

A new multidimensional integral transform concerning the multivariable

Aleph-function

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ABSTRACT

In the present document, we use the multidimensional integral transform introduced by Chandel et al [2] concerning the multivariable Aleph-function defined by Ayant [1]. Some interesting special cases are also discussed.

KEYWORDS : Aleph-function of several variables, multidimensional integral transform, Multivariable I-function, Aleph-function of two variables, I-function of two variables.

1.Introduction

Chandel et al [2] introduce a new multidimensional integral transform defined by :

$$R_{\alpha_1, \dots, \alpha_r}^{(a,b)} \{ \} = \frac{\Gamma(\alpha_1 + \dots + \alpha_r) \Gamma(1/2 + a - b + \alpha_1 + \dots + \alpha_r) 2^{2a+2\alpha_1+\dots+2\alpha_r-1}}{\Gamma(\alpha_1) \dots \Gamma(\alpha_r) \Gamma(2a + 2\alpha_1 + \dots + 2\alpha_r) \Gamma(1/2 - (a + b + \alpha_1 + \dots + \alpha_r))}$$

$$\int_0^\infty \dots \int_0^\infty (x_1 + \dots + \alpha_r)^a (1 + x_1 + \dots + x_r)^{-1/2} \left[(x_1 + \dots + x_r)^{1/2} + (1 + x_1 + \dots + x_r)^{1/2} \right]^{2b}$$

$$x_1^{\alpha_1-1} \dots x_r^{\alpha_r-1} \{ \} dx_1 \dots dx_r \quad (1.1)$$

where $0 < Re(a + \alpha_1 + \dots + \alpha_r) < 1/2 - Re(b)$, $Re(\alpha_i) > 0$, $i = 1, \dots, r$

and give two dimensional integral transforms concerning the multivariable H-function defined by Srivastava et al [6]. Here in the present document, we extend this work with the multivariable Aleph-function. The Aleph-function of several variables generalize the multivariable I-function defined by Sharma and Ahmad [4], itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occurring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have : $\aleph(z_1, \dots, z_r) = \aleph^{0, n: m_1, n_1, \dots, m_r, n_r}_{p_i, q_i, \tau_i; R: p_i(1), q_i(1), \tau_i(1); R^{(1)}; \dots; p_i(r), q_i(r), \tau_i(r); R^{(r)}} \left(\begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{matrix} \right)$

$$[(a_j; \alpha_j^{(1)}, \dots, \alpha_j^{(r)})_{1, n}] , [\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \dots, \alpha_{ji}^{(r)})_{n+1, p_i}] :$$

$$\dots \dots \dots , [\tau_i(b_{ji}; \beta_{ji}^{(1)}, \dots, \beta_{ji}^{(r)})_{m+1, q_i}] :$$

$$\left(\begin{matrix} [(c_j^{(1)}, \gamma_j^{(1)})_{1, n_1}], [\tau_{i(1)}(c_{ji}^{(1)}, \gamma_{ji}^{(1)})_{n_1+1, p_i^{(1)}}]; \dots ; [(c_j^{(r)}, \gamma_j^{(r)})_{1, n_r}], [\tau_{i(r)}(c_{ji}^{(r)}, \gamma_{ji}^{(r)})_{n_r+1, p_i^{(r)}}] \\ [(d_j^{(1)}, \delta_j^{(1)})_{1, m_1}], [\tau_{i(1)}(d_{ji}^{(1)}, \delta_{ji}^{(1)})_{m_1+1, q_i^{(1)}}]; \dots ; [(d_j^{(r)}, \delta_j^{(r)})_{1, m_r}], [\tau_{i(r)}(d_{ji}^{(r)}, \delta_{ji}^{(r)})_{m_r+1, q_i^{(r)}}] \end{matrix} \right)$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \psi(s_1, \dots, s_r) \prod_{k=1}^r \theta_k(s_k) z_k^{s_k} ds_1 \dots ds_r \quad (1.2)$$

with $\omega = \sqrt{-1}$

$$= \frac{\Gamma(1/2 + a - b + \alpha_1 + \dots + \alpha_r)}{4^{\zeta_1 + \dots + \zeta_r} \Gamma(2a + 2\alpha_1 + \dots + 2\alpha_r) \Gamma(1/2 - (a + b + \alpha_1 + \dots + \alpha_r))} \times \quad (2.2)$$

$$\frac{\Gamma(2(a + \alpha_1 + \dots + \alpha_r + \zeta_1 + \dots + \zeta_r)) \Gamma(1/2 - (a + b + \alpha_1 + \dots + \alpha_r) + \eta_1 - \zeta_1 + \dots + \eta_r - \zeta_r)}{\Gamma(1/2 + a - b + \alpha_1 + \zeta_1 + \eta_1 + \dots + \alpha_r + \zeta_r + \eta_r)}$$

$$R_{\alpha_1, \dots, \alpha_r}^{(a,b)} \left\{ x_1^{\lambda_1} \dots x_r^{\lambda_r} (x_1 + \dots + x_r)^{\zeta_1 + \dots + \zeta_r} \right.$$

$$\left. \left[(x_1 + \dots + x_r)^{1/2} + (1 + x_1 + \dots + x_r)^{1/2} \right]^{-2(\eta_1 + \dots + \eta_r)} \right\}$$

$$= \frac{\Gamma(\alpha_1 + \dots + \alpha_r) \prod_{i=1}^r \Gamma(\alpha_i + \lambda_i) \Gamma(1/2 + a - b + \alpha_1 + \dots + \alpha_r) 4^{-(\zeta_1 + \lambda_1 + \dots + \zeta_r + \lambda_r)}}{\Gamma(\alpha_1) \dots \Gamma(\alpha_r) \Gamma(\alpha_1 + \lambda_1 + \dots + \alpha_r + \lambda_r) \Gamma(2(a + \alpha_1 + \dots + \alpha_r)) \Gamma(1/2 - a - \alpha_1 - \dots - \alpha_r)}$$

$$\Gamma(2(a + \alpha_1 + \zeta_1 + \lambda_1 + \dots + \alpha_r + \zeta_r + \lambda_r))$$

$$\frac{\Gamma(1/2 - ab + \alpha_1 - \lambda_1 - \alpha_r - \lambda_r + \eta_1 - \zeta_1 + \dots + \eta_r - \zeta_r)}{\Gamma(1/2 + a - b + \alpha_1 + \lambda_1 + \zeta_1 + \eta_1 + \dots + \alpha_r + \lambda_r + \zeta_r + \eta_r)} \quad (2.3)$$

valid if $0 < \text{Re}(a + \zeta_1 + \alpha_1 + \lambda_1 + \dots + \zeta_r + \alpha_r + \lambda_r) < \text{Re}(1/2 - b + \eta_1 + \dots + \eta_r)$

$\text{Re}(\alpha_i) > 0, \text{Re}(\lambda_i) > 0, i = 1, \dots, r$

3. Main integrals

In this section, making an appeal to (2.2) and (2.3), we derive the following results involving the multivariable Aleph-function defined by Ayant [1]

$$R_{\alpha_1, \dots, \alpha_r}^{(a,b)} \left\{ \mathbb{N}_{U:W}^{0,n;V} \left(\begin{matrix} z_1 (x_1 + \dots + x_r)^{\zeta_1} [(x_1 + \dots + x_r)^{1/2} + (1 + x_1 + \dots + x_r)^{1/2}]^{-2\eta_1} \\ \vdots \\ z_r (x_1 + \dots + x_r)^{\zeta_r} [(x_1 + \dots + x_r)^{1/2} + (1 + x_1 + \dots + x_r)^{1/2}]^{-2\eta_r} \end{matrix} \right) \right.$$

$$\left. \begin{matrix} A : C \\ \dots \\ B : D \end{matrix} \right\} = \frac{\Gamma(1/2 + a - b + \alpha_1 + \dots + \alpha_r)}{\Gamma(2a + 2\alpha_1 + \dots + 2\alpha_r) \Gamma(1/2 - a - b - \alpha_1 - \dots - \alpha_r)} \mathbb{N}_{U_{21}:W}^{0,n+2;V} \left(\begin{matrix} 4^{-\zeta_1} z_1 \\ \vdots \\ 4^{-\zeta_r} z_r \end{matrix} \right)$$

$$\left. \begin{matrix} (1-2a-2\alpha_1 - \dots - 2\alpha_r; 2\zeta_1, \dots, 2\zeta_r), (1/2 + a + b + \alpha_1 + \dots + \alpha_r; \eta_1 - \zeta_1, \dots, \eta_r - \zeta_r), A : C \\ \vdots \\ (1/2-a+b-\alpha_1 - \dots - \alpha_r; \zeta_1 + \eta_1, \dots, \zeta_r + \eta_r), B : D \end{matrix} \right) \quad (3.1)$$

where $U_{21} = p_i + 2; q_i + 1; \tau_i; R$

Provided that

a) $0 < \text{Re}(a + \alpha_1 + \dots + \alpha_r) < 1/2 - \text{Re}(b), \text{Re}(\alpha_i) > 0, i = 1, \dots, r$

b) $\left| \frac{\text{arg} z_k}{4\zeta_k} \right| < \frac{1}{2} A_i^{(k)} \pi$, where $A_i^{(k)}$ is given in (1.3)

$$R_{\alpha_1, \dots, \alpha_r}^{(a,b)} \left\{ \begin{matrix} \mathfrak{N}_{U:W}^{0,n:V} \left(\begin{matrix} z_1 x_1^{\lambda_1} (x_1 + \dots + x_r)^{\zeta_1} [(x_1 + \dots + x_r)^{1/2} + (1 + x_1 + \dots + x_r)^{1/2}]^{-2\eta_1} \\ \vdots \\ z_r x_r^{\lambda_r} (x_1 + \dots + x_r)^{\zeta_r} [(x_1 + \dots + x_r)^{1/2} + (1 + x_1 + \dots + x_r)^{1/2}]^{-2\eta_r} \end{matrix} \right) \end{matrix} \right.$$

$$\left. \begin{matrix} A : C \\ \vdots \\ \vdots \\ B : D \end{matrix} \right\}$$

$$= \frac{\Gamma(\alpha_1 + \dots + \alpha_r) \prod_{i=1}^r \Gamma(\alpha_i + \lambda_i) \Gamma(1/2 + a - b + \alpha_1 + \dots + \alpha_r)}{\Gamma(\alpha_1) \dots \Gamma(\alpha_r) \Gamma(\alpha_1 + \lambda_1 + \dots + \alpha_r + \lambda_r) \Gamma(2(a + \alpha_1 + \dots + \alpha_r)) \Gamma(1/2 - a - b - \alpha_1 - \dots - \alpha_r)}$$

$$\mathfrak{N}_{U_{21}:W}^{0,n+2:V} \left(\begin{matrix} 4^{-\zeta_1 - \lambda_1} z_1 \\ \vdots \\ \vdots \\ 4^{-\zeta_r - \lambda_r} z_r \end{matrix} \left| \begin{matrix} (1-2a-2\alpha_1 - \dots - 2\alpha_r; 2(\zeta_1 + \lambda_1), \dots, 2(\zeta_r + \lambda_r)), \\ \vdots \\ \vdots \\ (1/2-a+b-\alpha_1 - \dots - \alpha_r; \zeta_1 + \eta_1 + \lambda_1, \dots, \zeta_r + \eta_r + \lambda_r) \end{matrix} \right. \right)$$

$$\left. \begin{matrix} (1/2+a+b+\alpha_1 + \dots + \alpha_r; \eta_1 - \zeta_1 - \lambda_1, \dots, \eta_r - \zeta_r - \lambda_r), A : C \\ \vdots \\ \vdots \\ B : D \end{matrix} \right) \quad (3.2)$$

where $U_{21} = p_i + 2; q_i + 1; \tau_i; R$

Provided that

a) $0 < \text{Re}(a + \alpha_1 + \dots + \alpha_r) < 1/2 - \text{Re}(b), \text{Re}(\alpha_i) > 0, \text{Re}(\lambda_i) > 0, i = 1, \dots, r$

b) $\left| \frac{\text{arg} z_k}{4\zeta_k + \lambda_k} \right| < \frac{1}{2} A_i^{(k)} \pi$, where $A_i^{(k)}$ is given in (1.3)

4. Special cases

a) For $\lambda_i = 0, i = 1, \dots, r$, (3.2) reduces to (3.1)

b) For $\eta_i = \zeta_i, i = 1, \dots, r$, (3.1) reduces

$$R_{\alpha_1, \dots, \alpha_r}^{(a,b)} \left\{ \begin{array}{l} N_{U:W}^{0,n;V} \left(\begin{array}{l} z_1(x_1 + \dots + x_r)^{\zeta_1} [(x_1 + \dots + x_r)^{1/2} + (1 + x_1 + \dots + x_r)^{1/2}]^{-2\zeta_1} \\ \vdots \\ z_r(x_1 + \dots + x_r)^{\zeta_r} [(x_1 + \dots + x_r)^{1/2} + (1 + x_1 + \dots + x_r)^{1/2}]^{-2\zeta_r} \end{array} \right) \\ A : C \\ \vdots \\ B : D \end{array} \right\} = \frac{\Gamma(1/2 + a - b + \alpha_1 + \dots + \alpha_r)}{\Gamma(2a + 2\alpha_1 + \dots + 2\alpha_r)\Gamma(1/2 - a - b - \alpha_1 - \dots - \alpha_r)} N_{U_{11}:W}^{0,n+1;V} \left(\begin{array}{l} 4^{-\zeta_1} z_1 \\ \vdots \\ 4^{-\zeta_r} z_r \end{array} \right) \\ \left. \begin{array}{l} (1-2a-2\alpha_1 - \dots - 2\alpha_r; 2\zeta_1, \dots, 2\zeta_r), A : C \\ \vdots \\ (1/2-a+b-\alpha_1 - \dots - \alpha_r; 2\zeta_1, \dots, 2\zeta_r), B : D \end{array} \right) \quad (4.1)$$

where $U_{11} = p_i + 1; q_i + 1; \tau_i; R$

Provided that

a) $0 < Re(a + \alpha_1 + \dots + \alpha_r) < 1/2 - Re(b), Re(\alpha_i) > 0, i = 1, \dots, r$

b) $\left| \frac{arg z_k}{4\zeta_k} \right| < \frac{1}{2} A_i^{(k)} \pi$, where $A_i^{(k)}$ is given in (1.3)

5. Multivariable I-function

If $\tau_i, \tau_{i(1)}, \dots, \tau_{i(r)} \rightarrow 1$, the Aleph-function of several variables degenerate to the I-function of several variables. The two formulas have been derived in this section for multivariable I-functions defined by Sharma et al [3].

$$R_{\alpha_1, \dots, \alpha_r}^{(a,b)} \left\{ \begin{array}{l} I_{U:W}^{0,n;V} \left(\begin{array}{l} z_1(x_1 + \dots + x_r)^{\zeta_1} [(x_1 + \dots + x_r)^{1/2} + (1 + x_1 + \dots + x_r)^{1/2}]^{-2\eta_1} \\ \vdots \\ z_r(x_1 + \dots + x_r)^{\zeta_r} [(x_1 + \dots + x_r)^{1/2} + (1 + x_1 + \dots + x_r)^{1/2}]^{-2\eta_r} \end{array} \right) \end{array} \right.$$

$$\left. \begin{array}{l} A : C \\ \dots \\ \dots \\ B : D \end{array} \right\} = \frac{\Gamma(1/2 + a - b + \alpha_1 + \dots + \alpha_r)}{\Gamma(2a + 2\alpha_1 + \dots + 2\alpha_r)\Gamma(1/2 - a - b - \alpha_1 - \dots - \alpha_r)} I_{U_{21}:W}^{0,n+2;V} \left(\begin{array}{c} 4^{-\zeta_1} z_1 \\ \vdots \\ \vdots \\ 4^{-\zeta_r} z_r \end{array} \middle| \right. \\
(1-2a-2\alpha_1 - \dots - 2\alpha_r; 2\zeta_1, \dots, 2\zeta_r), (1/2 + a + b + \alpha_1 + \dots + \alpha_r; \eta_1 - \zeta_1, \dots, \eta_r - \zeta_r), A : C \\
\vdots \\
\vdots \\
(1/2-a+b-\alpha_1 - \dots - \alpha_r; \zeta_1 + \eta_1, \dots, \zeta_r + \eta_r), B : D \left. \right) \quad (5.1)$$

under the same notations and conditions that (3.1)

$$R_{\alpha_1, \dots, \alpha_r}^{(a,b)} \left\{ I_{U:W}^{0,n;V} \left(\begin{array}{c} z_1 x_1^{\lambda_1} (x_1 + \dots + x_r)^{\zeta_1} [(x_1 + \dots + x_r)^{1/2} + (1 + x_1 + \dots + x_r)^{1/2}]^{-2\eta_1} \\ \vdots \\ \vdots \\ z_r x_r^{\lambda_r} (x_1 + \dots + x_r)^{\zeta_r} [(x_1 + \dots + x_r)^{1/2} + (1 + x_1 + \dots + x_r)^{1/2}]^{-2\eta_r} \end{array} \middle| \right. \right.$$

$$\left. \begin{array}{l} A : C \\ \dots \\ \dots \\ B : D \end{array} \right\}$$

$$= \frac{\Gamma(\alpha_1 + \dots + \alpha_r) \prod_{i=1}^r \Gamma(\alpha_i + \lambda_i) \Gamma(1/2 + a - b + \alpha_1 + \dots + \alpha_r)}{\Gamma(\alpha_1) \dots \Gamma(\alpha_r) \Gamma(\alpha_1 + \lambda_1 + \dots + \alpha_r + \lambda_r) \Gamma(2(a + \alpha_1 + \dots + \alpha_r)) \Gamma(1/2 - a - b - \alpha_1 - \dots - \alpha_r)}$$

$$I_{U_{21}:W}^{0,n+2;V} \left(\begin{array}{c} 4^{-\zeta_1 - \lambda_1} z_1 \\ \vdots \\ \vdots \\ 4^{-\zeta_r - \lambda_r} z_r \end{array} \middle| \begin{array}{l} (1-2a-2\alpha_1 - \dots - 2\alpha_r; 2(\zeta_1 + \lambda_1), \dots, 2(\zeta_r + \lambda_r)), \\ \vdots \\ \vdots \\ (1/2-a+b-\alpha_1 - \dots - \alpha_r; \zeta_1 + \eta_1 + \lambda_1, \dots, \zeta_r + \eta_r + \lambda_r) \end{array} \right)$$

$$(1/2+a+b+\alpha_1 + \dots + \alpha_r; \eta_1 - \zeta_1 - \lambda_1, \dots, \eta_r - \zeta_r - \lambda_r), A : C \\
\vdots \\
\vdots \\
B : D \left. \right) \quad (5.2)$$

under the same conditions and notations that (3.2) with $\tau_i, \tau_{i(1)}, \dots, \tau_{i(r)} \rightarrow 1$

6. Aleph-function of two variables

If $r = 2$, we obtain the Aleph-function of two variables defined by K.Sharma [5], and we have the following two relations.

$$\begin{aligned}
 & R_{\alpha_1, \alpha_2}^{(a, b)} \left\{ \aleph_{U:W}^{0, n:V} \left(\begin{array}{c} z_1(x_1 + x_2)^{\zeta_1} [(x_1 + x_2)^{1/2} + (1 + x_1 + x_2)^{1/2}]^{-2\eta_1} \\ \vdots \\ z_2(x_1 + x_2)^{\zeta_2} [(x_1 + x_2)^{1/2} + (1 + x_1 + x_2)^{1/2}]^{-2\eta_2} \end{array} \middle| \begin{array}{c} A : C \\ \dots \\ B : D \end{array} \right) \right\} \\
 &= \frac{\Gamma(1/2 + a - b + \alpha_1 + \alpha_2)}{\Gamma(2a + 2\alpha_1 + 2\alpha_2)\Gamma(1/2 - a - b - \alpha_1 - \alpha_2)} \aleph_{U_{21}:W}^{0, n+2:V} \left(\begin{array}{c} 4^{-\zeta_1} z_1 \\ \vdots \\ 4^{-\zeta_2} z_2 \end{array} \middle| \begin{array}{c} (1-2a-2\alpha_1 - 2\alpha_2; 2\zeta_1, 2\zeta_2), (1/2 + a + b + \alpha_1 + \alpha_2; \eta_1 - \zeta_1, \eta_2 - \zeta_2), A : C \\ \vdots \\ (1/2-a+b-\alpha_1 - \alpha_2; \zeta_1 + \eta_1, \zeta_2 + \eta_2), B : D \end{array} \right) \quad (6.1)
 \end{aligned}$$

under the same notations and conditions that (3.1) with $r = 2$

$$\begin{aligned}
 & R_{\alpha_1, \alpha_2}^{(a, b)} \left\{ \aleph_{U:W}^{0, n:V} \left(\begin{array}{c} z_1 x_1^{\lambda_1} (x_1 + x_2)^{\zeta_1} [(x_1 + x_2)^{1/2} + (1 + x_1 + x_2)^{1/2}]^{-2\eta_1} \\ \vdots \\ z_2 x_2^{\lambda_2} (x_1 + x_2)^{\zeta_2} [(x_1 + x_2)^{1/2} + (1 + x_1 + x_2)^{1/2}]^{-2\eta_2} \end{array} \middle| \begin{array}{c} A : C \\ \dots \\ B : D \end{array} \right) \right\} \\
 &= \frac{\Gamma(\alpha_1 + \alpha_2) \prod_{i=1}^2 \Gamma(\alpha_i + \lambda_i) \Gamma(1/2 + a - b + \alpha_1 + \alpha_2)}{\Gamma(\alpha_1) \Gamma(\alpha_2) \Gamma(\alpha_1 + \lambda_1 + \alpha_2 + \lambda_2) \Gamma(2(a + \alpha_1 + \alpha_2)) \Gamma(1/2 - a - b - \alpha_1 - \alpha_2)}
 \end{aligned}$$

$$\aleph_{U_{21}:W}^{0, n+2:V} \left(\begin{array}{c} 4^{-\zeta_1 - \lambda_1} z_1 \\ \vdots \\ 4^{-\zeta_2 - \lambda_2} z_2 \end{array} \middle| \begin{array}{c} (1-2a-2\alpha_1 - 2\alpha_2; 2(\zeta_1 + \lambda_1), 2(\zeta_2 + \lambda_2)), \\ \vdots \\ (1/2-a+b-\alpha_1 - \alpha_2; \zeta_1 + \eta_1 + \lambda_1, \zeta_2 + \eta_2 + \lambda_2) \end{array} \right)$$

$$\left. \begin{array}{l} (1/2+a+b+\alpha_1 + \alpha_2; \eta_1 - \zeta_1 - \lambda_1, \eta_2 - \zeta_2 - \lambda_2), A : C \\ \vdots \\ \vdots \\ B : D \end{array} \right) \quad (6.2)$$

under the same conditions and notations that (3.2) with $r = 2$

7. I-function of two variables

If $\tau_i, \tau'_i, \tau''_i \rightarrow 1$, then the Aleph-function of two variables degenerate in the I-function of two variables defined by sharma et al [4] and we obtain the same formulas with the I-function of two variables.

$$R_{\alpha_1, \alpha_2}^{(a,b)} \left\{ I_{U:W}^{0,n;V} \left(\begin{array}{l} z_1(x_1 + x_2)^{\zeta_1} [(x_1+x_2)^{1/2} + (1 + x_1 + x_2)^{1/2}]^{-2\eta_1} \\ \vdots \\ \vdots \\ z_2(x_1 + x_2)^{\zeta_2} [(x_1 + x_2)^{1/2} + (1 + x_1 + x_2)^{1/2}]^{-2\eta_2} \end{array} \middle| \begin{array}{l} A : C \\ \vdots \\ \vdots \\ B : D \end{array} \right) \right\}$$

$$= \frac{\Gamma(1/2 + a - b + \alpha_1 + \alpha_2)}{\Gamma(2a + 2\alpha_1 + 2\alpha_2)\Gamma(1/2 - a - b - \alpha_1 - \alpha_2)} I_{U_{21}:W}^{0,n+2;V} \left(\begin{array}{l} 4^{-\zeta_1} z_1 \\ \vdots \\ \vdots \\ 4^{-\zeta_2} z_2 \end{array} \middle| \begin{array}{l} A : C \\ \vdots \\ \vdots \\ B : D \end{array} \right)$$

$$\left. \begin{array}{l} (1-2a-2\alpha_1 - 2\alpha_2; 2\zeta_1, 2\zeta_2), (1/2 + a + b + \alpha_1 + \alpha_2; \eta_1 - \zeta_1, \eta_2 - \zeta_2), A : C \\ \vdots \\ \vdots \\ (1/2-a+b-\alpha_1 - \alpha_2; \zeta_1 + \eta_1, \zeta_2 + \eta_2), B : D \end{array} \right) \quad (7.1)$$

under the same notations and conditions that (3.1) with $r = 2$

$$R_{\alpha_1, \alpha_2}^{(a,b)} \left\{ I_{U:W}^{0,n;V} \left(\begin{array}{l} z_1 x_1^{\lambda_1} (x_1 + x_2)^{\zeta_1} [(x_1+x_2)^{1/2} + (1 + x_1 + x_2)^{1/2}]^{-2\eta_1} \\ \vdots \\ \vdots \\ z_2 x_2^{\lambda_2} (x_1 + x_2)^{\zeta_2} [(x_1 + x_2)^{1/2} + (1 + x_1 + x_2)^{1/2}]^{-2\eta_2} \end{array} \middle| \begin{array}{l} A : C \\ \vdots \\ \vdots \\ B : D \end{array} \right) \right\}$$

$$= \frac{\Gamma(\alpha_1 + \alpha_2) \prod_{i=1}^2 \Gamma(\alpha_i + \lambda_i) \Gamma(1/2 + a - b + \alpha_1 + \alpha_2)}{\Gamma(\alpha_1) \Gamma(\alpha_2) \Gamma(\alpha_1 + \lambda_1 + \alpha_2 + \lambda_2) \Gamma(2(a + \alpha_1 + \alpha_2)) \Gamma(1/2 - a - b - \alpha_1 - \alpha_2)}$$

$$I_{U_{21}:W}^{0,n+2;V} \left(\begin{array}{c} 4^{-\zeta_1 - \lambda_1} z_1 \\ \cdot \\ \cdot \\ 4^{-\zeta_2 - \lambda_2} z_2 \end{array} \middle| \begin{array}{l} (1-2a-2\alpha_1 - 2\alpha_2; 2(\zeta_1 + \lambda_1), 2(\zeta_2 + \lambda_2)), \\ \cdot \\ \cdot \\ (1/2-a+b-\alpha_1 - \alpha_2; \zeta_1 + \eta_1 + \lambda_1, \zeta_2 + \eta_2 + \lambda_2) \end{array} \right. \\ \left. \begin{array}{l} (1/2+a+b+\alpha_1 + \alpha_2; \eta_1 - \zeta_1 - \lambda_1, \eta_2 - \zeta_2 - \lambda_2), A : C \\ \cdot \\ \cdot \\ B : D \end{array} \right) \quad (7.2)$$

under the same conditions and notations that (3.2) with $r = 2$

8. Conclusion

In this paper we have evaluated two multidimensional integral transform concerning the multivariable Aleph-function. The formulas established in this paper is of very general nature as it contains multivariable Aleph-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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