# Closed Form Solutions of Poiseuille and Couette- Poiseuille Flow of Non-Newtonian Fluid Through Parallel Plates

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***Abstract***

*The velocity profiles with analytical solutions for the flow rates have been obtained through worked out solutions and are found to be accurate. The solution attributes to poiseuille and couette-poiseuille flow of a third grade fluid between two parallel plates. Such analytical solutions are almost equivalent to the corresponding numerical solutions. They are found to be rich in quality and comparitatively better than the approximate analytical solutions those were brought out in recent times. The impact of several parameters in respect of velocity profile and flow rate has been studied extensively in detail to conform to the process for further Research.*

**Keywords** — *poiseuille, couette-poiseuille flow, third grade fluid, Numerical solution, Closed form solution*

## I. INTRODUCTION

The characteristics, mainly simulates to have engineering applications viz. Polymer solutions, pastes and semi liquids (slurry) exhibit features of non-Newtonian engineering fluids. These fluids show different peculiar features with elastic temperament as shear thinning / thickening. Redundancy cannot be ruled out in explaining the equations relevant to Navier-stokes formulations about their rheological behavior approximately. Non-Newtonian flow behavior [1-4] has been suggestive to present models on rheological behavior. This has a characteristic of showing fluid model which also keeps a similar rheological pattern whereas the third grade fluid is categorized as a sub class under differential type fluid models. The capacity to store number of non-Newtonian effects are construed as the ability that has been retained as a source to innovation which can be applied for persuasive research which is ideal for investigation on the applied aspects of Thermodynamics [5-7], existence and uniqueness solutions [8-10] and certain other basic flow situations[11-16].

In recent times same of the nascent development, which are profiled have been adopted as analytical tools which are nomenclated as 'Adomian Decomposing' Method (ADM), 'Homotopy perturbation Method' (HPM), 'Hamotopy Analysis Method' (HAM) are applied to bring out solutions for solving basic flow problems of third grade fluid where velocity profiles are derived through solutions with similar characteristics [15-19 ] unlike the flow rate of velocity profile solutions for expression were not taken or secured to measure similar values distinctively.

In this paper exact analytical solutions for the velocity profile and flow rates have been obtained for the Poiseuille and couette-Poiseuille flow of a third grade fluid between two parallel plates. The observations are in compliance with the impacts of different parameters on velocity profiles and flow rates have also been discussed.

## II. MATHEMATICAL MODEL

Mathematical model of the present flow problem is given by the following equations and the details of their derivatives can be found elsewhere (13, 17,18)

| **Nomenclature** | |
|---|---|
| a | velocity of the plate(m/s) |
| A | dimensionless velocity of the plate,A= $\dfrac{a}{\left(-\dfrac{dp}{dy}\right)\dfrac{h^2}{\mu}}$ |
| B | dimensionless parameters considered in (17) |
| $C_1, C_2$ | constants of integration |
| 2h | separation between the two plates (m) |

| | |
|---|---|
| $K_1, K_2, K_3$ | constants |
| p | pressure ( $N/m^2$ ) |
| $\hat{p}$ | modified pressure ( $N/m^2$ ) |
| q | fluid flow rate per unit width of the plate ( $m^2/s$ ) |
| Q | dimensionless fluid flow rate per unit width of the plate |
| T | constant term |
| U | dimensionless fluid velocity |
| $U_0$ | Dimensionless maximum fluid velocity |
| $U_x, U_y, U_z$ | Fluid velocity in the x, y and z coordinates, respectively (m/s) |
| X | dimensionless distance in x direction |
| $X^*$ | Dimensionless distance where maximum fluid velocity occurs |
| x,y,z | distances in x,y and z directions ,respectively(m) |
| | Greek symbols |
| $\alpha_i, \beta_i$ | Material module (kg/m,kg.s/m) |
| $\beta$ | Dimensionless parameter |
| $\mu$ | Fluid viscosity (kg/m.s) |
| $\rho$ | Fluid density ( $kg/m^3$ ) |

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial x}\left[(2\alpha_1 + \alpha_2)\left(\frac{\partial u_y}{\partial x}\right)^2\right] \tag{1a}$$

$$\frac{\partial p}{\partial y} = \mu\frac{\partial^2 u_y}{\partial x^2} + 6\beta_3\left(\frac{\partial u_y}{\partial x}\right)\frac{\partial^2 u_y}{\partial x^2} \tag{1b}$$

$$\frac{\partial p}{\partial z} = 0 \tag{1c}$$

By defining a modified pressure as in [13,17,19]

$$\hat{p} = \hat{p} - (2\alpha_1 + \alpha_2)\left(\frac{\partial u_y}{\partial x}\right)^2$$

The above equations can be simplified into the following forms

$$\frac{\partial \hat{p}}{\partial x} = 0 \tag{2a}$$

$$\frac{\partial \hat{p}}{\partial x} = \mu\frac{\partial^2 u}{\partial x^2} + 6\beta_3\left(\frac{\partial u}{\partial x}\right)^2\frac{\partial^2 u}{\partial x^2} \tag{2b}$$

$$\frac{\partial \hat{p}}{\partial z} = 0 \tag{2c}$$

It is found that the Eqs. (2a)-(2c) remain same when compared with the different flow situations arising out of the present flow problem. However, the boundary condition problem, differ for each and result in different solutions .

## III. EXACT SOLUTIONS

### 3.1 Case 1:Pure poiseuille flow of 3rd grade fluid between two parallel plates

In this case the flow of fluid between parallel plates due to the due to the pressure gradient and the flow is given by Eq.(2b) along with the following BCs:

$$\text{BC I}: \mathrm{U}(h)=0 \text{ at } \mathrm{X}=h(\text{upper stationary plate}) \qquad (3a)$$
$$\text{BC II}: \mathrm{U}(-h)=0 \text{ at } \mathrm{X}=-h(\text{lower stationary plate}) \qquad (3b)$$

Introducing the below dimensionless parameters

$$X=\frac{x}{h}, \quad \beta=\beta_3\left(-\frac{\mathrm{d}\hat{\mathrm{p}}}{\mathrm{dy}}\right)^2\frac{h^2}{\mu^3}, \quad U=\frac{u}{\left(-\frac{\mathrm{d}\hat{\mathrm{p}}}{\mathrm{dy}}\right)\frac{h^2}{\mu}}$$

The Eqs.(2b),(3a) and (3b) becomes forms

$$\frac{d^2U}{dX^2}+6\beta\left(\frac{dU}{dX}\right)^2\frac{d^2U}{dX^2}=-1 \qquad (4a)$$
$$\text{BC I}: U(1)=0(\text{upper stationary plate}) \qquad (4b)$$
$$\text{BC II }: U(-1)=0(\text{lower stationary plate}) \qquad (4c)$$

BC II can be replaced by BC II'

i.e

$$\text{BC II}': \frac{dU}{dX}=0 \;\; at \quad X=0 \quad (\text{middle of the two plates}) \qquad (4c')$$

By using the transformation

$$\frac{dU}{dX}=f(U),$$

Where $f$(U) is some unknown function of U[20],the Eq.(4a)is reduced to the following form:

$$w'(1+6\beta w)=-2 \qquad (5)$$

Where $w=f^2(U)$ and $w'=\frac{dw}{dX}$ Eq.(5) is almost close the following exact solution:

$$w + 3\beta w^2 = 2U + C_1 \qquad (6)$$

$C_1$ is the constant of integration and by using BC II' it is found to be $C_1 = 2U_0 . U_0$ is the unknown dimensionless velocity at the centerline ,i.e $U_0 = U(0)$ and can be found by using BC I, put $C_1$ in Eq.(6) and solve for w; to get

$$w = f(U)^2 = \frac{dU}{dX} = (U')^2 = \frac{-1 \pm \sqrt{1 + 24\beta(U_0 - U)}}{6\beta} \qquad (7)$$

The express for U' is given by the negative sign of radical is omitted to avoid imaginary value of U':

$$\frac{dU}{dX} = U' = \pm\sqrt{\frac{-1 \pm \sqrt{1 + 24\beta(U_0 - U)}}{6\beta}} \qquad (8)$$

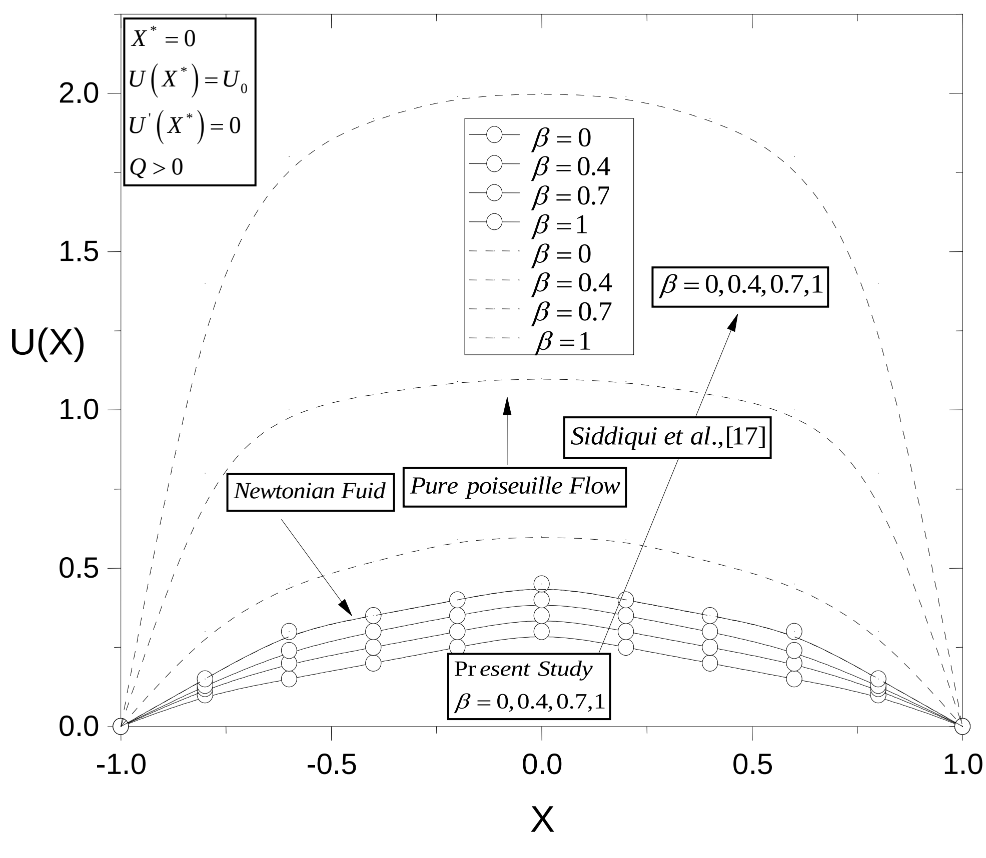


**Fig 1.Dimensionless Velocity Profiles for Pure Poiseuille Flow of 3rd Grade Fluid Between Two Parallel Plates, Solid Lines: Exact Solution; Open Circle: Numerical Solution; Broken Lines: Siddiqui Et Al.[17].**

For this case

$$\begin{cases} U'<0 & ;-1\le X<0 \\ U'<0 & ;0<X\le 1 \\ U'=0 & ;X=0 \end{cases}$$

[See Fig.1].since U is symmetric about X=0, hence one can select the region $0\le X\le 1$.for which the Eq. (8) becomes:

$$U'=\frac{dU}{dX}=\sqrt{\frac{-1+\sqrt{1+24(U_0-U)}}{6\beta}} \text{ for } 0\le X\le 1 \tag{9}$$

The explicit form of U can be obtained from equation(9)

$$U=\frac{1-T^2+24U_0\beta}{24\beta} \tag{10}$$

where

$$T=-1+\frac{2^{1/3}}{\left(2+K_1^2X^2+\sqrt{4K_1^2X^2+K_1^4X_1^4}\right)^{1/3}}+\frac{\left(2+K_1^2X^2+\sqrt{4K_1^2X^2+K_1^4X_1^4}\right)^{1/3}}{2^{1/3}}$$

And $K_1=-3\sqrt{6\beta}.U_0$ is found by BC I and is given as follows:

$$U_0=\frac{1}{12\beta}-\frac{-1+108\beta}{24\beta\left(-1-270\beta+1458\beta^2+6\sqrt{3}\sqrt{8\beta+324\beta^2+4374\beta^3+19683\beta^4}\right)^{1/3}}+\frac{\left(-1-270\beta+1458\beta^2+6\sqrt{3}\sqrt{8\beta+324\beta^2+4374\beta^3+19683\beta^4}\right)^{1/3}}{24\beta} \tag{11}$$

In design of pumps and piping flow rate is essential. The quantity 'Q' indicate the flow rate per unit width of plates and is given by

$$Q=\int_{-1}^{1}UdX \tag{12}$$

where $Q=\dfrac{q}{\left(\dfrac{d\hat{p}}{dy}\right)\dfrac{h^3}{\mu}}$ due to the complex from of U $\left[Eq.(10)\right]$

Using the symmetry, we get

$$Q=2\int_0^1 UdX=\int_{U_0}^{0}U\frac{1}{\frac{dU}{dX}}dU \tag{13}$$

From Eqs.(9)and (13),it can be observe that flow rate Q is in positive 'Y' direction

$$Q=-\frac{\sqrt{-1+\sqrt{1+24\beta U_0}}\left(-1+\sqrt{1+24\beta U_0}-24U_0\beta(13+5\sqrt{1+24\beta U_0}\right)}{315\sqrt{6}\beta^{3/2}} \qquad (14)$$

### 3.1.1. Discussion and comparison of results

For a limiting case of $\beta$ =0, the fluid behaves as a Newtonian fluid and one finds the established results.[1]: $U=\frac{1}{2}\left(1-X^2\right), U_0=\frac{1}{2}$ and $Q=\frac{2}{3}$ similarly, for $\beta=\infty$, one can conclude that with the increase in $\beta$ ,U decreases and so as Q. this fact is also evident from the Fig.1.

The results are compared with the HPM results of.[17].inspite of the obtained results it should be noted that dimensionless variables defined in .[17]are slightly different yet for B=1 (a dimensionless parameter considered in .[17]the relevant equation and the related BCs of .[17] become similar to those in the present study i.e.Eqs.(26) and (28) of .[17] and the present Eqs.(4a)-(4c)are identical to B=1.Moreover ,in .[17] heat transfer effects were considered for the present problem ,while the present work shall exclusively focused on the isothermal situation .Inspiteof the variation, the current contrast occurred while comparing which is substantiated as the works of siddiqui et al. .[17]have assumed that the equation of momentum is unique by itself which is not dependent on temperature and will be handled exclusively. It culminates into having the same velocity profiles whether the heat transfer effects are for? yet it can be observed that the assumption as viewed to consider in .[17]is not correct for higher temperature gradients as the density and viscosity will vary ,and the momentum equations cannot be handled in segregation. Hence, the solutions found in.[17] are only ideal to situations where lesser effects of temperature are prevalent .the HPM solution of .[17]for the velocity profile is shown below for B=1.

**Table 1-Comparison of the Flow Rates Per Unit Width of the Plate For The Pure Poiseuille Flow of 3rd Grade Fluid Between Two Parallel Plates.**

| $\beta$ | Q | | | %Error | |
|---|---|---|---|---|---|
| | *HPM solution siddiqui et at.* [17] | Exact solution | Numerical Solution | HPM Solution Siddiqui et al. [17] | Exact solution |
| | A=0 | | | | |
| 0 | 0.66667 | 0.66667 | 0.66667 | 0 | **0** |
| 0.4 | 0.89524 | 0.52465 | 0.52465 | -70.635 | **0** |
| 0.7 | 1.78667 | 0.48019 | 0.48019 | -272.073 | **0** |
| 1 | 3.29524 | 0.45013 | 0.45013 | -632.071 | **0** |

$$U_{HPM}=\frac{1}{2}(1-X^2)-\frac{1}{2}\beta(1-X^4)+2\beta^2(1-X^6) \qquad (15)$$

For the same values of $\beta$ as considered in.[17], the velocity profiles, obtained by the exact, numerical and HPM solutions, are shown in fig.A close match is found between the velocity profiles obtained by exact and numerical solutions , whereas HPM velocity profiles match only for $\beta$ =0, depicting an opposite trend as $\beta$ increases . Table 1 compares the values of Q obtained by the exact, numerical and HPM solution resulted in the following expression for Q:

$$Q_{HPM}=\int_{-1}^{1}U_{HPM}\,dX=\frac{2}{3}-\frac{4}{5}\beta+\frac{24}{7}\beta^2 \qquad (16)$$

### 3.2. Case 2: Couette-poiseuille flow of 3rd grade fluid between two parallel plates

In this case fluid flow between parallel plates due to moving of upper plate and the flow is given by equation (2b) with the BC'S The governing equation for this situation will remain same as that of the previous case $[Eq.(2b)]$ ,however, the following different BCs will be used:

$$\text{BC I}: \text{U}(h) = a \quad at\ X = h(\text{upper moving plate}) \tag{17a}$$

$$\text{BC I}: \text{U}(-h) = 0 \quad at \quad \text{X} = -h(\text{Lower stationary plate}) \tag{17b}$$

Positive (negative) value of' a' indicates that the upper plate moves in positive (negative)'Y' direction .Eqs.(2b),(17a) and (17b) are transformed into following dimensionless forms ;A is the dimensionless plate velocity:

$$\frac{d^2U}{dX^2} + 6\beta\left(\frac{dU}{dX}\right)^2 \frac{d^2U}{dX^2} = -1 \tag{18a}$$

$$\text{BC I}: U(1) = A \quad at\ X = 1(\text{Upper moving plate}) \tag{18b}$$

$$\text{BC II}: \text{U}(\text{-}1) = 0 \quad at\ X = -1\ (\text{Lower stationary plate}) \tag{18c}$$

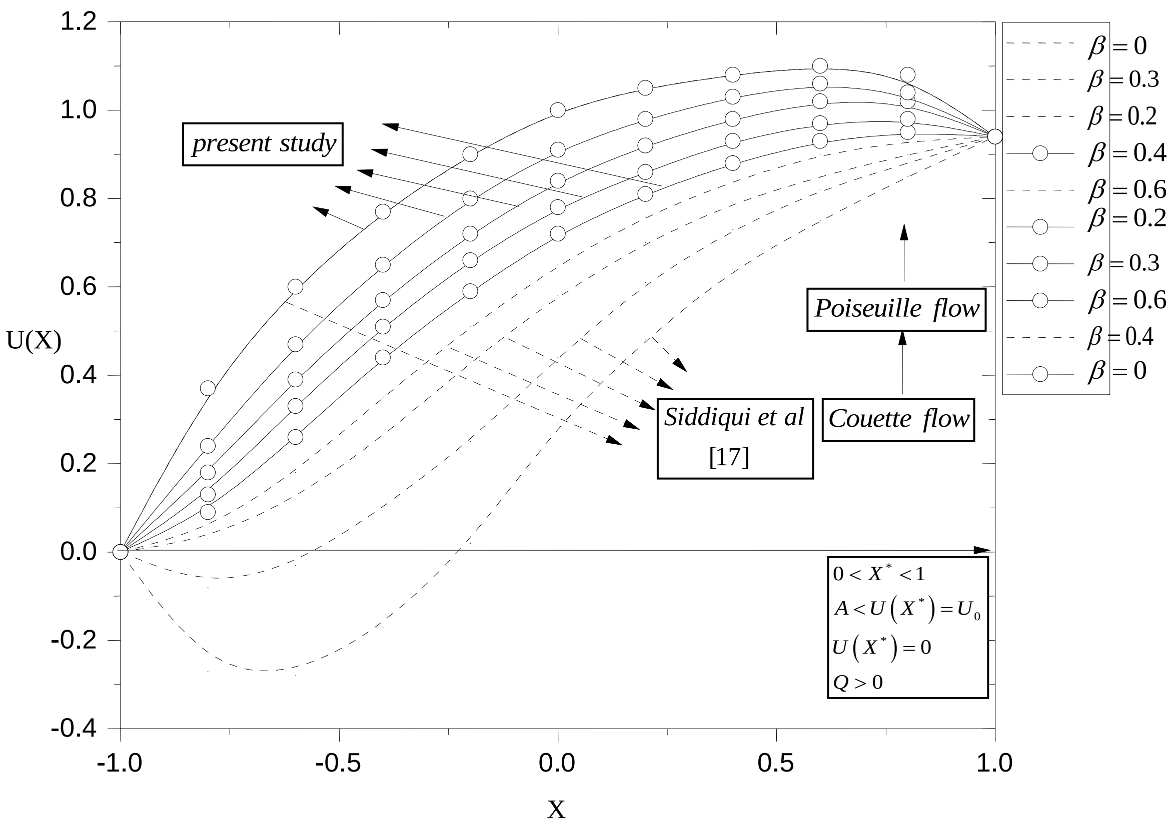


**Fig.2a.Dimensionsless Velocity Profiles For Coquette-Poiseuille Flow Of 3rd Grade Fluid Between Two Parallel Plates (Upper Moving Plate Is Moving Slowly In The Positive Y Direction), Solid Lines; Exact Solution; Open Circle: Numerical Solutions Broken Lines: Siddiqui Et Al. .[17]**

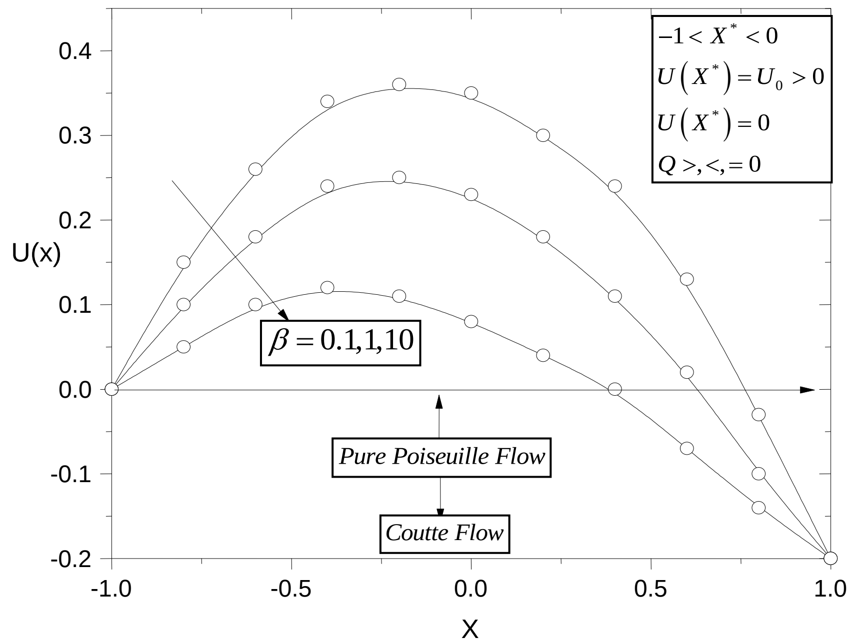


**Fig.2b. Dimensionless Velocity Profiles For Coquette-Poiseuille Flow Of 3rd Grade Fluid Between Two Parallel Plates (Upper Plate Is Moving Slowly In The Negative Direction), Solid Lines: Exact Solution; Open Circle: Numerical Solution.**

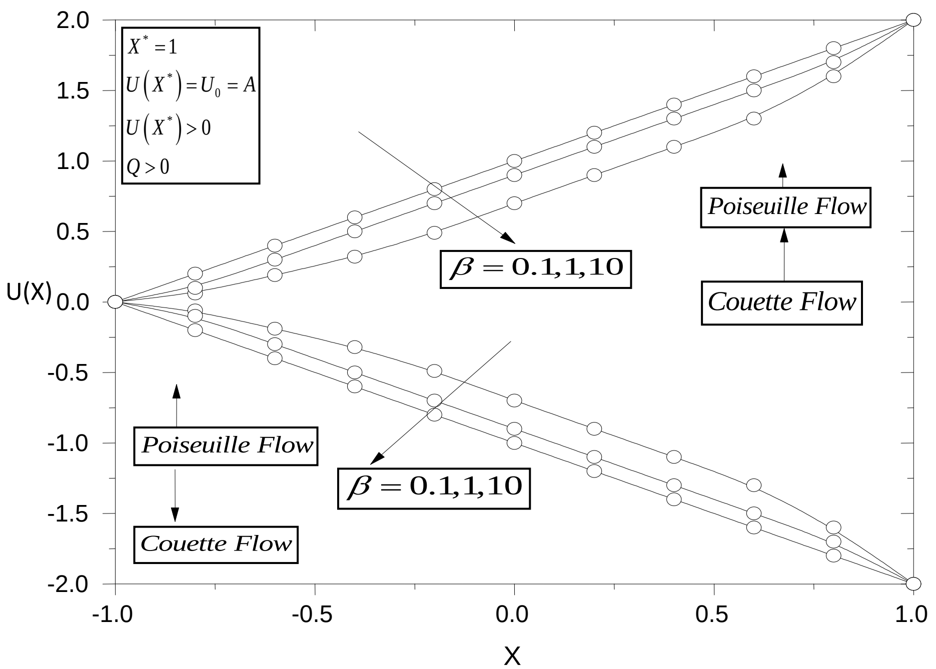


**Fig .2c.Dimensionless Velocity Profiles For Coquette-Poiseuille Flow Of 3rd Grade Fluid Between Two Parallel Plates (Upper Plate Is Moving Quickly In The Negative Direction),Solid Lines : Exact Solution ;Open Circle :Numerical Solution.**

$U'$ is given by:

$$U'=\frac{dU}{dX}=\pm\sqrt{\frac{-1+\sqrt{1+12\beta(C_1-2U)}}{6\beta}} \tag{19}$$

Where $C_1$ is the constant of integration .Unlike the previous care ,the velocity profile for this configuration will not be symmetric around X=0,i.e U'(X=0)$\neq$0.insted ,U'=0 at some unknown position $X=X^*$,where the fluid velocity will be maximum $(U^0)$ .In fact, $X^*$ depends on the plate velocity and can lie inside or outside the region of interest $(-1\leq X\leq 1)$ .Due to this ,one needs to consider the whole region ,i.e. $-1\leq X\leq 1$ .following three situations may arise in the present case:

(i) Upper plate moves in positive y direction but $A<U_0$ or $0<X^*<1$ (see Fig.2a).however, if the upper plate moves in the negative y direction but with $U_0$ >0, then -1< $X^*$ <0 [see Fig.2b].In both these situations, $U'\left(X^*\right)=0$.

(ii) Upper plate moves in the positive y direction but with a velocity higher enough that thus $\mathrm{U_0}$=A Thus $\mathrm{X^*}$ .However $\mathrm{U}'\left(\mathrm{X}^*\mathrm{=1}\right)\mathrm{>0}\left[\text{See Fig.2c}\right]$

(iii) Upper plate moves in the negative y direction but with a velocity higher enough that $U_0=0$ .hence. $X^*=-1$ and $U'(X^*=-1)<0\left[See\ Fig.2c\right]$.

**3.2.1 Case2 (a):** Upper plate moves with a show velocity in any direction ($|A|$ is small)

Solution of this can be obtained by the same methodology of case .1.However .for brevity we present the solutions and the details may be obtained from the authors.

$$U=\frac{1-T^2+24U_0\beta}{24\beta} \tag{20}$$

Where

$$T=-1+\frac{2^{1/3}}{\left(2+K_1^2(X-X^*)^2+\sqrt{4K_1^2(X-X^*)^2+K_1^4(X-X^*)^4}\right)^{1/3}}+\frac{\left(2+K_1^2(X-X^*)^2+\sqrt{4K_1^2(X-X^*)^2+K_1^4(X-X^*)^4}\right)^{1/3}}{2^{1/3}}$$

And $K_1=-3\sqrt{6\beta}$

The unknown $U_0$ and $X^*$ can be found from the BCs.Q is given by:

$$Q=-\frac{\sqrt{-1+\sqrt{1+24\beta U_0}}\left(-1+\sqrt{1+24\beta U_0}-24\beta U_0\left(13+5\sqrt{1+24\beta U_0}\right)\right)}{630\sqrt{6}\beta^{3/2}}+\frac{\sqrt{-1+\sqrt{1+24(U_0-A)}}}{630\sqrt{6}\beta^{3/2}}$$
$$\times\left[1-\sqrt{1+24(U_0-A)}+18A\beta\left(6+5\sqrt{1+24(U_0-A)}\right)+24\beta U_0\left(13+5\sqrt{1+24(U_0-A)}\right)\right] \tag{21}$$

Depending on the value of A,Q can be positive or negative, which signifies that the net flow is in the positive and negative y directions ,respectively.

**3.2.1.1 Discussion and comparison of the results.**

Figs .2a and 2b reveal an agreement between the velocity profiles obtained by exact and numerical solutions. For $\beta=\infty:\mathrm{U}(X)+\frac{A}{2}(1+X)$ and Q=A ,whereas ,for $\beta=0$ (Newtonian fluid): $U(X)=\frac{A}{2}\left(1+A+AX-X^2\right)$ and $Q=\frac{2}{3}+A$ .this means that with the increase in $\beta$ ,the flow tends to be a coquette flow and results in the decrease in Q,since A>0, Q will be maximum for Newtonian fluid . This fact is also supported by Figs.2a and 2b.

The results of this case have also been compared against the available HPM solution .[17].the HPM solution for velocity profile $(U_{HPM})$ .given by Eq.(72) in .[17], is reproduced below for B=1:

$$U_{HPM}=\frac{(1+X)}{2}+\frac{1}{2}(1-X^2)+\frac{\beta}{4}\left(3(-1+X^2)+4(X-X^3)+2(-1+X^4)\right) \qquad (22)$$

Similarly, the HPM expression of flow rate is given below:

**Table 2-Comparison of the Rates Per Unit Width of the Plate for the Coquette-Poiseuille Flow of 3rd Grade Fluid Between Two Plates**

| $\beta$ | Q | | | %Error | |
|---|---|---|---|---|---|
| | HPM solution siddiqui et al.[17] | Exact solution | Numerical solution | HPM solution siddiqui et al.[17] | Exact solution |
| | A=1 | | | | |
| 0 | 1.66667 | 1.66667 | 1.66667 | 0 | 0 |
| 0.2 | 1.30667 | 1.4866 | 1.4866 | 12.103 | 0 |
| 0.3 | 1.12667 | 1.44032 | 1.44032 | 21.77641 | 0 |
| 0.4 | 0.946667 | 1.40413 | 1.40413 | 32.57982 | 0 |
| 0.6 | 0.586667 | 1.34904 | 1.34904 | 56.51226 | 0 |
| $\beta$ | A=-1 | | | | |
| 0 | - | -0.333333 | -0.333333 | - | 0 |
| 0.2 | - | -0.513405 | -0.513405 | - | 0 |
| 0.3 | - | -0.559677 | -0.559677 | - | 0 |
| 0.4 | - | -0.595873 | -0.595873 | - | 0 |
| 0.6 | - | -0.650963 | -0.650963 | - | 0 |
| | | | | | |

Not evaluated.

$$Q_{HPM}=\int_{-1}^{1}U_{HPM}\,dX=\frac{5}{3}-\frac{9}{5}\beta \qquad (23)$$

Discrepancies in $U_{HPM}$ and $Q_{HPM}$ are visible in Fig .2a and Table 2 respectively.

**3.2.2 Case 2(b):** Upper plate moves in positive y direction with a high velocity (A>0 and A is large )
For this case, one finds the following explicit relation for U.

$$U=\frac{1-T^2+12C_1\beta}{24\beta} \qquad (24)$$

*Where*

$$T=-1+\frac{3(2)^{1/3}}{\left(K_3+\sqrt{-2916+K_3^{\,2}}\right)^{1/3}}+\frac{\left(K_3+\sqrt{-2916+K_3^{\,2}}\right)^{1/3}}{3(2)^{1/3}},$$

$$K_1=-3\sqrt{6\beta},$$

$$K_2=\sqrt{-1+\sqrt{1+12C_1\beta}}\left(2+\sqrt{1+12C_1\beta}\right) \textit{and}$$

$$K_3=54+27K_1^{\,2}+54K_1K_2+27K_2^{\,2}+54K_1^{\,2}X+54K_1K_2X+24K_1^{\,2}X^2$$

Unknown constants can be found from the BCs, and Q is given as follows:

$$Q=-\frac{\sqrt{-1+\sqrt{1+12C_1\beta}}\left(-1+\sqrt{1+12C_1\beta}-12C_1\beta\left(13+5\sqrt{1+12C_1\beta}\right)\right)}{630\sqrt{6}\beta^{3/2}}+\frac{\sqrt{-1+\sqrt{1+12C_1\beta-24A\beta}}}{630\sqrt{6}\beta^{3/2}}$$
$$\times\left[-1+\sqrt{1+12C_1\beta-24A\beta}-12C_1\beta\left(13+\sqrt{1+12C_1\beta-24A\beta}\right)-18A\beta\left(6+5\sqrt{1+12C_1\beta-24A\beta}\right)\right]$$
*(25)*

Since A>0, Q>0 and corresponds to the net flow in the positive y direction.

**3.2.2.1 Discussion of results.** Fig.2C shows that the velocity profiles obtained by the exact and numerical solutions are in a good agreement. Limiting values of $\beta(=\infty,0)$ yield the same expressions of U and Q as those in sub case 2(a).

**3.2.3 Case 2(c):** Upper plate moves in negative y direction with a high velocity $\left(\mathrm{A}<0 \text{ and} |\mathrm{A}| \text{ Is large}\right)$

For this sub case, the following explicit relation for U is obtained:

$$U=\frac{1-T^2+12C_1\beta}{24\beta} \tag{26}$$

where

$$T=-1+\frac{3(2)^{1/3}}{\left(K_3+\sqrt{-2916+K_3^{\,2}}\right)^{1/3}}+\frac{\left(K_3+\sqrt{-2916+K_3^{\,2}}\right)^{1/3}}{3(2)^{1/3}}$$

$$K_1=-3\sqrt{6\beta},K_2=\sqrt{-1+\sqrt{1+12C_1\beta}}\left(2+\sqrt{1+12C_1\beta}\right)\mathit{and}$$

$$K_3=54+27K_1^{\,2}-54K_1K_2+27K_2^{\,2}+54K_1^{\,2}X-54K_1K_2X+24K_1^{\,2}X^2$$

The unknown constants can be found from the BCs and Q is given by:

$$Q=-\frac{\sqrt{-1+\sqrt{1+12C_1\beta}}\left(-1+\sqrt{1+12C_1\beta}-12C_1\beta\left(13+5\sqrt{1+12C_1\beta}\right)\right)}{630\sqrt{6}\beta^{3/2}}-\frac{\sqrt{-1+\sqrt{1+12C_1\beta-24A\beta}}}{630\sqrt{6}\beta^{3/2}}$$
$$\times\left[-1+\sqrt{1+12C_1\beta-24A\beta}-12C_1\beta\left(13+\sqrt{1+12C_1\beta-24A\beta}\right)-18A\beta\left(6+5\sqrt{1+12C_1\beta-24A\beta}\right)\right] \tag{27}$$

**3.2.3.1.Discussion of results .**Fig.2c validates the velocity profiles obtained by the exact solution .limiting values of $\beta(=\infty,0)$ gives the same expressions of U and Q as in the sub case 2(a).Since A<0,Q<0 for $\beta=\infty$.However for $\beta=0,Q>0$ or Q<0 depending on the magnitude of A.As A<0, $|Q|\left(=\left|\frac{2}{3}+A\right|\right)$ will be minimum for Newtonian fluid.

## CONCLUSION

An attempt has been made to discuss the closed form solution and numerical solution for the velocity profile of Poiseuille and Couette-Poiseuille flow of a third grade fluid between parallel plates. We have observed that the observation are in compliance with the velocity profiles and flow rates as discussed in [17] Siddiqui etal.

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