Some New Multiplicative Connectivity Kulli-Basava Indices

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Abstract: *A topological index for a graph is used to determine some property of the graph of molecular by a single number. In this paper, we introduce the multiplicative sum connectivity Kulli-Basava index, multiplicative product connectivity Kulli-Basava index, multiplicative ABC Kulli-Basava index and multiplicative GA Kulli-Basava index of a graph. We compute these multiplicative connectivity Kulli-Basava indices of regular, wheel and helm graphs.*

Keywords: *Multiplicative sum connectivity Kulli-Basava index, multiplicative product connectivity Kulli-Basava index, multiplicative ABC Kulli-Basava index, multiplicative GA Kulli-Basava index, graph.*

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I. Introduction

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v will be denoted by uv. The degree of an edge $e=uv$ in a graph G is defined by $d_G(e)= d_G(u)+ d_G(v) – 2$. Let $S_e(v)$ denote the sum of the degrees of all edges incident to a vertex v. For undefined term and notation, we refer [1]. Topological indices or graph indices have their applications in various disciplines of Science and Technology.

In [2], Kulli introduced the first and second multiplicative Kulli-Basava indices of a graph, defined as

$$KB_1II(G)=\prod_{uv\in E(G)}\left[S_e(u)+S_e(v)\right],$$

$$KB_2II(G)=\prod_{uv\in E(G)}\left[S_e(u)S_e(v)\right].$$

Recently, the connectivity Kulli-Basava indices [3], square Kulli-Basava index [4], multiplicative *F*-Kulli-Basava index [5], first and second hyper Kulli-Basava indices [6], *F*-Kulli-Basava index [7] multiplicative hyper Kulli-Basava indices [8] were introduced and studied.

We introduce the multiplicative sum connectivity Kulli-Basava index and multiplicative product connectivity Kulli-Basava index of a graph, defined as

$$SKBII(G)=\prod_{uv\in E(G)}\frac{1}{\sqrt{S_e(u)+S_e(v)}},$$

$$PKBII(G)=\prod_{uv\in E(G)}\frac{1}{\sqrt{S_e(u)S_e(v)}}.$$

Also we propose the multiplicative atom bond connectivity Kulli-Basava index, multiplicative geometric-arithmetic Kulli-Basava index and multiplicative reciprocal Kulli-Basava index of a graph, defined as

$$ABCKBII(G)=\prod_{uv\in E(G)}\sqrt{\frac{S_e(u)+S_e(v)-2}{S_e(u)S_e(v)}},$$

$$GAKBII(G)=\prod_{uv\in E(G)}\frac{2\sqrt{S_e(u)S_e(v)}}{S_e(u)+S_e(v)},$$

$$RKBII(G)=\prod_{uv\in E(G)}\sqrt{S_e(u)S_e(v)}.$$

Recently, some connectivity indices were studied [9, 10, 11, 12, 14, 15, 16,17].

Finally, we introduce the general first and second Kulli-Basava indices of a graph, defined as

$$KB_1^aII(G)=\prod_{uv\in E(G)}\left[S_e(u)+S_e(v)\right]^a,$$

$$KB_2^a II(G) = \prod_{uv \in E(G)} \left[S_e(u) S_e(v) \right]^a,$$

where a is a real number.

Recently some multiplicative topological indices were studied [18, 19, 20, 21].In this paper, some multiplicative connectivity Kulli-Basava indices of regular, wheel, helm graphs are computed.

II. Results for regular graphs

A graph G is an r-regular graph if the degree of every vertex of G is r.

Theorem 1. Let G be an r-regular graph with n vertices and m edges. Then

$$KB_1^a II(G) = [4r(r-1)]^{am}. \tag{1}$$

Proof: If G is an r-regular graph with n vertices and m edges, then $S_e(u) = 2r(r - 1)$ for any vertex u in G. Thus,

$$KB_1^a II(G) = \prod_{uv \in E(G)} \left[S_e(u) + S_e(u) \right]^a = [2r(r-1) + 2r(r-1)]^{am}$$

$$= [4r(r-1)]^{am}$$

Corollary 1.1. If G is an r-regular graph with n vertices and m edges, then

$$SKBII(G) = \left[\frac{1}{\sqrt{4r(r-1)}} \right]^m.$$

Proof: Put $a = -½$ in equation (1), we get the desired result.

Corollary 1.2. If K_n is a complete graph with n vertices, then

$$SKBII(K_n) = \left[\frac{1}{2\sqrt{(n-1)(n-2)}} \right]^{\frac{n(n-1)}{2}}.$$

Proof: Put $r = n - 1$, $m = \frac{n(n-1)}{2}$ and $a = -\frac{1}{2}$ in equation (1), we get the desired result.

Corollary 1.3. If C_n is a cycle with n vertices, then

$$SKBII(C_n) = \left[\frac{1}{2\sqrt{2}} \right]^n.$$

Proof: Put $r = 2$, $m = n$ and $a = -\frac{1}{2}$ in equation (1), we get the desired result.

Theorem 2. Let G be an r-regular graph with n vertices and m edges. Then

$$KB_2^a II(G) = \left[4r^2 (r-1)^2 \right]^{am}. \tag{2}$$

Proof: Let G be an r-regular graph with n vertices and m edges. Then $S_e(u) = 2r(r - 1)$ for any vertex u in G. Therefore

$$KB_2^a II(G) = \prod_{uv \in E(G)} \left[S_e(u) S_e(v) \right]^a = [2r(r-1) 2r(r-1)]^{am} = \left[4r^2 (r-1)^2 \right]^{am}$$

Corollary 2.1. If G is an r-regular graph with n vertices and m edges, then

$$PKBII(G) = \left[\frac{1}{2r(r-1)} \right]^m.$$

Proof: Put $a = -½$ in equation (2), we get the desired result.

Corollary 2.2. If K_n is a complete graph with n vertices and m edges, then

$$PKBII(K_n) = \left[\frac{1}{2(n-1)(n-2)} \right]^{\frac{n(n-1)}{2}}.$$

Proof: Put $r = n - 1$, $m = \frac{n(n-1)}{2}$ and $a = -\frac{1}{2}$ in equation (2), we get the desired result.

Corollary 2.3. If C_n is a cycle with n vertices, then

$$PKBII\left(C_n\right)=\left[\frac{1}{4}\right]^n.$$

Proof: Put $r = 2$, $m = n$ and $a = -\frac{1}{2}$ in equation (2), we get the desired result.

Corollary 2.4. If G is an r-regular graph with n vertices and m edges, then

$$RKBII\left(G\right)=\left[2r\left(r-1\right)\right]^m.$$

Proof: Put $a = ½$ in equation (2), we get the desired result.

Corollary 2.5. If K_n is a complete graph with n vertices and m edges, then

$$RKBII\left(K_n\right)=\left[\frac{1}{2\left(n-1\right)\left(n-2\right)}\right]^{\frac{n(n-1)}{2}}.$$

Proof: Put $a = -\frac{1}{2}$, $r = n - 1$ $m=\frac{n\left(n-1\right)}{2}$ in equation (2), we get the desired result.

Corollary 2.6. If C_n is a cycle with n vertices and m edges, then

$$RKBII\left(C_n\right)=4^n.$$

Proof: Put $r = 2$, $m = n$, $a = -\frac{1}{2}$, in equation (2), we get the desired result.

Theorem 3. Let G be an r-regular graph with n vertices and m edges. Then

$$ABCKBII\left(G\right)=\left(\frac{\sqrt{4r\left(r-1\right)-2}}{2r\left(r-1\right)}\right)^m.$$

Proof: If G is an r-regular graph with n vertices and m edges, then $S_e(u) = 2r(r - 1)$ for any vertex u in G. Therefore

$$ABCKBII\left(G\right)=\prod_{uv\in E(G)}\sqrt{\frac{S_e\left(u\right)+S_e\left(v\right)-2}{S_e\left(u\right)S_e\left(v\right)}}$$

$$=\left(\sqrt{\frac{2r\left(r-1\right)+2r\left(r-1\right)-2}{2r\left(r-1\right)2r\left(r-1\right)}}\right)^m=\left(\frac{\sqrt{4r\left(r-1\right)-2}}{2r\left(r-1\right)}\right)^m$$

Theorem 4. Let G be an r-regular graph with n vertices and m edges. Then

$GAKBII(G) = 1$.

Proof: If G is an r-regular graph with n vertices and m edges, then $S_e(u) = 2r(r - 1)$ for any vertex u in G. Thus

$$GAKBII\left(G\right)=\prod_{uv\in E(G)}\frac{2\sqrt{S_e\left(u\right)S_e\left(v\right)}}{S_e\left(u\right)+S_e\left(v\right)}$$

$$=\left(\frac{2\sqrt{2r\left(r-1\right)2r\left(r-1\right)}}{2r\left(r-1\right)+2r\left(r-1\right)}\right)^m=1$$

III. Results for wheel graphs

A wheel W_n is the join of C_n and K_1. Then W_n has n+1 vertices and $2n$ edges. The vertices of C_n are called rim vertices and the vertex K_1 is called apex.

Lemma 5. Let W_n be a wheel with n+1 vertices and $2n$ edges. Then

$E_1 = \{uv$ □ $E(G) \mid S_e(u) = n(n+1), S_e(v) = n+9\}$, $|E_1| = n$.

$E_2 = \{uv$ □ $E(G) \mid S_e(u) = S_e(v) = n + 9\}$, $|E_2| = n$.

Theorem 6. Let W_n be a wheel with n+1 vertices and $2n$ edges, n□3. Then

$$KB_1^aII\left(W_n\right)=\left(n^2+2n+9\right)^{an}\times\left(2n+18\right)^{an}. \quad (3)$$

Proof: Let W_n be a wheel with n+1 vertices and $2n$ edges. By using definition and Lemma 5, we deduce

$$KB_1^a II(W_n) = \prod_{uv\in E(W_n)} \left[S_e(u)+S_e(v)\right]^a = \left[n(n+1)+n+9\right]^{an} \times \left[(n+9)+(n+9)\right]^{an}$$
$$= \left(n^2+2n+9\right)^{an} \times (2n+18)^{an}$$

Corollary 6.1. If W_n is a wheel graph with n+1 vertices and 2n edges, then

$$SKBII(W_n) = \left(\frac{1}{\sqrt{n^2+2n+9}}\right)^n \times \left(\frac{1}{\sqrt{2n+18}}\right)^n.$$

Proof: Put $a = -\frac{1}{2}$ in equation (3), we get the desired result.

Theorem 7. Let W_n be a wheel with n+1 vertices and 2n edges, n□3. Then

$$KB_2^a II(W_n) = \left[n(n+1)\right]^{an} \times (n+9)^{3an}. \tag{4}$$

Proof: Let W_n be a wheel with n+1 vertices and 2n edges. By using definition and Lemma 5, we derive

$$KB_2^a II(W_n) = \prod_{uv\in E(W_n)} \left[S_e(u)S_e(v)\right]^a = \left[n(n+1)(n+9)\right]^{an} \times \left[(n+9)(n+9)\right]^{an}$$
$$= \left[n(n+1)\right]^{an} \times (n+9)^{3an}.$$

Corollary 7.1. Let W_n be a wheel with n+1 vertices and 2n edges. Then

i) $$PKBII(W_n) = \left(\frac{1}{\sqrt{n(n+1)}}\right)^n \times \left(\frac{1}{n+9}\right)^{\frac{3}{2}n}.$$

ii) $$RKBII(W_n) = \left(\sqrt{n(n+1)}\right)^n \times (n+9)^{\frac{3}{2}n}.$$

Proof: Put $a = -\frac{1}{2}, \frac{1}{2}$ in equation (4), we get the desired results.

Theorem 8. Let W_n be a wheel with n vertices and 2n edges. Then

$$ABCKBII(W_n) = \left(\frac{n^2+2n+7}{n(n+1)(n+7)}\right)^{\frac{n}{2}} \times \left(\frac{2n+16}{(n+9)^2}\right)^{\frac{n}{2}}$$

Proof: By using definition and Lemma 5, we obtain

$$ABCKBII(W_n) = \prod_{uv\in E(G)} \sqrt{\frac{S_e(u)+S_e(v)-2}{S_e(u)S_e(v)}}$$
$$= \left(\sqrt{\frac{n(n+1)+n+9-2}{n(n+1)(n+9)}}\right)^n \times \left(\sqrt{\frac{n+9+n+9-2}{(n+9)(n+9)}}\right)^n$$
$$= \left(\frac{n^2+2n+7}{n(n+1)(n+9)}\right)^{\frac{n}{2}} \times \left(\frac{2n+16}{(n+9)^2}\right)^{\frac{n}{2}}.$$

Theorem 9. Let W_n be a wheel with n+1 vertices and 2n edges. Then

$$GAKBII(W_n) = \left(\frac{2\sqrt{n(n+1)(n+9)}}{n^2+2n+9}\right)^n.$$

Proof: By using definition and Lemma 5, we obtain

$$GAKBII(W_n) = \prod_{uv\in E(W_n)} \frac{2\sqrt{S_e(u)S_e(v)}}{S_e(u)+S_e(v)}$$

$$=\left(\frac{2\sqrt{n(n+1)(n+9)}}{n(n+1)+(n+9)}\right)^{n}\times\left(\frac{2\sqrt{(n+9)(n+9)}}{(n+9)+(n+9)}\right)^{n}$$
$$=\left(\frac{2\sqrt{n(n+1)(n+9)}}{n^{2}+2n+9}\right)^{n}.$$

IV. Results for helm graphs

A helm graph H_n is a graph obtained from W_n by attaching an end edge to each rim vertex. We see that H_n has $2n+1$ vertices and $3n$ edges.

Lemma 10. Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. Then H_n has three types of edges as

$E_1 = \{uv \in E(H_n) \mid S_e(u) = n(n+2), S_e(v) = n+17\}$, $|E_1| = n$.
$E_2 = \{uv \in E(H_n) \mid S_e(u) = S_e(v) = n + 17\}$, $|E_2| = n$.
$E_3 = \{uv \in E(H_n) \mid S_e(u) = n+17, S_e(v) = 3\}$, $|E_3| = n$.

Theorem 11. Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. Then

$$KB_1^aII(H_n)=(n^2+3n+17)^{an}\times(2n+34)^{an}\times(n+20)^{an}. \tag{5}$$

Proof: Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. Then

$$KB_2^aII(H_n)=\prod_{uv\in E(H_n)}\left[S_e(u)+S_e(v)\right]^a$$
$$=\left[n(n+1)+n+17\right]^{an}\times\left[(n+17)+(n+17)\right]^{an}\times\left[n+17+3\right]^{an}$$
$$=(n^2+3n+17)^{an}\times(2n+34)^{an}\times(n+20)^{an}$$

Corollary 11.1. If H_n is a helm graph with $2n+1$ vertices and $3n$ edges, then

$$SKBII(H_n)=\left(\frac{1}{n^2+3n+17}\right)^{\frac{n}{2}}\times\left(\frac{1}{2n+34}\right)^{\frac{n}{2}}\times\left(\frac{1}{n+20}\right)^{\frac{n}{2}}$$

Proof: Put $a = -\frac{1}{2}$ in equation (5), we get the desired result.

Theorem 12. Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. Then

$$KB_2^aII(H_n)=[n(n+2)(n+17)]^{an}\times(n+17)^{2an}\times[3(n+17)]^{an}. \tag{6}$$

Proof: By using definition and Lemma 10, we deduce

$$KB_2^aII(H_n)=\prod_{uv\in E(H_n)}\left[S_e(u)S_e(v)\right]^a$$
$$=[n(n+2)(n+17)]^{an}\times[(n+17)(n+17)]^{an}\times[(n+17)3]^{an}$$
$$=[n(n+2)(n+17)]^{an}\times(n+17)^{2an}\times[3(n+17)]^{an}.$$

Corollary 12.1. Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. Then

i) $$PKBII(H_n)=\left(\frac{1}{n(n+2)(n+17)}\right)^{\frac{n}{2}}\times\left(\frac{1}{n+17}\right)^{n}\times\left(\frac{1}{n(n+17)}\right)^{\frac{n}{2}}.$$

ii) $$RKBII(H_n)=[n(n+1)(n+17)]^{\frac{n}{2}}\times(n+17)^{n}\times[3(n+17)]^{\frac{n}{2}}.$$

Proof: Put $a = -\frac{1}{2}, \frac{1}{2}$ in equation (6), we get the desired results.

Theorem 13. Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. Then

$$ABCKBII(H_n)=\left(\frac{n^2+3n+15}{n(n+2)(n+17)}\right)^{\frac{n}{2}}\times\left(\frac{2n+32}{(n+17)^2}\right)^{\frac{n}{2}}\times\left(\frac{n+18}{3(n+17)}\right)^{\frac{n}{2}}$$

Proof: By using definition and Lemma 10, we deduce

$$ABCKBII(H_n) = \prod_{uv \in E(H_n)} \sqrt{\frac{S_e(u)+S_e(v)-2}{S_e(u)S_e(v)}}$$

$$= \left(\frac{n(n+2)+n+17-2}{n(n+2)(n+17)}\right)^{\frac{n}{2}} \times \left(\frac{n+17+n+17-2}{(n+17)(n+17)}\right)^{\frac{n}{2}} \times \left(\frac{n+17+3-2}{3(n+17)}\right)^{\frac{n}{2}}$$

$$= \left(\frac{n^2+3n+15}{n(n+2)(n+17)}\right)^{\frac{n}{2}} \times \left(\frac{2n+32}{(n+17)^2}\right)^{\frac{n}{2}} \times \left(\frac{n+18}{3(n+17)}\right)^{\frac{n}{2}}.$$

Theorem 14. If H_n is a helm graph with $2n+1$ vertices and $3n$ edges, then

$$GAKBII(H_n) = \left(\frac{2\sqrt{n(n+2)(n+17)}}{n^2+3n+17}\right)^n \times \left(\frac{2\sqrt{3(n+17)}}{n+20}\right)^n.$$

Proof: By using definition and Lemma 5, we obtain

$$GAKBII(H_n) = \prod_{uv \in E(H_n)} \frac{2\sqrt{S_e(u)S_e(v)}}{S_e(u)+S_e(v)}$$

$$= \left(\frac{2\sqrt{n(n+2)(n+17)}}{n(n+2)+(n+17)}\right)^n \times \left(\frac{2\sqrt{(n+7)(n+17)}}{n+17+n+17}\right)^n \times \left(\frac{2\sqrt{(n+17)3}}{n+17+3}\right)^n$$

$$= \left(\frac{2\sqrt{n(n+2)(n+17)}}{n^2+3n+17}\right)^n \times \left(\frac{2\sqrt{3(n+17)}}{n+20}\right)^n.$$

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