Solving Graphically Sine Wave Second Order Boundary Value Problem using Laplace Transform and Finite Difference Method

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Abstract
In this paper the FDM & LT method has been established for the numerical solution of a two-point second order boundary value problem's (BVP) are analyzed. Numerical solutions of both methods were implemented and are tabulated. Finally it was observed that the finite-difference method is numerically more strengthen and converges the nearer to LT solution by taking the lengthen intervals.

Keywords - *Boundary value problem (BVP), Laplace Transform (LT), Finite difference Method (FDM).*

I.INTRODUCTION

In Mathematics, Two-point bvp's have received a major concentration due to its priority in many areas of sciences and engineering field. Different types of differential equations arise very frequently in optimal control, aerodynamics, fluid mechanics, chemical–reactor theory, quantum mechanics, reaction-diffusion process and geophysics .

Different types of logical and computational ideas used for the result of differential equations are given in the survey literature; Differential Transform Method **[1-6],** Bernoulli Polynomials **[7]**, Adomain Decomposition Method **[8-13]**, Block Method **[14-16]**, Modified Picard Technique **[17]**, Sinc Collocation Method **[18],** Runge-kutta 4th Order Method **[19]**, Cubic Spline Method **[20]**, Homotopy Perturbation Method **[21-23]**.

In this Paper, we use FDM for the result of two-point boundary value problems has been extensively used **[24-27]**. In the example problems the step length is elongated and it is examined that the access strengthen the convergence of the result when compared with the explicit from Laplace Transforms (which gives a close form of solution), see Table 1.

II.EVALUATION OF FDM

Consider the second order BVP as,

$$\chi'' + s(\varrho)\chi' + t(\varrho)\zeta = u(\varrho), \;\; \varrho \in [\gamma, \delta] \tag{1}$$

With the boundary conditions

$$\zeta(\gamma) = \mathbf{M} \;\text{ and }\; \zeta(\delta) = \mathbf{N} \tag{2}$$

The period $[\boldsymbol{v}, \boldsymbol{w}]$ is partitioned into into n equal subintervals. The subintervals length is denoted as $\hbar$,

i.e) $\boldsymbol{h} = \frac{\delta-\gamma}{\boldsymbol{n}}$ **(3)**

Let us examine the following points

$$\boldsymbol{\gamma = \varrho_0,\; \varrho_1 = \varrho_0 + h, \varrho_2 = \varrho_0 + 2h, \ldots\ldots.. \varrho_m = \varrho_0 + mh, \ldots\ldots., \varrho_n = \varrho_0 + nh} \tag{4}$$

The analytical solution at any point $\boldsymbol{\varphi_m}$ is indicated by $\boldsymbol{\zeta_m}$ and abstract solution is denoted as $\boldsymbol{\chi(\varrho_m)}$.

The Central difference approximation for the differential equation is given below

$$\left.\begin{aligned} \chi_m{}' &= \frac{1}{2\hbar}[\chi_{m+1} - \chi_m] \\ \chi_m{}'' &= \frac{1}{\hbar^2}[\chi_{m+1} - 2\chi_m + \chi_{m-1}] \end{aligned}\right\} \tag{5}$$

Substitute (5) in (1)

$$\frac{1}{\hbar^2}[\chi_{m+1} - 2\chi_m + \chi_{m-1}] + s(\varrho)\frac{1}{2\hbar}[\chi_{m+1} - \chi_m] + t(\varrho)\chi_m = u(\varrho)$$

(6)

$$2[\chi_{m+1} - 2\chi_m + \chi_{m-1}] + \hbar s(\varrho)[\chi_{m+1} - \chi_{m-1}] + 2\hbar^2 t(\varrho)\chi_m = 2\hbar^2 u(\varrho)$$

(7)

Equation (7) can be written as

$$i_m\chi_{m-1} + j_m\chi_m + k_m\chi_{m+1} = l_m, \;\; m = 1,2,3 \ldots\ldots\ldots\ldots\ldots \tag{8}$$

Where

$$\left.\begin{aligned} i_m &= 2 - \hbar s(\varrho) \\ j_m &= -4 + 2\hbar^2 t(\varrho) \\ k_m &= 2 + \hbar s(\varrho) \\ l_m &= 2\hbar^2 w(\varrho) \end{aligned}\right\}$$

(9)

The following equations are obtained from (8)

$$i_1\chi_0 + j_1\chi_1 + k_1\chi_2 = l \quad \textbf{(10)}$$

$$\boldsymbol{i_2\chi_1 + j_2\chi_2 + k_2\chi_3 = l_2} \quad \textbf{(11)}$$

etc.

Hence the result of the above equations to a logical order of equations of the form $A\chi = l$ for the undistinguished $\chi_1, \chi_2, \chi_3, \ldots . \chi_{n-1}$, where A is the co-efficient matrix determine the logical order of equations above provide the results of the bvp's.

III.NUMERICAL EXAMPLES

A. Problem 3.1:

Evaluate the two-point BVP of $\chi''(\varrho) + \chi(\varrho) = 0, \chi'(0) = 1, \chi(\pi/2) = 0$, by Laplace transform and Finite difference method.

Solution:

Given: $\chi''(\varrho) + \chi(\varrho) = 1, \chi'(0) = 1, \chi(\pi/2) = 0$ **(12)**

The general solution of (12) is

$\chi(\varrho) = \sin\varrho$ **(13)**

Solving by Laplace Transform

Equation (12) gives

$L\{\chi''\} + L\{\chi\} = L\{0\}$ **(14)**

$s^2\chi - s\chi(0) - \chi'(0) + \chi = 0$ **(15)**

Let $L\{\chi(0)\} = \varsigma$ **(16)**

Substituting equation (16) in (15), we get

$s^2\chi - s\chi(0) - 1 + \chi = 0$ **(17)**

And simplifying, we obtain

$\chi = \frac{s\varsigma}{(s^2+1)} + \frac{1}{s^2+1}$ **(18)**

Converting into partial fraction ,

$\chi(\omega) = \frac{s\varsigma}{s^2+1} + \frac{\mathbf{1}}{\mathbf{s^2+1}}$ **(19)**

Taking inverse Laplace transform

$\chi(\omega) = \sin\varrho + \varsigma\cos\varrho$ **(20)**

Using $\chi(\pi/2) = 0$, we obtain

$0 = \sin\frac{\pi}{2} + \varsigma\cos\frac{\pi}{2}$ **(21)**

Which gives $\varsigma = 0$, then

$\chi(\varrho) = \sin\varrho + 0(\cos\varrho)$ **(22)**

Then, $\boldsymbol{\chi(\varrho) = \sin\varrho}$ **(23)**

Which is explicit solution

Solving by Finite difference Method

The following steps are written using equation **(12)**

i.e) $n = 10, h = \frac{\nu - \upsilon}{10} = \frac{\pi/2 - 0}{10} = \frac{\pi}{20}$ **(24)**

From the above we have

$\chi(0) = 1,\ \chi(\pi/20) = ?, \chi(2\pi/20) = ?, \ldots \ldots \ldots \ldots \ldots \chi(\pi/2) = 0$ **(25)**

Equation (12) is expressed by central difference approximations as follows

$\frac{400}{\pi^2}[\chi_{m+1} - 2\chi_m + \chi_{m-1}] + \chi_m = 0$ **(26)**

For

$m = 1,\ \chi_0 = 1: \quad (-800 + \pi^2)\chi_1 + 400\chi_2 = 0$ **(27)**

$$m = 2: \quad 400\chi_1 + (-800 + \pi^2)\chi_2 + 400\chi_3 = 0 \tag{28}$$
$$m = 3: \quad 400\chi_2 + (-800 + \pi^2)\chi_3 + 400\chi_4 = 0 \tag{29}$$
$$\vdots$$
$$m = 9: \quad 400\chi_{10} + (-800 + \pi^2)\chi_9 + 400\chi_8 = 0 \tag{30}$$

Deriving the logical of equations (27-30) gives the result of the bvp's; and the comparison with the nearer form result is presented in table 1.

IV. Table 1 NUMERICAL SOLUTION OF PROBLEM 1

n	LAPLACE TRANSFORM	FDM	ERROR
0	0	0	0
$\pi/20$	0.156434465	0.156594616	1.60149^{-04}
$2\pi/20$	0.309016994	0.309325411	3.08417^{-04}
$3\pi/20$	0.453990499	0.454423909	-4.3341^{-04}
$4\pi/20$	0.587785252	0.588309948	-5.24696^{-04}
$5\pi/20$	0.707106781	0.70768002	-5.73239^{-04}
$6\pi/20$	0.809016994	0.809588788	-5.71794^{-04}
$7\pi/20$	0.891006524	0.89152754	-5.1523^{-04}
$8\pi/20$	0.951056516	0.951457303	-4.00787^{-04}
$9\pi/20$	0.98768834	0.987916584	-2.28244^{-04}
$\pi/2$	1	1	0

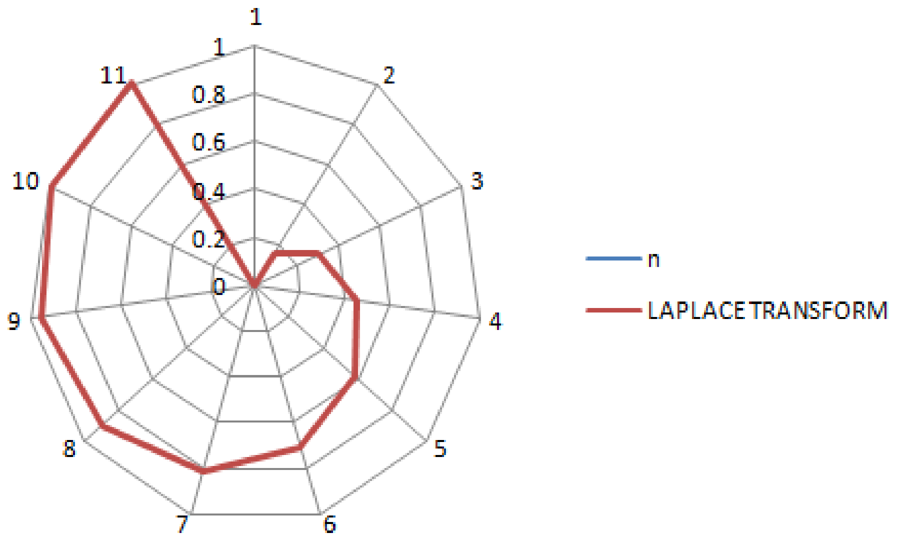

Fig 1 : n value and Laplace Transform

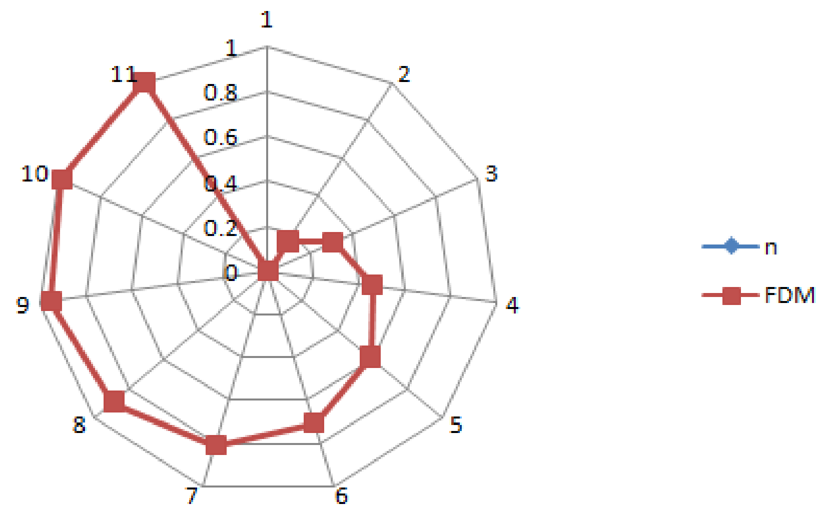

Fig 2: n value and FDM

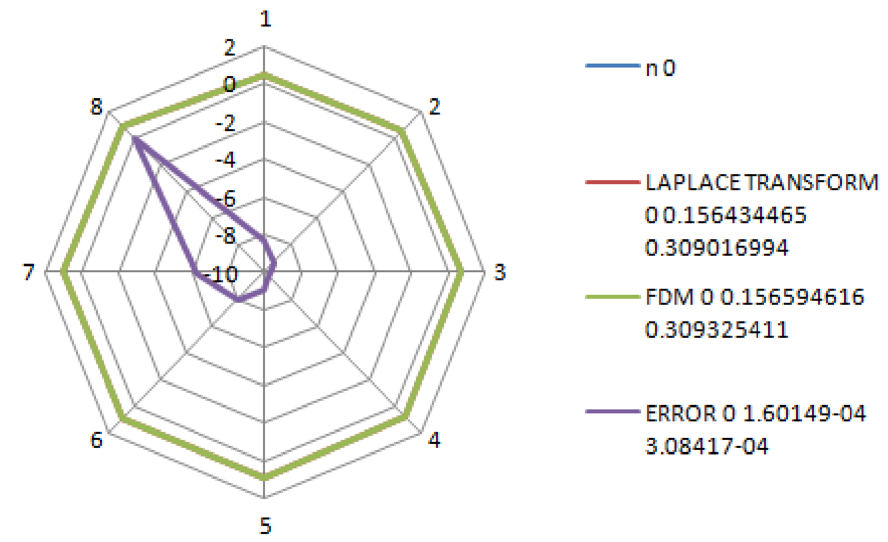

Fig 3: Comparison of n value, LT, FDM and Error

V.CONCLUSION

In this paper, LTM and FDM technique are proposed to solve two point boundary value problems. The step length is extended in FDM to enhance the convergence of the method; the results are compared with the close form solution of LT in table

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