Numerical Experiments on Quarter of an Elliptic Plate with Exponential Thickness Variation

Neetu Singh[#1], Vipin Saxena[*2]

[#1]Research Scholar, Applied Mathematics, Babasaheb Bhimrao Ambedkar University

[*2]Professor, Computer Science, Babasaheb Bhimrao Ambedkar University

Babasaheb Bhimrao Ambedkar University Lucknow-226025 (UP) India

Abstract— *A study of transverse vibrations of plates plays an important role in the design of naval architecture, engineering design, aircraft design, etc. Due to wide variety of its application, first few frequencies play crucial role for getting best structural design. The present work is related to consider a plate in the form of quarter of an elliptic and a variable thickness in the form of exponent is considered. A well-known Rayleigh Ritz method is used for mathematical solution of the problem in the form of eigenvalue problem. The solution of eigenvalue form is further computed through generalized Jacobin method which gives first few frequencies. The aim of this paper is to compute first three frequencies and computed results are compared with the existing results for the uniform thickness. The new computed results are represented through tables and graphs. Convergence up-to five significant digits are also presented*.

Keywords — *Rayleigh-Ritz Method, quarter elliptic Plate, eigen value, Jacobi Method.*

I. INTRODUCTION

Variational methods are used for solutions of thin plates and one of the important methods is the Rayleigh-Ritz technique for computing the first few frequencies for the thin plates. During the survey, it is found that a study of an exponential thickness variation is done by some researchers on different shapes of thin plates. Let us describe source of the important reference related to present work. The frequencies of vibration of flat circular plates fixed at the circumference have discussed by Carrington [1]. Authors [2-4] have also analysed frequencies of vibration problems for different types of plate like, elliptical plate, circular plate, etc. by Rayleigh-Ritz technique. Shibaolay [5] has briefly discussed transverse vibration of an elliptic plate with clamped boundary condition. Wah [6] has demonstrated vibrations of circular plate with different boundary conditions of the plate as clamped or simply supported. Free vibration of a clamped elliptic plate was discussed by Micnitt [7]. Wilkison [8] has analysed frequencies of exponentially varying thickness. Many of researchers discussed frequencies, mode shapes and nodal radii for various shapes of the plates. Vibration of a circular plate with variable thickness is also explained by Leissa [9]. Cheung and Cheung [10] have discussed the flexural vibration of rectangular and polygonal plates. Soni [11] has defined axisymmetric vibration of a circular plate which is taken as clamped or simply supported plate in the field. Leissa [12] is the best source of vibration of plate with frequencies, mode shapes by Rayleigh-Ritz method. Mukhopadhyay [13] has investigated the case of semi analytic solution for free vibration of annular sector plate. Srinivasan and Threuvenkatchari [14] used integral equation technique solve for free vibration of annular sector plate. Kim and Dickinson [15] have obtained free transverse vibration of annular and circular, thin, sectorial plates subject to certain complicating effect. Singh and Chakraverty [16] have also demonstrated transverse vibration of circular and elliptic plates with quadratically varying thickness. Singh and Saxena [17] have defined quarter of circular plate with variable thickness by Rayleigh-Ritz method. Hassan and Makery [18] have found the first four frequencies of elliptic plate using the boundary conditions like clamped and simple-supported. Leissa [19] has analysed Mathematics of eigenvalue through Mathematical programming for Pseudos-Pectral method and found the eigenvalues of axisymmetric Mindin plate and Timoshenko beams by Lee and Schutz [20].The entire coverage on the research done on thin plates is available in monograph of Lesissa'[21] which is excellent source of information on vibration of a specific plate i.e. circular plates with variable thickness. Gupta et al. [22] have introduced vibration of analysis for non-homogeneous circular plate of nonlinear thickness variation by differential quadrature method. Rayleigh-Ritz method is defined the free vibration analysis of super elliptical plates with constant and variables thickness by Ceribasi and Altany [23]. Lakshmi et al. [24] have discussed vibration analysis for the elliptical with clamped plate.

On the basis of above, it is observed that quarter of an elliptical plate with exponential thickness variation is still not studied by the researchers, therefore, the present work is in this direction and Rayleigh-Ritz method is used for computation of the first three frequencies for vibration of quarter of elliptic plate. The thickness of thin plate is considered as exponential. Combinations of boundary conditions are considered as clamped, simply-supported and completely free which leads to total combinations of twenty seven cases, some of the important cases are reported here along with frequencies and mode shapes are also depicted through programming language.

II. MATERIAL AND METHODS

Classical Plate Theory

According to Kirchhoff plate theory, (classical plate theory) is based on some important points, which are given below.

1) Thickness of plate is considered as small in comparison of other dimensions.

2) Rotatory of inertia is considered as negligible.

3) Normal to the undeformed and deformed middle surface remains straight and unstretched due to effect of light,

4) Plate is taken as negligibly small of normal stresses in the transform direction.

Boundary Conditions

The plate is shown below in figure 1, where R is the domain of the plate, N is normal to the boundary of plate as shown in the figure and the boundary conditions are defined below in brief.

1) ***Clamped Boundary Condition*** when the boundary of plate considers as clamped then there is no deflection in the plate represented as W and N is normal to the plate then on the curve surface of the plate, the following boundary condition of the plate is applicable.

$$W = 0 \tag{1}$$

and $\frac{\partial W}{\partial N} = 0$ on the curved surface (C) (2)

2) ***Supported Boundary Condition*** when the curved boundary of plate is simply-supported then there will be a little deflection represent as W. Let M represents bending moment with respect to the normal of the plate then the following boundary condition of applicable.

$$W = 0 \tag{3}$$

and $\frac{\partial M}{\partial N} = 0$ on the curved surface of plate (C) (4)

3) ***Free Boundary Condition*** when the boundary of plate is considered as completely-free then there will be deflection (W), bending moment (M) and shear force (Q) with respect to normal of the plate then the following boundary condition is applicable.

$$M_N = 0 \tag{5}$$

$Q_N + \frac{\partial M_{NT}}{\partial T} = 0$ on the curved surface of plate (C) (6)

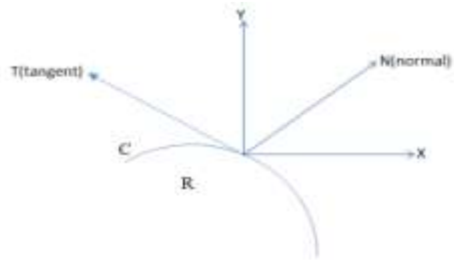

Fig 1 Portion of path C with normal and tangential

On the basis of the above different kind of thin plates are consider by the various authors namely circular, elliptic, square, rectangular, skew, rhombus, etc. From the literature, it is observed that some authors have also considered plate with small whole inside the plate for observing the behaviour of plate. During vibration of thin plate stiffness of the plate is considered by varying the thickness as linear, quadratic, exponential, etc.

Rayleigh-Ritz Method

In the Rayleigh-Ritz method, we equate the maximum kinetic with the maximum strain energy to obtain the Rayleigh quotient which is given by the following equation.

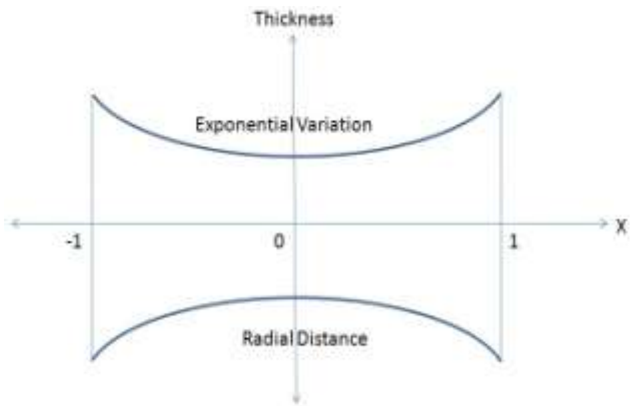

Fig 2 Exponential thickness variation for quarter of an elliptic plate.

$$\omega^2 = \frac{E}{12\rho\left(1-\nu^2\right)}\left[\frac{\iint\limits_R h^3\left[\left(\nabla^2 W\right)^2 + 2(1-\nu)\left\{\left(W^{x'y'}\right)^2 - W^{x'x'}W^{y'y'}\right\}\right]dx'dy'}{\iint\limits_R hW^2 dx'dy'}\right], \tag{7}$$

where E, ρ ,ν and h are Young's modulus, density, Poisson ratio and variable thickness respectively. R is domain of the plate. Let us consider non-dimensional variables x and y which are given below by following equation.

$$x' = x/a \text{ , } y' = y/b \text{ , } m = b/a \tag{8}$$

where m is the aspect ratio. On the basis of these non-dimensional parameters the deflection of plate is given by equation (9).

$$W(x,y)=\sum_{j=1}^{N} C_j\phi_j(x,y), \tag{9}$$

Where ϕ_j are taken to satisfy the boundary conditions of given problem and C_j $\left(j\in\Delta_N=\{1,2,.......N\}\right)$ are and putting the value of W(x, y) in equation (7) an eigenvalue problem is given below;

$$\sum_{j=1}^{N}\left(a_{ij}-\lambda^2 b_{ij}\right)C_j=0, \quad i\in\Delta_N\ , \tag{10}$$

where

$$a_{ij}=\iint_R H^3\begin{bmatrix}\phi_i^{xx}\phi_j^{xx}+\phi_i^{yy}\phi_j^{yy}+\nu\\ \left(\phi_i^{xx}\phi_j^{yy}+\phi_i^{yy}\phi_j^{xx}\right)+2\left(1-\nu\right)\phi_i^{xy}\phi_j^{xy}\end{bmatrix}dxdy, \tag{11}$$

$$b_{ij}=\iint_R H\phi_i\phi_j\,dxdy, \tag{12}$$

$$\lambda^2=\left[12\rho a^4\left(1-\nu^2\right)\omega^2\right]\Big/\left(Eh_0^2\right), \tag{13}$$

where h_0 is thickness of given elliptic plate. Let us consider a basis function ϕ_i which satisfies the boundary conditions

$$\phi_i\left(x,y\right)=f\left(x,y\right)x^{m_i+p}y^{n_i+q}, \tag{14}$$

$$f\left(x,y\right)=\left[1-\left(x^2+\frac{y^2}{m^2}\right)\right]^r \tag{15}$$

Since ϕ_i satisfies the boundary conditions, therefore $f\left(x,y\right)$ also satisfies the given boundary conditions. The parameter r is controlling type of boundary conditions i.e. r=0, 1, 2 shows that the plate is completely-free, simply-supported and clamped, respectively. The variables m_i+p and n_i+q are non-negative integers given by following table.

The thickness of the plate is given by following equation and represented in.

$$H=e^{\alpha r} \tag{16}$$

By putting the values of f and H in equations (10), (11) and (12) and solving the expressions of a_{ij} and b_{ij} which contains all the integrals in closed form given by following formula.

TABLE I

Non negative value of m_i and n_i

m_i	n_i
0	0
0	1
1	0
2	0
1	1
0	2
3	0
2	1
1	2
0	3
4	0
3	1
2	2
1	3
0	4
5	0
4	1
3	2
2	3
1	4
0	5
6	0
5	1
4	2
3	3
2	4
1	5
0	6

$$\iint_R x^k y^l r^m\left(1-r^2\right)^n dxdy=\frac{G\left(\frac{k+l+m}{2}+1\right)G\left(\frac{k+1}{2}\right)G\left(\frac{l+1}{2}\right)G\left(n+1\right)}{4G\left(\frac{k+l+m}{2}+n+2\right)G\left(\frac{k+l}{2}+1\right)}, \tag{17}$$

where G stands for the gamma function.

III. Computation of Numerical results

On the basis of above formulation of the problem, a program has been defined for extensive numerical computations and for all the calculation, the following values of the constants and variables have been considered.

1) The value of Poisson's ratio ν is taken as 0.3 for isotropic plate.
2) F, S, C can take value 0, 1, 2 for a completely free, simply-supported and clamped plate, respectively.
3) The thickness is controlled by the parameter α which is taken as variable from -1 to +1.
4) The values of m are considered as 0.25, 0.5, and 0.75 for quarter of elliptic plate.

By the use of above parameters, we solved all the integral and computed a_{ij} and b_{ij} as given in the equation (10) & (11) respectively. By the use of the generalized Jacobi method, eigenvalue problem can be solved and first few frequencies have been computed in some selected cases as described below in brief. Tables 2, 3 and 4 demonstrate the first three frequencies for various boundary conditions for various value of α running from -1 to +1. From the tables, it is observed that when taper parameter α is increasing from -1 to +1 then all the frequencies are increasing in the all cases.

TABLE II

First three frequencies for quarter of on elliptic plates (m=0.25, ν =0.3, N=28)

α	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
CCC	228.690	257.590	290.052	326.586	367.758	414.227	466.766	526.308	593.991	671.211	759.679
	260.340	295.900	336.340	382.300	434.524	493.921	561.646	639.179	728.375	831.508	951.331
	301.352	344.989	394.730	451.425	516.361	591.307	678.463	780.416	900.154	1041.152	1207.506
CCS	174.981	195.158	217.410	241.968	269.075	299.004	332.075	368.675	409.287	454.517	505.131
	203.911	230.406	260.337	294.113	332.242	375.373	424.330	480.136	544.035	617.530	702.423
	242.908	276.970	315.676	359.998	411.185	470.702	540.224	621.673	717.288	829.714	962.103
FCC	219.564	243.459	269.882	299.040	332.404	369.642	411.741	459.478	513.746	575.577	646.156
	239.307	269.121	302.957	341.322	384.813	434.118	490.040	553.524	625.705	707.960	801.971
	270.995	308.197	350.821	399.366	454.463	517.005	588.227	669.746	763.574	872.155	998.426
SSS	108.613	123.143	139.340	157.333	177.262	199.287	223.603	250.542	280.144	313.081	349.788
	139.851	159.377	181.474	206.461	234.724	266.739	303.078	344.410	391.518	445.322	506.925
	178.969	204.895	234.786	269.416	309.655	356.412	410.624	473.275	545.469	628.552	724.291
SSC	149.604	171.345	196.055	224.117	255.976	292.152	333.264	380.040	433.347	494.202	563.787
	184.417	211.498	242.431	277.743	318.041	364.042	416.615	476.811	545.868	625.190	716.332
	226.420	260.229	299.076	343.609	394.711	453.603	521.783	600.874	692.512	798.358	920.280
FSC	132.576	151.797	173.689	198.376	226.251	257.789	293.559	334.238	380.616	433.604	494.238
	163.990	187.666	214.603	245.283	280.272	320.220	365.871	418.066	477.767	546.090	624.343
	201.189	230.469	264.216	303.078	347.078	398.757	457.038	523.589	599.825	687.577	789.068
FSS	93.448	105.874	119.439	134.176	150.126	167.331	185.848	205.742	227.100	250.039	274.727
	121.466	137.833	156.196	176.804	199.937	225.92	255.133	288.038	325.203	367.342	415.339
	155.532	177.447	202.540	231.18	263.799	300.956	343.384	392.013	447.970	512.586	587.444
CSS	111.226	126.351	143.260	162.106	183.052	206.290	232.043	260.589	292.278	327.563	367.028
	144.572	165.047	188.282	214.626	244.472	278.275	316.588	360.107	409.716	466.532	531.946
	185.716	213.082	244.271	279.675	319.909	365.913	418.974	480.695	552.977	638.034	738.469
FFC	21.833	27.208	33.835	41.982	51.964	64.147	78.951	96.853	118.385	144.127	172.699
	43.488	51.173	60.354	71.329	84.449	100.125	118.834	141.123	167.604	198.955	235.904
	69.709	80.964	94.138	109.598	127.782	149.198	174.421	204.083	238.865	279.517	326.894
FFS	3.473	4.277	5.269	6.493	8.004	9.869	12.176	15.029	18.563	22.946	28.388
	23.483	26.795	30.651	35.169	40.493	46.808	54.339	63.370	74.249	87.409	103.384
	81.175	93.166	107.035	123.01	141.373	162.494	186.873	215.16	248.181	286.961	332.735
FCS	167.145	182.685	199.244	217.04	236.243	256.993	279.417	303.642	329.798	358.042	388.571
	184.92	206.076	229.857	256.472	286.200	319.394	356.495	398.058	444.776	497.525	557.402
	215.041	243.496	275.77	312.281	353.608	400.519	454.001	515.293	585.927	667.777	763.110
SCC	226.712	254.647	285.931	321.052	360.556	405.073	455.328	512.170	576.594	649.773	733.090
	255.726	289.938	328.808	372.970	423.172	480.296	545.375	619.609	704.410	801.464	912.829
	294.311	336.506	384.842	440.312	504.124	577.66	662.494	760.461	873.786	1005.222	1158.18
CFS	18.825	20.814	23.116	25.813	29.006	32.823	37.427	43.018	49.847	58.223	68.528
	44.370	50.046	56.534	63.980	72.573	82.545	94.188	107.863	124.017	143.201	166.091
	78.236	88.546	100.298	113.770	129.276	147.167	167.852	191.829	219.738	252.317	290.133
SFC	34.497	40.693	48.225	57.377	68.479	81.915	98.127	117.615	140.942	168.729	201.696
	60.424	69.925	81.072	94.177	109.61	127.802	149.253	174.54	204.325	239.367	280.526
	94.661	108.553	124.613	143.232	164.855	189.085	219.217	253.302	293.189	340.023	395.074
SFS	15.543	17.264	19.289	21.701	24.606	28.130	32.436	37.721	44.230	52.264	62.196
	38.025	43.114	48.955	55.696	63.518	72.651	83.374	96.052	111.113	129.102	150.684
	69.239	78.603	89.323	101.620	115.768	132.115	151.116	173.348	199.501	230.382	266.909
SCS	173.327	192.670	213.887	237.195	262.817	290.996	322.000	356.142	393.803	435.456	481.713
	199.945	225.178	253.647	285.801	322.144	363.245	409.755	462.445	522.25	590.329	668.126
	236.595	270.076	308.864	353.709	405.439	465.013	533.601	612.671	704.077	810.136	933.707
CFC	38.914	45.437	53.311	62.821	74.304	88.148	104.805	124.787	148.675	177.112	210.808
	67.485	77.637	89.497	103.381	119.661	138.777	161.244	187.667	218.745	255.291	298.247
	104.169	119.128	136.34	156.184	179.136	205.755	236.665	272.589	314.419	363.319	420.825
CSC	152.697	175.124	200.652	229.685	262.689	300.213	342.897	391.504	446.937	510.279	582.826
	189.772	217.938	250.122	286.882	328.851	376.748	431.399	493.793	565.171	647.127	741.694
	233.997	269.15	309.539	355.783	408.502	468.535	537.206	616.463	708.849	817.419	945.712

TABLE III

First three frequencies for quarter of on elliptic plates (m=0.5, ν =0.3, N=28)

α	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
CCC	65.517	74.385	84.470	95.938	108.975	123.796	140.642	159.789	181.554	206.303	234.457
	89.340	102.489	117.551	134.787	154.487	176.974	202.612	231.815	265.662	302.925	346.105
	114.991	136.888	161.241	185.846	214.037	246.289	283.142	325.247	373.431	428.767	492.633
CCS	50.831	57.30	64.635	72.879	82.163	92.618	104.390	117.652	132.608	149.796	168.646
	72.567	83.115	95.196	109.024	124.837	142.910	163.558	187.154	214.145	229.080	280.629
	102.127	117.961	136.252	157.354	181.667	209.650	241.855	278.971	321.874	345.831	429.733
FCC	55.906	62.133	69.121	76.983	85.852	95.882	107.258	120.196	134.950	151.814	171.127
	70.596	79.965	90.641	102.813	116.700	132.557	150.674	171.382	195.055	222.117	253.047
	94.041	107.562	123.073	140.881	161.332	184.816	211.767	242.671	278.082	318.647	365.140
SSS	33.674	38.136	43.201	48.939	55.429	62.764	71.052	80.422	91.032	103.074	116.786
	54.723	62.556	71.538	81.831	93.617	107.107	122.541	140.206	160.437	183.631	210.253
	83.143	95.655	110.071	126.662	145.741	167.694	192.995	222.218	256.018	295.077	339.958
SSC	45.105	51.532	58.900	67.340	76.999	88.047	100.682	115.129	131.649	150.537	172.132
	68.541	78.575	90.091	103.293	118.415	135.715	155.488	178.065	203.821	233.189	266.670
	99.432	114.380	131.529	151.195	173.733	199.530	229.002	262.627	300.992	344.839	395.055
FSC	34.525	39.645	45.441	52.004	59.449	67.909	77.546	88.549	101.142	115.582	132.161
	53.620	61.148	69.772	79.664	91.018	104.058	119.041	136.255	156.028	178.728	204.762
	78.817	90.155	103.173	118.128	135.321	155.096	177.837	203.970	233.957	268.314	307.617
CFC	19.482	22.071	25.102	28.666	32.868	37.832	43.702	50.642	58.840	68.505	79.87
	42.614	48.296	54.768	62.144	70.560	80.170	91.151	103.706	118.069	134.506	153.325
	68.355	77.958	88.974	101.592	116.020	132.481	151.196	172.389	196.304	223.240	253.611
SCS	48.57	54.467	61.088	68.521	76.861	86.222	96.731	108.539	121.826	136.811	153.762
	68.019	77.648	88.661	101.248	115.626	132.047	150.801	172.229	196.727	224.765	256.900
	95.589	110.151	126.938	146.297	168.64	194.454	224.312	258.881	298.938	345.384	399.241
SFS	9.091	10.213	11.488	12.948	14.635	16.600	18.908	21.640	24.840	28.797	33.502
	27.092	30.661	34.713	39.320	44.569	50.561	57.417	65.279	74.316	84.726	96.744
	50.659	57.440	65.186	74.019	84.073	95.489	108.425	123.059	139.602	158.315	179.512
FCS	42.432	46.492	50.924	55.765	61.056	66.884	73.187	80.150	87.815	96.281	105.673
	55.659	62.684	70.647	79.685	89.958	101.654	114.99	130.223	147.653	167.637	190.602
	77.176	88.148	100.781	115.332	132.096	151.409	173.662	199.318	228.919	263.158	302.814
FSS	24.358	27.649	31.269	35.240	39.587	44.343	49.548	55.254	61.528	68.456	76.150
	41.337	46.842	53.113	60.27	68.452	77.826	88.583	100.947	115.182	131.597	150.56
	64.288	73.351	83.788	95.831	109.742	125.815	144.384	165.833	190.609	219.234	252.307
FFC	6.565	8.153	10.105	12.498	15.419	18.972	23.277	28.468	34.696	42.124	50.930
	19.434	22.731	26.595	31.124	36.435	42.662	49.959	58.502	68.493	80.155	93.734
	41.771	47.86	54.869	62.937	72.229	82.934	95.27	109.480	125.832	144.611	166.117
FFS	1.841	2.226	2.693	3.261	3.952	4.795	5.828	7.094	8.651	10.569	12.935
	12.943	14.979	17.336	20.072	23.253	26.963	31.298	36.381	42.354	49.392	57.701
	32.589	37.161	42.403	48.429	55.371	63.381	72.633	83.318	95.642	109.83	126.106
SCC	62.87	71.07	80.375	90.934	102.921	116.53	131.985	149.542	169.495	192.181	217.983
	84.284	96.44	110.362	126.276	144.453	165.197	188.848	215.795	246.483	281.421	321.207
	113.969	131.269	151.145	173.957	200.116	230.084	264.398	303.687	348.694	400.302	459.577
SFC	14.856	17.181	19.926	23.176	27.063	31.622	37.074	43.549	51.223	60.292	70.970
	35.606	40.586	46.288	52.824	60.324	68.936	78.833	90.209	103.285	118.311	135.566
	64.688	73.765	84.106	95.810	109.001	123.881	140.686	159.666	181.092	205.297	232.737
CSS	36.272	41.212	46.829	53.206	60.436	68.627	77.904	88.413	100.330	113.866	129.280
	59.532	68.214	78.172	89.584	102.655	117.616	134.735	154.316	176.715	202.35	231.722
	89.935	103.658	119.48	137.729	158.749	182.892	210.519	242.021	277.878	318.713	365.309
CFS	13.023	14.355	15.859	18.00	19.528	21.787	24.412	27.484	31.106	35.406	40.542
	33.379	37.594	42.357	47.745	53.849	60.777	68.657	77.639	87.902	99.653	113.133
	53.950	61.161	69.407	78.824	89.558	101.774	115.657	131.42	149.311	169.635	192.782
CSC	48.132	55.101	63.093	72.251	82.736	94.734	108.458	124.148	142.083	162.579	185.994
	73.884	84.830	97.391	111.789	128.271	147.117	168.638	193.184	221.145	252.961	289.142
	100.872	116.272	135.136	158.239	186.412	215.471	247.373	283.623	324.68	371.114	423.779

TABLE IV

First three frequencies for quarter of on elliptic plates (m=0.75, ν =0.3, N=28)

α	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
CCC	35.417	40.336	45.960	52.385	59.723	68.098	77.653	88.548	100.965	115.108	131.209
	56.506	65.088	74.951	86.271	99.244	114.085	131.029	150.328	172.259	197.122	225.272
	66.865	79.818	95.653	114.965	135.788	153.362	172.960	194.980	219.794	247.866	279.773
CCS	27.760	31.449	35.645	40.415	45.836	51.996	58.934	66.947	75.991	86.287	98.028
	46.658	53.747	61.912	71.307	82.106	94.501	108.705	124.954	143.510	164.673	188.798
	70.052	79.737	90.199	101.773	114.677	129.106	145.283	163.489	184.085	207.530	234.394
FCC	25.564	28.474	31.765	35.495	39.732	44.556	50.060	56.355	63.571	71.856	81.380
	38.368	43.605	49.608	56.492	64.392	73.556	83.858	95.788	109.465	125.129	143.050
	59.071	67.891	78.034	89.692	103.076	117.975	131.851	147.130	164.188	183.252	204.583
SSS	18.996	21.472	24.308	27.550	31.255	35.483	40.310	45.822	52.124	59.342	67.628
	36.649	42.028	48.217	55.333	63.508	72.890	83.644	95.956	110.034	126.108	144.431
	56.749	63.923	71.972	81.001	91.126	102.476	115.208	129.518	145.645	163.887	184.606
SSC	25.036	28.529	32.554	37.184	42.507	48.620	55.638	63.688	72.916	83.490	95.595
	45.016	51.680	59.336	68.123	78.194	89.719	102.888	117.907	135.002	154.420	176. 436
	67.798	76.586	86.473	97.599	110.115	124.189	140.014	157.811	177.831	200.408	225.903
FSC	16.225	18.644	21.396	24.530	28.108	32.200	36.893	42.287	48.496	55.653	63.906
	31.578	35.924	40.929	46.699	53.352	61.024	69.865	80.047	91.760	105.215	120.646
	52.563	60.162	68.374	77.046	86.600	97.256	109.168	122.498	137.418	154.126	172.849
CSC	28.407	32.406	37.000	42.275	48.325	55.261	63.208	72.307	82.719	94.624	108.227
	50.193	57.729	66.387	76.322	87.703	100.719	115.574	132.487	151.690	173.426	197.956
	67.419	76.986	88.776	103.084	116.986	132.112	149.118	168.275	189.895	214.357	242.131
CSS	21.873	24.790	28.123	31.928	36.270	41.223	46.872	53.317	60.677	69.091	78.728
	41.206	47.379	54.484	62.658	72.051	82.829	95.180	109.304	125.417	143.745	164.532
	60.348	68.119	76.829	86.596	97.551	109.849	123.680	139.277	156.938	177.044	200.080
CFC	16.063	18.002	20.230	22.799	25.769	29.211	33.212	37.867	43.289	49.605	56.958
	34.459	39.333	44.925	51.328	58.643	66.975	76.429	87.110	99.125	112.590	127.646
	43.695	48.979	54.954	61.720	69.400	78.137	88.108	99.527	112.648	127.765	145.206
CFS	11.659	12.830	14.139	15.608	17.262	19.132	21.257	23.686	26.478	29.708	33.471
	27.368	31.133	35.455	40.409	46.082	52.561	59.936	68.287	77.674	88.129	99.678
	35.898	40.000	44.602	49.770	55.587	62.152	69.595	78.086	87.856	99.213	112.529
SCC	32.293	36.637	41.600	47.270	53.746	61.139	69.577	79.204	90.186	102.707	116.978
	51.405	59.082	67.899	78.012	89.596	102.847	117.981	135.234	154.868	177.175	202.479
	77.135	88.830	101.468	114.509	129.027	145.290	163.527	183.990	206.967	232.794	261.867
SCS	25.097	28.282	31.899	36.004	40.662	45.949	51.949	58.764	66.512	75.336	85.405
	42.139	48.401	55.602	63.877	73.379	84.279	96.771	111.076	127.440	146.141	167.493
	65.588	75.617	85.603	96.324	108.263	108.263	136.567	153.342	172.200	193.463	217.538
SFC	11.090	12.763	14.695	16.935	19.541	22.581	26.134	30.291	35.157	40.847	47.492
	29.581	33.692	38.404	43.798	49.965	57.004	65.025	74.149	84.507	96.241	109.511
	37.604	42.496	48.066	54.412	61.646	69.896	79.308	90.051	102.316	116.324	132/329
SFS	7.430	8.404	9.493	10.718	12.102	16.935	19.541	17.539	19.936	22.733	26.020
	23.241	26.350	29.908	33.977	38.404	43.798	49.965	56.811	64.575	73.343	83.217
	30.137	33.864	38.080	42.854	48.066	54.412	61.383	69.331	78.413	88.830	100.835
FCS	19.271	21.156	23.237	25.535	28.075	30.886	34.005	37.477	41.356	45.713	50.633
	30.589	34.618	39.227	44.505	50.561	57.516	65.514	74.717	85.316	97.528	111.607
	49.252	56.601	65.081	74.866	86.056	96.066	106.568	118.195	131.086	145.395	161.316
FSS	11.371	12.911	14.615	16.500	18.585	20.894	23.460	26.321	29.526	33.138	37.235
	24.818	28.085	31.840	36.165	41.153	46.913	53.569	61.268	70.175	80.481	92.405
	43.453	49.276	55.331	61.193	69,193	77.252	86.183	96.086	107.071	119.279	132.885
FFC	3.690	4.568	5.544	6.961	8.565	10.513	12.870	15.707	19.108	23.161	27.963
	14.221	16.617	19.409	22.661	26.447	30.851	35.970	41.915	48.807	56.785	65.996
	23.203	26.940	31.212	36.085	41.635	47.948	55.122	63.269	72.515	83.001	
FFS	1.297	1.558	1.871	2.247	2.700	3.248	3.910	4.714	5.694	6.890	8.354
	10.017	11.636	13.508	15.675	18.185	21.095	24.473	28.397	32.962	38.275	44.460
	17.094	19.770	22.804	26.232	30.097	34.444	39.323	44.793	50.915	57.610	65.417

This is became of stiffness of the plate is increasing. These frequencies are also represented thought figures 3, 4 and 5. The three cases are represented through different colours.

Red colour represents CCC, blue colour represents CCS and FCC shows yellow colour. From the figures, it is observed that the three frequencies are in increasing form. Similar representations stand for figures 4 and 5 also.

To check the validity of computed results, source frequencies are compared in the case of clamped quarter of an elliptic plate. It is observed that the frequencies are matching up-to five significant digits.

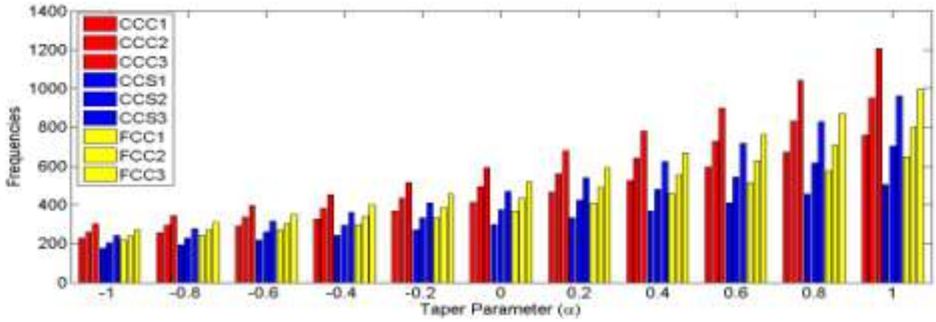

Figure 3 variation of frequency with CCC, CCS, FCC

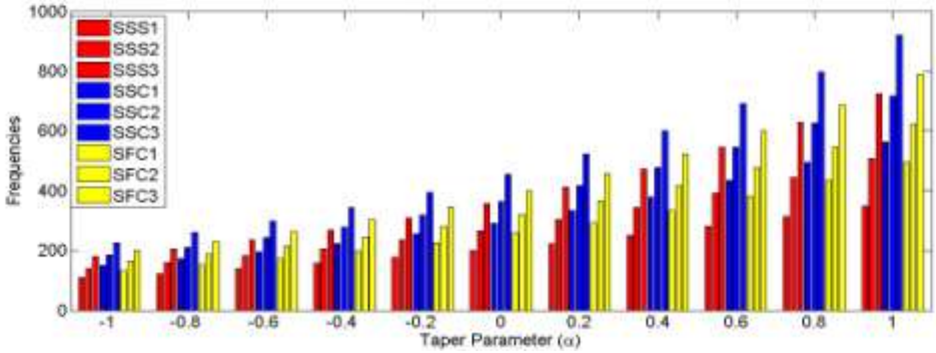

Figure 4 variation of frequency with SSS, SSC, FSC

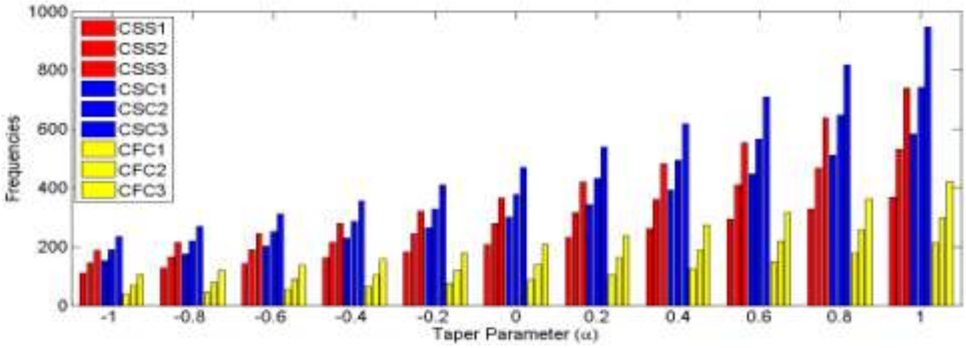

Figure 5 variation of frequency with CSS, CSC, CFC

Table V
Comparison of result for a quarter of circular plate with CCC (α =0) m=1

Ref	First three frequencies		
	I	II	III
Present	48.788	87.787	104.872
[17]	48.788	87.787	104.872
[10]	48.820	87.860	104.970
[13]	48.200	86.890	103.020
[14]	48.700	88.130	105.060
[15]	48.786	87.779	104.890

For checking the convergence of results the first three frequencies are computed by creating a matrix of order 5, 10, 15, 20, 25 and 28 and sound the first three frequencies are converging up-to five significant

Table VI
Convergence of result for CCC plate with (α =0)

N	λ_1	λ_2	λ_3
5	49.377	91.754	119.070
10	48.809	87.924	105.837
15	48.794	87.862	104.919
20	48.790	87.795	104.886
25	48.789	87.791	104.876
28	48.788	87.787	104.872

IV. Concluding Remarks

From the above work, it is observed that Rayleigh-Ritz technique is an efficient method used to solve the complex vibrational problems. The first three frequencies have been computed in some selected cases by varying the boundary conditions at the three edges of quarter of elliptic plate. Convergence of result shows the convergence of data up-to five significant digits. It is also concluded that as thickness variation is increasing then the frequencies are increasing in all the cases due to increase in the stiffness of the plate. In the case of CCC, the first three frequencies have been compared with the existing result available in the literature and up-to five significant digits; it is observed that the frequencies are matching up-to four significant digit.

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