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# Quantitative Study of Time Dilation

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***Abstract****: This paper demonstrates one among the fundamental consequences of special relativity theory particularly, the time dilation property. The scientific conceptualization and mathematics of time dilation explain in broader sense. Mathematical derivation of time dilation derives using Lorentz transformations. Time dilation encompasses the age-old mystery of life within the space (Twin Paradox) and the time constraints that they follow that ought to with the time standards followed by our universe. Einstein proposed the idea of time dilation in 1905. With his concept of time, this paper also discussed the facts behind time travel or if it is possible in future and explode the mathematics of time!*

**Keywords —** *Time dilation, special relativity, Lorentz transformation, time travel, mathematical derivation*

## I. INTRODUCTION

Time dilation is very important and really popular aspect of Lorentz transformation. For outlining time dilation perhaps, the better place to start out is definition of time. What is time? What is it that clocks are measuring? It seems to measure unseen medium, flow which never stops or slow for anyone. We experience time is merely an illusion. Time is self -evident. Even if we don't understand what exactly time is, we will precisely measure what time it is. Numerous standards have been set up to keep our lives running smoothly. In the early 1900s, Einstein presented a revolutionary picture of space and time. Time experience by a clock depends on how briskly it is moving, as the speed of a clock increases, the speed of ticking decreases. Time dilation is the elongation of the time interval between two events for an observer in an inertial frame that is moving with respect to the rest frame of the events occurs at the same position.

## II. SPECIAL RELATIVITY THEORY

In 1905 the theory of special relativity was proposed by Albert Einstein. It explains the behaviour of things that moves very fast- as in, significant fraction of the speed of light. where regular Newtonian physics doesn't always apply. It called special relativity as a result of it solely applies to specific situations where the distinct frames of reference aren't accelerating. They are called inertial reference frames.

Special relativity theory built around two main assumptions/postulates:[1]

I. The laws of physics are the same in all inertial reference frames.

II. The speed of light in a vacuum is the same for all observes. (approximately 300,000 kilometres per second)

According to second postulates, even if light source coming from an object which moving at the half the speed of light, the light itself still moving at about three hundred million m/s. This is where the thing starts to get weird. We already know that speed(v) multiplied by time(t) equals to distance(d) vt=d. but in special relativity, when it comes to light, speed is always constant.[8] Which would mean that the other two variables would have to change that time and distance. They do! When time changes called time dilation and when distance changes called length contraction.

## III. TIME DILATION

Time dilation occurs when another reference frame is moving relative to you, so time in that reference frames slows down relative to the time you measure. For e.g., a person stands on the side of his train that closer to the platform and he is facing a mirror on the opposite side, 5 meters away he shines a flashlight towards mirror, which reflects the light back towards him. From his point of view on the train the light travelled straight to the mirror and back, a distance of 10m, at the speed of light. But, another person(person2) outside the train looking through the window saw the light travel to the mirror and back, but meanwhile, the train was still moving. While the light travels toward the mirror, the mirror moved sideways relative to another person's(person2) spot on the platform. The result that person2 saw light travel diagonally, as though its path formed two sides of triangle. From person2 point of view, the light travelled a greater distance than it did from person on the train point of view.

According to special relativity, light speed was still c. even though it travelled a greater distance.so, if light travelled a greater distance at the same speed then it must have been traveling from longer. Here both persons timing the exact same series of events. But person2 measuring a longer time than person on the train. So, from person2's perspective on the platform, time has slowed down for the person on the train. That's time dilation. If we Measured distance of the light was traveling, we found that time slowed down for the person on the train by a factor of $1/\sqrt{1-v^2/c^2}$. this factor is called $\gamma$(gamma). It is always greater than1, because the velocity of the moving reference frame always has to be less than c, the speed of light.

### *A. Derivation of Time Dilation*:

Suppose there are two frame of reference S and S'. the relative velocity of both frames is v. An event is started at time $t_1$ and ended at time $t_2$. time difference of S event is $\Delta t = t_2 - t_1$, and time difference of S' event is $\Delta t' = t'_2 - t'_1$.

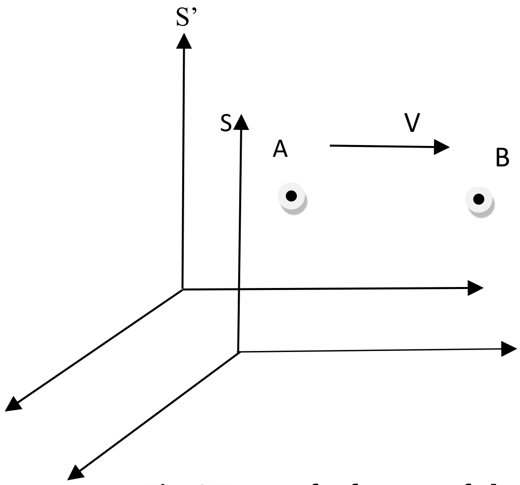


**Fig. 1 Frame of references of observers**

According to Lorentz transformation equation,

$$t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t' = \frac{t_2 - \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{t_1 - \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t' = \frac{t_2 - \frac{xv}{c^2} - t_1 + \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t' = \frac{t_2 - t_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

### *B. Mathematics behind time dilation:*

we have the equation $\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$

Where, for observers Albus and harry, Δt is the time elapsed for Albus, Δt' is the time elapsed for Harry, v is the relative velocity (the difference in velocity) and c is the speed of light.

**I.** *Albus leaves earth on a spacecraft and travels for 10 years at 10,000,000 m/s. After Albus's 10 years, how much time has passed for Harry back on earth?*

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t' = \frac{10}{\sqrt{1 - \frac{10{,}000{,}000^2}{300{,}000{,}000^2}}}$$

$$\Delta t' = \frac{10}{\sqrt{1 - \frac{1}{900}}}$$

$$\Delta t' = \frac{10}{\sqrt{\frac{899}{900}}}$$

$$\Delta t' \approx 10.006\ years$$

Which is only 2 days more than 10 years. So, travelling at such a huge speed for a very long time, it doesn't make much difference, that's why we don't see these effects in our day today lives.

**II.** *Suppose Albus travels for 10 years at 200,000,000 m/s. How much time has passed for Harry on earth?*

$$\Delta t' = \frac{100}{\sqrt{1 - \frac{200{,}000{,}000^2}{300{,}000{,}000^2}}}$$

$$\Delta t' = \frac{10}{\sqrt{1 - \frac{4}{9}}}$$

$$\Delta t' = \frac{10}{\sqrt{\frac{5}{9}}}$$

$$\Delta t' \approx 13.42\ years$$

Here, 3.5 years difference made if he travels 2/3 the speed of light.

**III.** Now, for reference, in I was about 3% the speed of light and II was about 67% the speed of light

Suppose observer Harry travels for 10 years at 99.9% the speed of light (v = 0.999c), how much time has passed on earth?

$$\Delta t' = \frac{10}{\sqrt{1 - \frac{(0.999c)^2}{c^2}}}$$

$$\Delta t' = \frac{10}{\sqrt{1 - \frac{0.999^2.c^2}{c^2}}}$$

$$\Delta t' = \frac{10}{\sqrt{1 - 0.999^2}}$$

$$\Delta t' \approx 223.66\ years$$

From I, II, III Here are two figures:

v = 0.9999c i.e., 99.99% the speed of light for 10 years is 707 years on earth and v = 0.99999c i.e., 99.999% the speed of light for 10 years is 2,236 years on earth.

We can conclude that until you start moving at speeds very close to the speed of light the effect is nonexistence, but when it close to the light speed, the effect becomes very pronounced.

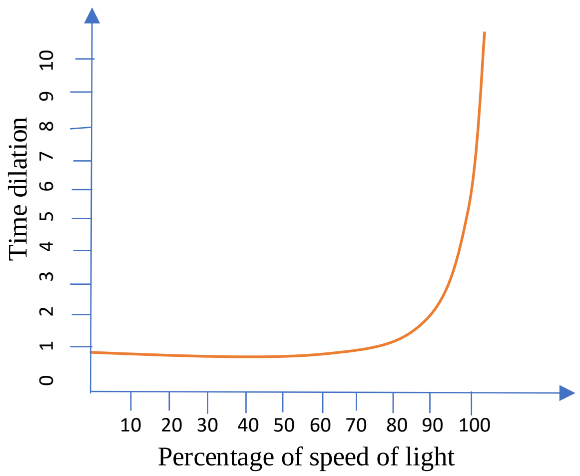


**Fig.2 Time dilation graph**

### *C. Twin paradox:*

The Twin Paradox was an experiment for theory of relativity performed by Einstein which involved identical twins, one of the twins will be launched in space boarded in a high-speed rocket and then return home to find out his twin older than him. This scenario was explained in special relativity i.e., the twin who was launched in space by the help of high-speed rocket was is continuous acceleration it means that twin was a non-inertial observer (real force was applied), there was no balance or proportionate similarity found in both of the twins.

#### *I. The twin paradox of the principle of relativity:*

Imagine that twin X remains at rest on the earth and twin Y travels with zero velocity (v), 8c to the nearest star, Alpha Proxima 4 light years from the earth and back. According to X this will take

$$t_X\ X = \frac{2L_0}{v} = \ 10$$

Which means that X is 10 years older at the reunion. X would predict the same about Twin Y is

$$t_Y Y = \sqrt{\frac{1 - v^2}{c^2 t_Y}} X = 6\ years$$

Older at the reunion. But according to principle of relativity Y could consider himself at rest and X as moving. Y would predict that he is 10 years older and X is 6 years older at the reunion. This contradiction named as twin paradox.

#### *II. Time dilation effect in twin paradox:*

Einstein resolution is that comparative to ceiling each second at the floor is little longer. From gravitational time shift time passes slower in farther down in the gravitational field. An observer staying in gravitational field will observe faster rate of time in higher up in a gravitational field. This will explain by time paradox.

Consider twin X and twin Y. Y observes twin A and the Earth and Alpha Proxima move with a velocity (v) 0,8c . Lorentz contracted distance,

$$\mathrm{L} = \mathrm{L}_0 \sqrt{1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}} = 2{,}4\ \text{ years}$$

According to Y the time taken by X's travel back is

$$t_Y\ Y = \frac{2L}{v} = 6\ years$$

Here, Y agrees with X's prediction that he ages by 6 years during X's travelling,

$$t_Y\ Y = \ t_X\ Y$$

From time dilation effect Y would predict that X ages by

$$t_Y\ X_{OUT-IN} = \sqrt{1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}} t_Y\ Y = \frac{2\sqrt{1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}} L_0}{v} = \left(1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}\right) \frac{2L_0}{v} = 3{,}6\ years$$

From above equation this is in conflict with X's prediction that he should age by 10 years $t_X\ X = 10$ years. As we observe, there is something missing in y's prediction of X's ageing as given. According to Y, when Y accelerates towards his twin brother, he experiences a field of gravity away from X, who is higher up in gravitational field than he is. Hence, as Y measures that X at the earth ages faster than him during time,

$$\Delta t_Y\, Y = \frac{2vL}{g}$$

When Y accelerates. When constant proper acceleration measured by Y it follows from general theory of relativity. So, the relation between twins ageing is given ass [4]

$$\Delta t_Y\, X = \left(1 + \frac{gL_0}{c^2}\right)\Delta t_Y Y$$

Which gives

$$\Delta t_Y\, X = \left(1 + \frac{gL_0}{c^2}\right)\frac{2v}{g} = \frac{2v}{g} + \frac{2vL_0}{c^2}$$

When twin on earth X calculated his own and Y's ageing during travel, he neglected the time X taken at Alpha Proxima to reverse his velocity. The expression calculated by Y for the ageing of X during the time experiences a gravitational field, then reduces to,

$$\Delta t_Y\, X = \frac{2vL_0}{c^2} = 2.0, 8.4\, years = 6{,}4\, years$$

Hence, Y correctly predicted the aging of X is,

$$t_Y\, X = t_Y\, X_{OUT-IN} + \Delta t_Y\, X = \left(1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}\right)\frac{2vL_0}{v} + \frac{2vL_0}{c^2} = \frac{2vL_0}{v} = 10\, years$$

This equation is in agreement with X's prediction.

However, we consider twin paradox in spacetime. Conclusion of this twin contradiction is: one who accelerate when they were away is youngest after reunion [2]. Eriksen and Grøn have thought of the more general scenario when each twins accelerate and found that the twin who has greatest acceleration is youngest after the travel.. [3]

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***Reference frame of Earth*** ***Reference frame of outbound traveller***

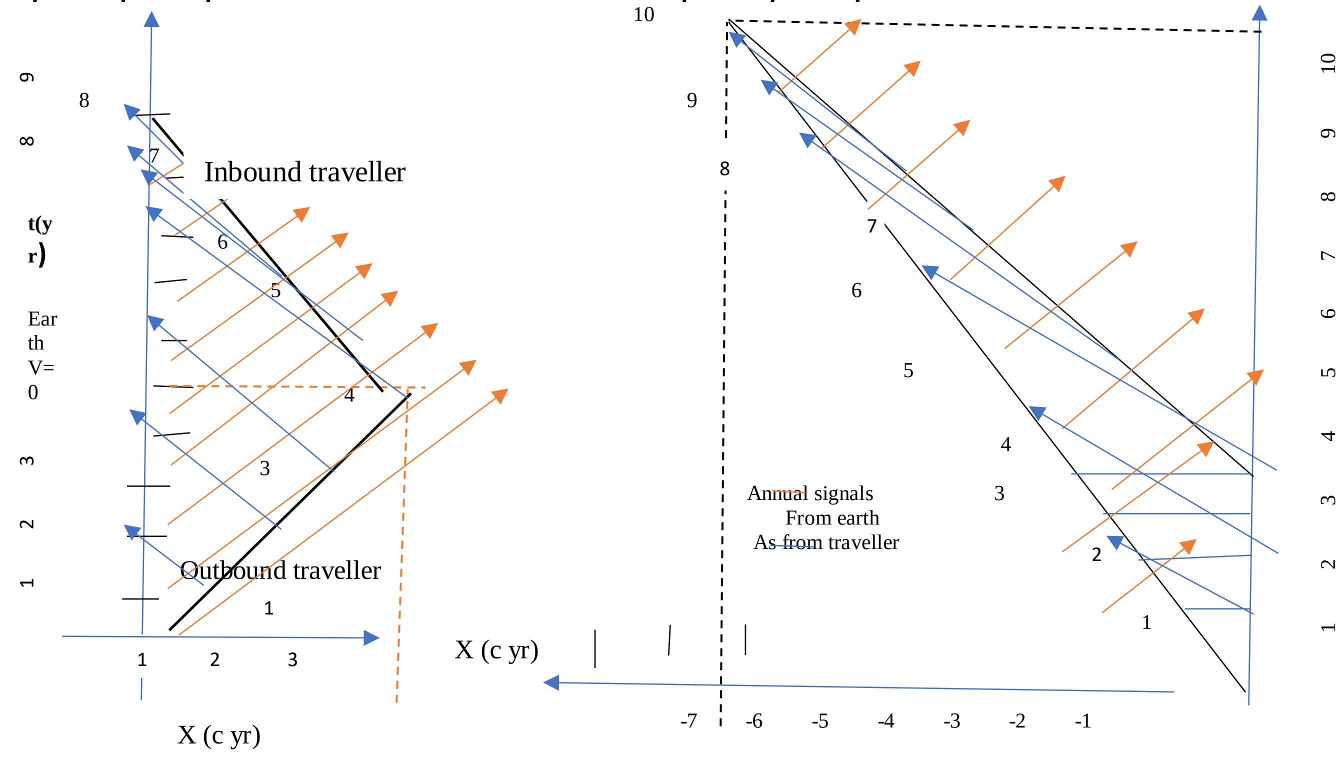


**Fig.3 Twin paradox age graph**

In the above equations, we only assume that X is not accelerated and Y is and show how we calculate Ageing of twins from both point of view considers themselves at rest. We cannot be sure about who is younger than another. This apparent contradiction is named as "twin paradox". But it is not a paradox just an example for how special relativity can be easily misunderstood. Langevin explains this different ageing rates as: "Only the traveller has undergone an acceleration that changed the direction of his velocity." He explains the spaceship departure and return to Earth in his example. Where, of all worldlines joining two events, longest proper time is taken by the one which is not accelerated. Earth and spaceship are not in symmetrical relationship in the twin experiment; a "turnaround "of ship makes the difference as it undergoes non inertial motion, but Earth has no such "turnaround". Because of there is no symmetry, special relativity is not contradicted after realization that the earthbound twin is older than one who return from the space

## III. Time-Warping in daily life

### *A. GPS*

We frequently use GPS feature in our daily life. Anyone who has smartphone has access to GPS (Global Positioning System) Every time we plan a route our phone connected to a satellite. Speed of satellite around the earth is 10000km/hr, it's about thousandth the speed of light. Even at this speed satellite experience time dilation. Roughly 4 microsecond every day it gets "older". Compare to people on earth satellite experience faster passage of time. If including the gravitational effects as it also causes time dilation, the figure is about 7 microseconds.

If this effect uncounted then after a day your location could be 8 km away from your actual location according to GPS. Thankfully satellite is programmed in the way that take these effects into consideration.

### *B. Colour of gold:*

Gold has characteristic yellow colour. You will predict gold has silver sheen if you do not take relativity in consideration, however the actual colour of gold leans to red end of the spectrum. The frequency of the light gold emits that is colour will find out by examining electrons in Gold atom. Electrons in the orbital closest to the nucleus move roughly half of the speed of the light to avoid being dragged by the protons.it causes lot of relativistic effects. Gold usually absorbs ultraviolet range of wavelength. Gold starts to absorb blue light i.e., smaller frequency. Only red colours are reflected into our eyes; hence it has glamourous, yellowy sheen.

## IV. TIME TRAVEL

The concept of traveling time i.e., traveling forward or back in time, with the hypothetical device named as time machine. It is generally understood that time traveling require a device. In 1895, '*The Time Machine'* novel published by H.G Wells and the idea of the time machine popularized. [4] traveling in the past physically may be uncertain but travel forward, outside the perception of time is an observer phenomenon. It is understood in the special relativity and general relativity frame work. However, it is not possible with current technology to make advanced and delay in one body more than few milliseconds compared to other body.

Special relativity has some theories that suggest, suitable space time geometries or specific spacetime [7] motion may allow time travel if these motions or geometries possible. In tachyonic antitelephone [5] which is hypothetical scenario where signal travels faster than light, it is received before it is sent, in all reference frames it could be said that signal moved backward in time. It explained by the mathematics of simultaneity in the relativity theory. In all reference frames the transmission event happens before reception and also if signal travels faster than time then only it receives before sent.[6] However, the current scientific consensus is that Faster-Than-Light communication is not possible and not achieve in any experiment till date.

## V. CONCLUSION

The present study and analysis show the substantial role of gravity in time dilation. In higher gravitational potential time runs slower. As time moves relative to observer, object in motion always experience time dilation. We can experience time dilation in daily life, but compare to speed of light we are unable to see the time wrap happens in daily life. Time travel is an important and thrilling concept explained with time dilation, while time travel does not appear possible till date. Advances in quantum theories perhaps provide understanding of time travel paradox. Research in time dilation leads to enhancement in the subject of astrophysics and mathematics related research.

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