subhemiring of a hemiring.

#### 2.1 Theorem
Union of any two anti-L-fuzzy soft subhemirings of a hemiring \(R\) is an anti-L-Fuzzy soft subhemiring of a hemiring \(R\).

**Proof:** Let \((F,A)\) and \((G,B)\) be any two anti-L-fuzzy subhemirings of a hemiring \(R\) and \(x_{(F,A)}\) and \(y_{(G,B)}\) in \(R\). Let \(f(A) = \{x_{(F,A)}: \mu_{(F,A)}(x_{(F,A)}) \in R\}\) and \((G,B) = \{x_{(G,B)}: \mu_{(G,B)}(x_{(G,B)}) \in R\}\) and also let \(H(C) = (F,B) \cup (G,B)\) be any two anti-L-fuzzy soft subhemirings of a hemiring \(R\) respectively then anti-product \((F,A) \times (G,B)\) is an anti-L-fuzzy soft subhemirings of \(R_1 \times R_2\).
PROOF: Let (F,A) and (G,B) be two anti-L-fuzzy soft subhemirings of the hemiring R₁ and R₂ respectively. Let \( x_{(F,A)} \) and \( y_{(F,A)} \) be in \( R_1 \), \( y_{(G,B)} \) and \( y_{(G,B)} \) be in \( R_2 \). Then \( (x_{(F,A)}), y_{(G,B)} \) and \( (x_{(F,A)}), y_{(G,B)} \) are in \( R_1 \times R_2 \). Now
\[
\mu_{(F,A)}(x_{(F,A)} + y_{(G,B)}) \leq \mu_{(F,A)}(x_{(F,A)}) \cup \mu_{(F,A)}(y_{(G,B)})
\]
Therefore
\[
\mu_{(F,A)}(x_{(F,A)} + y_{(G,B)}) \leq \mu_{(F,A)}(x_{(F,A)}) \cup \mu_{(F,A)}(y_{(G,B)})
\]

2.4 THEOREM : Let (F,A) be an anti-L-fuzzy soft subhemiring of a hemiring R. If \( (a, b) \) is a homomorphism. Then \( f(x+y) = f(x) + f(y) \) for all \( x \) and \( y \) in \( R \). Let \( (G,V) = f(F,A) \), where \( (F,A) \) is an anti-L-fuzzy soft subhemiring of \( R \). Therefore \( (a, b) \) is an anti-L-fuzzy soft subhemiring of \( R \).

2.7 THEOREM : Let (F,A) be an anti-L-fuzzy soft subhemiring of a hemiring \( (R,+,:) \), then the pseudo anti-L-fuzzy soft cost of \( (a(F,A))^p \) is an anti-L-fuzzy soft subhemiring of a hemiring \( R \). For every \( a \) in \( R \).

PROOF: Let (F,A) be an anti-L-fuzzy soft subhemiring of a hemiring R. For every \( x_{(F,A)} \) and \( y_{(F,A)} \) in \( R_1 \), we have
\[
\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \leq \mu_{(F,A)}(x_{(F,A)}) \cup \mu_{(F,A)}(y_{(F,A)})
\]
Therefore
\[
\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \leq \mu_{(F,A)}(x_{(F,A)}) \cup \mu_{(F,A)}(y_{(F,A)})
\]

2.8 THEOREM: Let \( (R,+,:), (R,+,:)^{'} \) be any two hemirings. The homomorphic image of an anti-L-fuzzy soft subhemiring of a hemiring R is an anti-L-fuzzy soft subhemiring of \( R' \). PROOF : Let \( f:R \to R' \) be a homomorphism. Then \( f(x+y) = f(x) + f(y) \) for all \( x \) and \( y \) in \( R \). Let \( (G,V) = f(F,A) \), where \( (F,A) \) is an anti-L-fuzzy soft subhemiring of \( R \). Therefore \( (a, b) \) is an anti-L-fuzzy soft subhemiring of \( R' \).

2.9 THEOREM: Let \( (R,+,:) \) and \( (R,+,:)^{'} \) be any two hemirings. The homomorphic preimage of an anti-L-fuzzy soft subhemiring of \( R' \) is an anti-L-fuzzy soft subhemiring of \( R \).

PROOF: Let \( (G,V) = f(F,A) \), where \( (G,V) \) is an anti-L-fuzzy soft subhemiring of \( R' \). Let \( x_{(F,A)} \) and \( y_{(F,A)} \) in \( R \). Then \( \mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) = \mu_{(G,V)}(f(x_{(F,A)}) + f(y_{(F,A)})) \leq \mu_{(G,V)}(f(x_{(F,A)}) \cup f(y_{(F,A)})) = \mu_{(F,A)}(x_{(F,A)}) \cup \mu_{(F,A)}(y_{(F,A)}) \). Therefore \( (a, b) \) is an anti-L-fuzzy soft subhemiring of \( R \).
\( \mu_{(F,A)}(x_{(F,A)}y_{(F,A)}) \leq \mu_{(F,A)}(x_{(F,A)}) \mu_{(F,A)}(y_{(F,A)}) \). Hence, \( (F,A) \) is an anti-L-fuzzy soft subhemiring of \( R \).

2.10 THEOREM: Let \( (R,+,\cdot) \) and \( (R',+,\cdot) \) be any two hemirings. The anti-homomorphic image of an anti-L-fuzzy soft subhemiring of \( R \) is an anti-L-fuzzy soft subhemiring of \( R' \).

**PROOF:** Let \( f: R \rightarrow R' \) be an anti-homomorphism. Then \( f(x+y) = f(x) + f(y) \) and \( f(xy) = f(y)f(x) \) for all \( x \) and \( y \) in \( R \). Let \( G,V = f((F,A)) \) where \( (G,V) \) is an anti-L-fuzzy soft subhemiring of \( R \). Now, for \( f(x_{(G,V)})(f(y_{(G,V)})) \) are in \( R' \).

Again, \( \mu_{G,V}(f(x_{(G,V)})) \mu_{G,V}(f(y_{(G,V)})) \) as \( f \) is an anti-homomorphism \( \leq \mu_{G,V}(f(x_{(G,V)})) \mu_{G,V}(f(y_{(G,V)})) \) which implies that \( \mu_{G,V}(f(x_{(G,V)})) \mu_{G,V}(f(y_{(G,V)})) \).

Hence, \((G,V)\) is an anti-L-fuzzy soft subhemiring of \( R' \).

We denote the composition of operations by \( \circ \) for the following:

2.11 THEOREM: Let \((F,A)\) be an anti-L-fuzzy soft subhemiring of a hemiring \( H \) and \( f \) is an isomorphism from a hemiring \( R \) onto \( H \). Then \((F,A)\) is an anti-L-fuzzy soft subhemiring of \( R \).

**PROOF:** Let \( x_{(F,A)} \) and \( y_{(F,A)} \) in \( R \) and \( (F,A) \) be an anti-L-fuzzy soft subhemiring of a hemiring \( H \). We have, \( \mu_{(F,A)}(f(x_{(F,A)})) \mu_{(F,A)}(f(y_{(F,A)})) = \mu_{(F,A)}(f(x_{(F,A)})) \mu_{(F,A)}(f(y_{(F,A)})) \). As \( f \) is an isomorphism \( \leq \mu_{(F,A)}(f(x_{(F,A)})) \mu_{(F,A)}(f(y_{(F,A)})) \). This implies that \( \mu_{(F,A)}(f(x_{(F,A)})) \mu_{(F,A)}(f(y_{(F,A)})) \).

And \( \mu_{(F,A)}(f(x_{(F,A)})) \mu_{(F,A)}(f(y_{(F,A)})) \). As \( f \) is an isomorphism \( \leq \mu_{(F,A)}(f(x_{(F,A)})) \mu_{(F,A)}(f(y_{(F,A)})) \). This implies that \( \mu_{(F,A)}(f(x_{(F,A)})) \mu_{(F,A)}(f(y_{(F,A)})) \).

Therefore \((F,A)\) is an anti-L-fuzzy soft subhemiring of a hemiring \( R \).

2.12 THEOREM: Let \((F,A)\) be an anti-L-fuzzy soft subhemiring of a hemiring \( H \) and \( f \) is an anti-isomorphism from a hemiring \( R \) onto \( H \). Then \((F,A)\) is an anti-L-fuzzy soft subhemiring of \( R \).

**PROOF:** Let \( x_{(F,A)} \) and \( y_{(F,A)} \) in \( R \) and \( (F,A) \) be an anti-L-fuzzy soft subhemiring of a hemiring \( H \). We have, \( \mu_{(F,A)}(f(x_{(F,A)})) \mu_{(F,A)}(f(y_{(F,A)})) = \mu_{(F,A)}(f(x_{(F,A)})) \mu_{(F,A)}(f(y_{(F,A)})) \). As \( f \) is an anti-isomorphism \( \leq \mu_{(F,A)}(f(x_{(F,A)})) \mu_{(F,A)}(f(y_{(F,A)})) \). This implies that \( \mu_{(F,A)}(f(x_{(F,A)})) \mu_{(F,A)}(f(y_{(F,A)})) \).

2.13 THEOREM: Let \((F,A)\) be an anti-L-fuzzy soft subhemiring of a hemiring \( R \). Then \((F,A)\) is isomorphic to \((F,A)\) in \( R \).

**PROOF:** Let \((F,A)\) be an anti-L-fuzzy soft subhemiring of a hemiring \( R \). Then \((F,A)\) is isomorphic to \((F,A)\) in \( R \).

The International Journal of Mathematics Trends and Technology (IJMTT) - Special Issue ICRTECITA April 2018

ISSN: 2231-5313 www.internationaljournalssrg.org Page 160
(F,A') be a L-fuzzy soft set in R defined by (F,A')

\[(x_{F,A'})(0) = (F,A)(x_{F,A}) + 1 - (F,A)(0)\]

for all \(x_{F,A}\) and \(y_{F,A}\) in R. Then there exists \(0 \in R\) such that (F,A)(0) = 1 if and only if

\[(F,A')(x_{F,A}) = (F,A)(x_{F,A})\]

PROOF: It is trivial.

2.16 THEOREM: Let (F,A) be an anti-L-fuzzy soft subhemiring of a hemiring R,

\[(F,A')(0) = (F,A)(0) + 1 - (F,A)(0)\]

for all x and y in R. Then there exists \(0 \in R\) such that (F,A')(x_{F,A}) = 1 if and only if \(x_{F,A} = 0\).

PROOF: It is trivial.

2.17 THEOREM: Let (F,A) be an anti-L-fuzzy soft subhemiring of a hemiring R,

\[(F,A') = (F,A)(x_{F,A}) + 1 - (F,A)(0)\]

for all \(x_{F,A}\) and \(y_{F,A}\) in R. Then \((F,A')^{(y)} = (F,A')\).

PROOF: Let \(x_{F,A}, y_{F,A} \in R\). We have, \((F,A')^{(y)} = (F,A') = (F,A')(x_{F,A}) + 1 - (F,A')(0) = (F,A)(x_{F,A}) + 1 - (F,A)(0) = (F,A)(x_{F,A}) + 1 - (F,A)(0) = (F,A')\) (Hence \((F,A')^{(y)} = (F,A')\).

2.18 THEOREM: Let (F,A) be an anti-L-fuzzy soft subsemiring of a hemiring R. Then \((F,A')\) is an anti-L-fuzzy soft subsemiring of a hemiring R, for all \(x_{F,A}\) and \(y_{F,A}\) in R.

PROOF: For any \(x_{F,A} \in R\), we have \((F,A') = (F,A)(x_{F,A}) + 1 - (F,A)(0)\) \[\leq \{1 / (F,A)(0)\} \cdot (F,A)(x_{F,A})V(F,A)(y_{F,A})\]. Similarly, \((F,A')(x_{F,A})V(F,A')(y_{F,A})\) \[\leq \{1 / (F,A)(0)\} \cdot (F,A)(x_{F,A})V(F,A)(y_{F,A})\]. Hence \((F,A')\) is an anti-L-fuzzy soft subsemiring of a hemiring R, for all \(x_{F,A}\) and \(y_{F,A}\) in R.

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