On Radio Geometric Mean $D$- Distance Number of Some Basic Graphs

K. John Bosco¹, S. Priya²

¹Assistant Professor, ²Research scholar (Reg.No: 20213232092003)
Department of Mathematics, St. Jude’s College, Thoothoor, Tamil Nadu, India

Abstract

A Radio geometric mean $D$-distance labeling of a connect graph $G$ is an injective function $f$ from the vertex set $V(G)$ to the $\mathbb{N}$ such that for two distinct vertices $u$ and $v$ of $G$, $d^0(u, v) + \sqrt{f(u)f(v)} \geq 1 + \text{diam}^D(G)$, where $d^0(u, v)$ denotes the $D$-distance between $u$ and $v$ $\text{diam}^D(G)$ denotes the $D$-diameter of $G$. The radio geometric mean $D$-distance number of $f$, $r\text{gmn}^D(f)$ is the maximum label assigned to any vertex of $G$. The radio geometric mean $D$-distance number of $r\text{gmn}^D(G)$ is the minimum value of $G$, $r\text{gmn}^D(G)$ is the minimum value of $r\text{gmn}^D(f)$ taken over all radio geometric mean $D$-distance labeling $f$ of $G$. In this paper we find the radio geometric mean $D$-distance number of some basic graphs.

Keywords: $D$-distance, Radio geometric mean $D$-distance, Radio geometric mean $D$-distance number.

I. INTRODUCTION

By a graph $G = (V(G), E(G))$ we mean a finite undirected graph without loops or multiple edges. The order and size of $G$ are denoted by $p$ and $q$ respectively. Radio labeling (multi-level distance labeling) can be regarded as an extension of distance two labeling which is motivated by the channel assignment problem introduced by Hale [3]. Chartrand et al [2] introduced the concept of radio labeling of graph. We are introduce the concept of radio geometric mean $Dd$–distance number of some basic graphs[6]

The concept of $D$-distance was introduced by D. Reddy Babu et al [11]. For a connected graph $G$, the $D$-length of a connected $u - v$ path is defined as $l^D(s) = l(s) + \text{deg}(v) + \text{deg}(u) + \sum \text{deg}(w)$ where the sum runs over all intermediate vertices $w$ of $s$ and $l(s)$ is length of the path. The $D$ – distance, $d^0(u, v)$ between $u$ and $v$ of connected graph $G$ is defined $d^0(u, v) = \min \{l^D(s)\}$ where the minimum is taken over all $u$-$v$ paths $s$ in $G$. In other words, $d^0(u, v) = \min \{l(s) + \text{deg}(v) + \text{deg}(u) + \sum \text{deg}(w)\}$ where the sum runs over all intermediate vertices $w$ in $s$ and minimum is taken over all $u$-$v$ paths $s$ in $G$. The lower bound is clear from the definition and the upper bound follows from the triangular inequality.

In [9], Radio geometric mean labeling was introduced by V. Hemalatha et al. A radio geometric mean labeling is a one to one mapping $f$ from $V(G)$ to $\mathbb{N}$ satisfying the condition $d(u, v) + \sqrt{f(u)f(v)} \geq 1 + \text{diam}(G)$, for every $u, v \in G$. The span of labeling $f$ is the maximum integer that $f$ maps to a vertex of $G$. The radio geometric mean number of $G$, $r\text{gmn}(G)$ is the lowest span taken over all radio geometric mean labeling of the graph $G$. The above condition is called radio geometric mean condition.

Further we are introduced the concept of radio geometric mean $D$-distance. The radio geometric labeling is a function $f: V(G) \rightarrow \mathbb{N}$ such that $d^0(u, v) + \sqrt{f(u)f(v)} \geq 1 + \text{diam}^D(G)$. It is denoted by $r\text{gmn}^D(G)$, where $r\text{gmn}^D(G)$ is called the radio geometric mean $D$- distance number. The radio geometric mean $D$-distance number of $f$, $r\text{gmn}^D(f)$ is the maximum label assigned to any vertex of $G$. The radio geometric mean $D$-distance number of, $r\text{gmn}^D(G)$ is the minimum value of $G$, $r\text{gmn}^D(G)$ is the minimum value of $r\text{gmn}^D(f)$ taken over all radio geometric mean $D$-distance labeling $f$ of $G$. In this paper we find the radio geometric mean $D$-distance number of some basic graphs.
II. MAIN RESULTS

Theorem 2.1
The Radio geometric mean D-distance number of a complete graph $K_n$, $rgmn^D(K_n) = n$.

Proof.
Let $V(K_n) = \{v_1, v_2, v_3, ..., v_n\}$ be the vertex set and $E(K_n) = \{v_i, v_j, 1 \leq i, j \leq n, i \neq j\}$ be the edge set. Its $\text{diam}^D(K_n) = 2n - 1$. We define the vertex label $f(v_i) = i, 1 \leq i \leq n$.

The radio geometric mean D-distance condition is

$$d^D(u, v) + \left|\sqrt{f(u)f(v)}\right| \geq 1 + \text{diam}^D(G),$$

Compute the pair $(v_i, v_j) = 2n - 1, 1 \leq i, j \leq n, i \neq j$ are adjacent

$$d^D(v_i, v_j) + \left|\sqrt{f(v_i)f(v_j)}\right| \geq 1 + \text{diam}^D(K_n),$$

$$\left|\sqrt{ij}\right| \geq 1$$

Hence, $rgmn^D(K_n) = n$.

Theorem 2.2
The Radio geometric mean D-distance number of a star graph $K_{1,n}$.

$rgmn^D(K_{1,n}) = n + 2$.

Proof.
Let $V(K_{1,n}) = \{v_0, v_1, v_2, v_3, ..., v_n\}$ be the vertex set, where $v_0$ is the central vertex and $E(K_{1,n}) = \{v_0v_i, 1 \leq i \leq n\}$ be the edge set. Its $\text{diam}^D(K_{1,n}) = n + 4$. We define the vertex label $f(v_0) = 2, f(v_i) = i + 2, 1 \leq i \leq n, f(v_j) = j + 2, 1 \leq j \leq n$.

By the radio geometric mean D-distance condition is

$$d^D(u, v) + \left|\sqrt{f(u)f(v)}\right| \geq 1 + \text{diam}^D(G),$$

For every pair of vertices $(u, v)$ where $u \neq v$.

Case(i): Compute the pair $(v_0, v_i) = n + 2, 1 \leq i \leq n$ are adjacent

$$d^D(v_0, v_i) + \left|\sqrt{f(v_0)f(v_i)}\right| \geq 1 + \text{diam}^D(K_{1,n}),$$

$$\left|\sqrt{(2)(i + 2)}\right| \geq 3$$

Case(ii): Compute the pair $(v_i, v_j) = n + 4, 1 \leq i, j \leq n, i \neq j$ are non adjacent

$$d^D(v_i, v_j) + \left|\sqrt{f(v_i)f(v_j)}\right| \geq 1 + \text{diam}^D(K_{1,n}),$$

$$\left|\sqrt{(i + 2)(j + 2)}\right| \geq 1$$

Hence, $rgmn^D(K_{1,n}) = n + 2$. 

104
Theorem 2.3

The Radio geometric mean $D$-distance number of a Fan graph $F_n$, 

$$rgmn^D (F_n) = 2n - 2, n \geq 6.$$ 

Proof.

Let $V(F_n) = \{v_0, v_1, v_2, v_3, ..., v_n\}$ be the vertex set and $E(F_n) = \{v_0v_j, v_iv_{i+1}, 1 \leq i \leq n - 1, 1 \leq j \leq n\}$ be the edge set. Its $diam^D(F_n) = n + 6$. we define the vertex label $f(v_0) = 2, f(v_i) = n + i - 2, 1 \leq i \leq n.$

By the radio geometric mean $D$-distance condition is

$$d^D(u, v) + \left\lfloor \sqrt{f(u)f(v)} \right\rfloor \geq 1 + diam^D(G),$$

For every pair of vertices $(u, v)$ where $u \neq v$.

**Case(i)**: Compute the pair $(v_0, v_i) = n + 3$ are adjacent. If $v_i$ is end vertices, 

$$d^D(v_0, v_i) + \left\lfloor \sqrt{f(v_0)f(v_i)} \right\rfloor \geq 1 + diam^D(F_n),$$

$$\left\lfloor \sqrt{(2)(n + i - 2)} \right\rfloor \geq 4$$

**Case(ii)**: Compute the pair $(v_0, v_i) = n + 4$ are adjacent. If $v_i$ is intermediate vertices, 

$$d^D(v_0, v_i) + \left\lfloor \sqrt{f(v_0)f(v_i)} \right\rfloor \geq 1 + diam^D(F_n),$$

$$\left\lfloor \sqrt{(2)(n + i - 2)} \right\rfloor \geq 3$$

**Case(iii)**: Compute the pair $(v_i, v_j) = n + 6$, are both end vertices $1 \leq i \leq n, i + 1 \leq j \leq n,$

$$d^D(v_i, v_j) + \left\lfloor \sqrt{f(v_i)f(v_j)} \right\rfloor \geq 1 + diam^D(F_n),$$

$$\left\lfloor \sqrt{(n + i - 2)(n + j - 2)} \right\rfloor \geq 1$$

**Case(iv)**: Compute the pair $(v_i, v_j) = 7$, intermediate adjacent vertices

$$d^D(v_i, v_j) + \left\lfloor \sqrt{f(v_i)f(v_j)} \right\rfloor \geq 1 + diam^D(F_n),$$

$$\left\lfloor \sqrt{(n + i - 2)(n + j - 2)} \right\rfloor \geq n$$

Hence, $rgmn^D (F_n) = 2n - 2, n \geq 6.$

**Note:** $rgmn^D (F_n) = n + 4$ if $n = 1, 2, 3, 4, 5.$

Theorem 2.4

The Radio geometric mean $D$-distance number of a Double Fan graph $DF_n$, 

$$rgmn^D (DF_n) = 2n, n \geq 5.$$
Proof.

Let $V(DF_n) = \{v_1, v_2, v_3, ..., v_n, w, u\}$ be the vertex set and let $v_1, v_2, v_3, ..., v_n$ be the path graph and $u, w$ are two vertex are joined to the end vertex of the path graph. Its $diam^D(DF_n) = 2n + 5$. we define the vertex label as

$$f(u) = 9, f(v_i) = n + i, 1 \leq i \leq n, f(w) = 10, f(v_j) = n + j, 1 \leq j \leq n.$$ 

By the radio geometric mean $D$-distance condition is

$$d^D(u, v) + \left[\sqrt{f(u)f(v)}\right] \geq 1 + diam^D(G),$$

For every pair of vertices $(u, v)$ where $u \neq v$.

**Case(i)**: Compute the pair $(u, v_i) = n + 4, i = 1, n$

$$d^D(u, v_i) + \left[\sqrt{f(u)f(v_i)}\right] \geq 1 + diam^D(DF_n)$$

$$\left[\sqrt{9(n + i)}\right] \geq n + 2$$

**Case(ii)**: Compute the pair $(w, v_i) = n + 4, i = 1, n$

$$d^D(w, v_i) + \left[\sqrt{f(w)f(v_i)}\right] \geq 1 + diam^D(DF_n)$$

$$\left[\sqrt{10(n + i)}\right] \geq n + 2$$

**Case(iii)**: Compute the pair $(v_i, v_j) = n + 8$, are both end vertices

$$d^D(v_i, v_j) + \left[\sqrt{f(v_i)f(v_j)}\right] \geq 1 + diam^D(DF_n)$$

$$\left[\sqrt{(n + i)(n + j)}\right] \geq n - 2$$

**Case(iv)**: Compute the pair $(u, w) = 2n + 5$, are both end vertices

$$d^D(u, w) + \left[\sqrt{f(u)f(w)}\right] \geq 1 + diam^D(DF_n)$$

$$\left[\sqrt{(9)(10)}\right] \geq 1$$

**Case(v)**: Compute the pair $(v_i, v_j) = 9$ both are intermediate adjacent vertices

$$d^D(v_i, v_j) + \left[\sqrt{f(v_i)f(v_j)}\right] \geq 1 + diam^D(DF_n)$$

$$\left[\sqrt{(n + i)(n + j)}\right] \geq 2n - 3$$

**Case(vi)**: Compute the pair $(u, v_i) = n + 5, 2 \leq i \leq n - 1$

$$d^D(u, v_i) + \left[\sqrt{f(u)f(v_i)}\right] \geq 1 + diam^D(DF_n)$$

$$\left[\sqrt{(9)(n + i)}\right] \geq n + 1$$

Hence, $rgmn^D(DF_n) = 2n, n \geq 5$.

**Note:** $rgmn^D(DF_n) = 2n - 2$ if $n = 1, 2, 3, 4$. 

K. John Bosco & S. Priya / IJMTT, 67(2), 103-108, 2021
Theorem 2.5
The Radio geometric mean $D$-distance number of a Subdivision of a star graph $S(K_{1,n})$.

$$\text{rgmn}^D \left(S(K_{1,n})\right) = 3n + 4, n \geq 3.$$ 

Proof.
Let $V \left(S(K_{1,n})\right) = \{v_0, v_1, v_2, v_3, \ldots, v_n, u_1, u_2, u_3, \ldots, u_n\}$ be the vertex set, where $v_0$ is the central vertex and $E \left(S(K_{1,n})\right) = \{v_0v_i, v_iu_i | 1 \leq i \leq n\}$ be the edge set. Its $\text{diam}^D(S(K_{1,n})) = n + 10$. We define the vertex label $f(v_0) = 7, f(v_i) = n + 2i + 3, 1 \leq i \leq n, f(u_i) = n + 2i + 4, 1 \leq i \leq n$.

By the radio geometric mean $D$-distance condition is

$$d^D(u, v) + \left[\sqrt{f(u)f(v)}\right] \geq 1 + \text{diam}^D(G),$$

For every pair of vertices $(u, v)$ where $u \neq v$.

Case (i): Compute the pair $(v_0, v_i) = n + 3, 1 \leq i \leq n$ are adjacent

$$d^D(v_0, v_i) + \left[\sqrt{f(v_0)f(v_i)}\right] \geq 1 + \text{diam}^D(S(K_{1,n})),$$

$$\left[\sqrt{(7)(n + 2i + 3)}\right] \geq 8$$

Case (ii): Compute the pair $(v_0, u_i) = n + 5, 1 \leq i \leq n$ are non adjacent

$$d^D(v_0, u_i) + \left[\sqrt{f(v_0)f(u_i)}\right] \geq 1 + \text{diam}^D(S(K_{1,n})),$$

$$\left[\sqrt{(7)(n + 2i + 4)}\right] \geq 6$$

Case (iii): Compute the pair $(v_i, v_j) = n + 6, 1 \leq i, j \leq n, i \neq j$ are non adjacent

$$d^D(v_i, v_j) + \left[\sqrt{f(v_i)f(v_j)}\right] \geq 1 + \text{diam}^D(S(K_{1,n})),$$

$$\left[\sqrt{(n + 2i + 3)(n + 2j + 3)}\right] \geq 5$$

Case (iv): Compute the pair $(v_i, u_j) = 4, if \ |i - j| = 1$ are non adjacent

$$d^D(v_i, u_j) + \left[\sqrt{f(v_i)f(u_j)}\right] \geq 1 + \text{diam}^D(S(K_{1,n})),$$

$$\left[\sqrt{(n + 2i + 3)(n + 2j + 4)}\right] \geq n + 7$$

Case (v): Compute the pair $(u_i, u_j) = 10, 1 \leq i, j \leq n, i \neq j$ are non adjacent

$$d^D(u_i, u_j) + \left[\sqrt{f(u_i)f(u_j)}\right] \geq 1 + \text{diam}^D(S(K_{1,n})),$$

$$\left[\sqrt{(n + 2i + 4)(n + 2j + 4)}\right] \geq 1$$

Case (vi): Compute the pair $(v_i, u_j) = n + 8, if \ |i - j| > 1$
\[d^p(v_i, u_j) + \left\lfloor \sqrt{f(v_i)f(u_j)} \right\rfloor \geq 1 + \text{diam}^p(S(K_{1,n}))\]

\[\left\lfloor \sqrt{(n+2i+3)(n+2j+4)} \right\rfloor \geq 3\]

Hence, \(rgmn^p(S(K_{1,n})) = 3n + 4, n \geq 3\).

**Note:** \(rgmn^p(S(K_{1,n})) = 2n + 7, \text{if } n = 1, 2\).

### III. Reference