

Domination of Irregular Graphs Using Wiener Index And Distance Matrix

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Abstract — In this paper, domination of irregular graphs using wiener index and distance matrix is discussed. Wiener index and distance matrix are most simplest and useful method to finding domination sets and domination number. The purpose of this paper is to minimize the distance to get a closest facility by using the method wiener index and distance matrix.

Keywords — irregular graphs, domination, wiener index, distance matrix.

I. INTRODUCTION

Domination have applications in many fields like network, project planning, computer etc. Domination arises in facility location problems, where the number of facilities are fixed and one attempt to minimize the distance that a person needs to travel to get the closest facility. In this paper, it is explained that the method of finding domination sets and domination number of irregular graphs using wiener index and distance matrix. From the paper [1], M.Sakthi, Domination of semiregular graphs using wiener index and distance matrix, International Journal of Mathematical Archive-10(7), 2019, 1-5, we have developed that for irregular graphs.

II. PRELIMINARIES

A. REGULAR GRAPH

A regular graph is a graph where each vertex has the same number of neighbors; i.e. every vertex has the same degree [2].

B. IRREGULAR GRAPH

A graph that is not regular. i.e. if atleast 2 vertices, such that both of them have different degrees, then graph cannot be regular.

C. WIENER INDEX

The Wiener index of a graph G, denoted by $W(G)$ is the sum of the distances between all (unordered) pair of vertices of G [3]. A set $S \subseteq V$ of vertices in a graph $G = (V, E)$ is called a dominating set if every vertex $v \in V$ is either an element of S or is adjacent to an element of S [4]. The domination number $\gamma(G)$ is the minimum cardinality among all dominating sets in G [5].

D. DISTANCE MATRIX

Distance matrix is a square matrix (two dimensional array) containing the distances, taken pairwise, between the elements of a set [6].

E. FINDING DOMINATION NUMBER AND SETS OF WIENER INDEX

1. Find $W_G(v_j)$ where $j=1,2,\dots,n$.
2. Write $D_i = \{v_j / W_G(v_j) \text{ has same value } f \text{ or } j\}$ where $i=1,2,\dots$
3. If $i=2$ then $\gamma(G) = |D_i|$, where D_i is the set of minimum value of $W_G(v_j)$ and $D = D_1$ and D_2 . Otherwise $i > 2$ then $D = D_i$ where D_i is the set of minimum value of $W_G(v_j)$ of v_j for all j and $\gamma(G) = |D|$ [1].

F. FINDING DOMINATION NUMBER AND SETS OF DISTANCE MATRIX

1. Find the shortest distances of each pair of vertices and table it. (i.e.) Find distancematrix of given graph.
2. Find the total distance of each rows and columns.
 $\sum r_j = a_{j1} + a_{j2} + \dots + a_{jn}$, $\sum c_j = a_{1j} + a_{2j} + \dots + a_{nj}$,
 $\sum r_j = \sum c_j = W_G(v_j)$ for all $j = 1, 2, \dots, n$.
3. Write $D_i = \{v_j / W_G(v_j) \text{ has same value for } j\}$ where $i, j = 1, 2, \dots, n$.
4. If $i=2$ then $\gamma(G) = |D_i|$, where D_i is the set of minimum value of $W_G(v_j)$ and $D = D_1$ and D_2 . Otherwise $i > 2$ then $D = D_i$ Where D_i is the set of minimum value of $W_G(v_j)$ of v_j for all j and $\gamma(G) = |D|$ [1].



III. EXAMPLE OF IRREGULAR GRAPHS

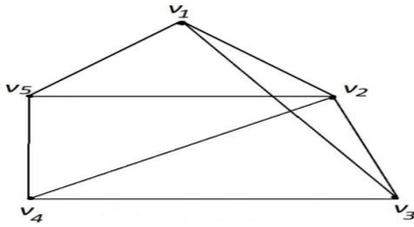


Fig 1: irregular

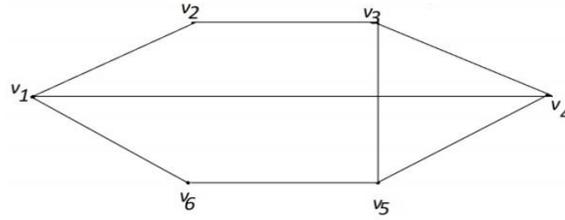


Fig 2: irregular

In fig 1: $W_G(v_1) = 5, W_G(v_2) = 4, W_G(v_3) = 5, W_G(v_4) = 5, W_G(v_5) = 5$, then $D_1 = \{v_2\}, D_2 = \{v_1, v_3, v_4, v_5\}$ we get $\gamma(G) = |D|=1, D(G) = \{v_2\}$

In fig 2: $W_G(v_1) = 7, W_G(v_2) = 8, W_G(v_3) = 7, W_G(v_4) = 7, W_G(v_5) = 7, W_G(v_6) = 8$ then $D_1 = \{v_1, v_3, v_4, v_5\}, D_2 = \{v_2, v_6\}$ we get $\gamma(G) = |D| = 4, D(G) = \{v_1, v_3, v_4, v_5\}$

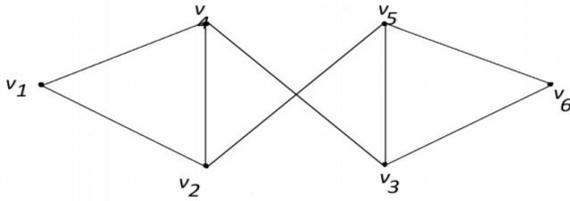


Fig 3: irregular

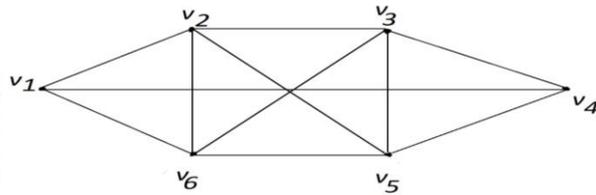


Fig 4: irregular

In fig 3: $W_G(v_1) = 9, W_G(v_2) = 7, W_G(v_3) = 7, W_G(v_4) = 7, W_G(v_5) = 7, W_G(v_6) = 9$ then $D_1 = \{v_2, v_3, v_4, v_5\}, D_2 = \{v_1, v_6\}$ we get $\gamma(G) = |D| = 4, D(G) = \{v_2, v_3, v_4, v_5\}$

In fig 4: $W_G(v_1) = 7, W_G(v_2) = 6, W_G(v_3) = 6, W_G(v_4) = 7, W_G(v_5) = 6, W_G(v_6) = 6$ then $D_1 = \{v_2, v_3, v_5, v_6\}, D_2 = \{v_1, v_4\}$ we get $\gamma(G) = |D| = 4, D(G) = \{v_2, v_3, v_5, v_6\}$

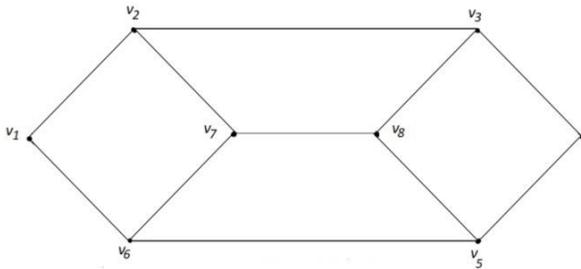


Fig 5: irregular

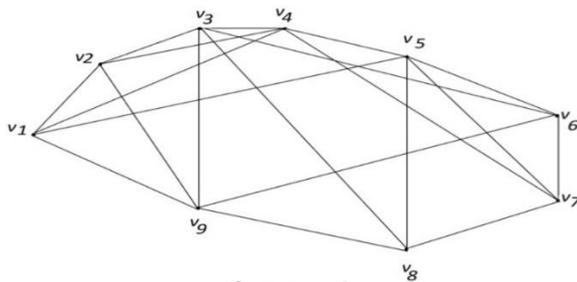


Fig 6: irregular

In fig 5: $W_G(v_1) = 14, W_G(v_2) = 12, W_G(v_3) = 12, W_G(v_4) = 14, W_G(v_5) = 12, W_G(v_6) = 12, W_G(v_7) = 12, W_G(v_8) = 12$ then $D_1 = \{v_2, v_3, v_5, v_6, v_7, v_8\}, D_2 = \{v_1, v_4\}$ we get $\gamma(G) = |D| = 6, D(G) = \{v_2, v_3, v_5, v_6, v_7, v_8\}$

In fig 6: $W_G(v_1) = 12, W_G(v_2) = 12, W_G(v_3) = 11, W_G(v_4) = 11, W_G(v_5) = 11, W_G(v_6) = 12, W_G(v_7) = 12, W_G(v_8) = 12, W_G(v_9) = 11$ then $D_1 = \{v_3, v_4, v_5, v_9\}, D_2 = \{v_1, v_2, v_6, v_7, v_8\}$ we get $\gamma(G) = |D| = 4, D(G) = \{v_3, v_4, v_5, v_9\}$

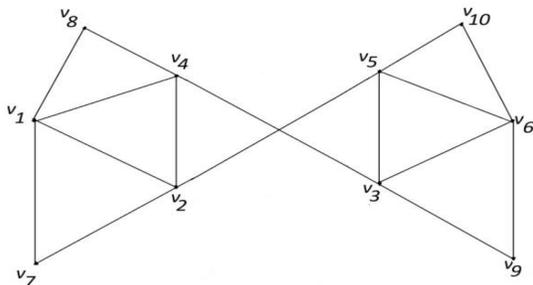


Fig 7: irregular

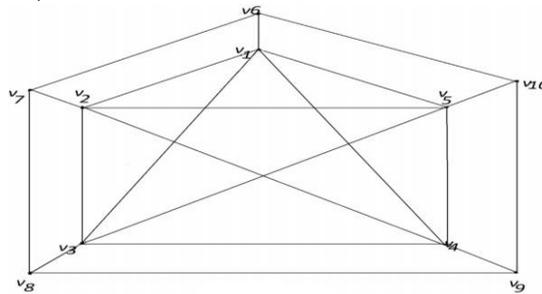


Fig 8: irregular

In fig 7: $W_G(v_1) = 17, W_G(v_2) = 15, W_G(v_3) = 15, W_G(v_4) = 15, W_G(v_5) = 15, W_G(v_6) = 17, W_G(v_7) = 21, W_G(v_8) = 21, W_G(v_9) = 21, W_G(v_{10}) = 21$ then $D_1 = \{v_2, v_3, v_4, v_5\}, D_2 = \{v_1, v_6\}, D_3 = \{v_7, v_8, v_9, v_{10}\}$ we get $\gamma(G) = |D| = 4, D(G) = \{v_2, v_3, v_4, v_5\}$
In fig 8: $W_G(v_1) = 13, W_G(v_2) = 13, W_G(v_3) = 13, W_G(v_4) = 13, W_G(v_5) = 13, W_G(v_6) = 15, W_G(v_7) = 15, W_G(v_8) = 15, W_G(v_9) = 15, W_G(v_{10}) = 15$ then $D_1 = \{v_1, v_2, v_3, v_4, v_5\}, D_2 = \{v_6, v_7, v_8, v_9, v_{10}\}$ we get $\gamma(G) = |D| = 5, D(G) = \{v_1, v_2, v_3, v_4, v_5\}$

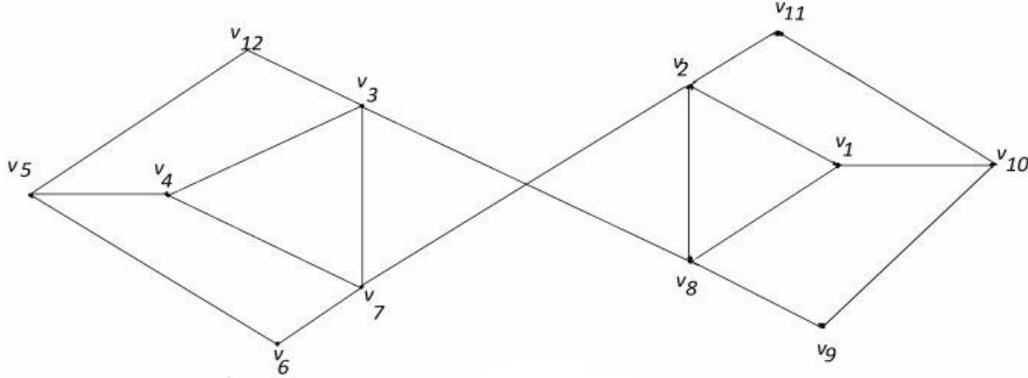


Fig 9: irregular

In fig 9: $W_G(v_1) = 24, W_G(v_2) = 20, W_G(v_3) = 20, W_G(v_4) = 24, W_G(v_5) = 30, W_G(v_6) = 27, W_G(v_7) = 20, W_G(v_8) = 20, W_G(v_9) = 27, W_G(v_{10}) = 30, W_G(v_{11}) = 27, W_G(v_{11}) = 27$ then $D_1 = \{v_2, v_3, v_7, v_8\}, D_2 = \{v_1, v_4\}, D_3 = \{v_6, v_9, v_{11}, v_{12}\}, D_4 = \{v_5, v_{10}\}$ we get $\gamma(G) = |D| = 4, D(G) = \{v_2, v_3, v_7, v_8\}$

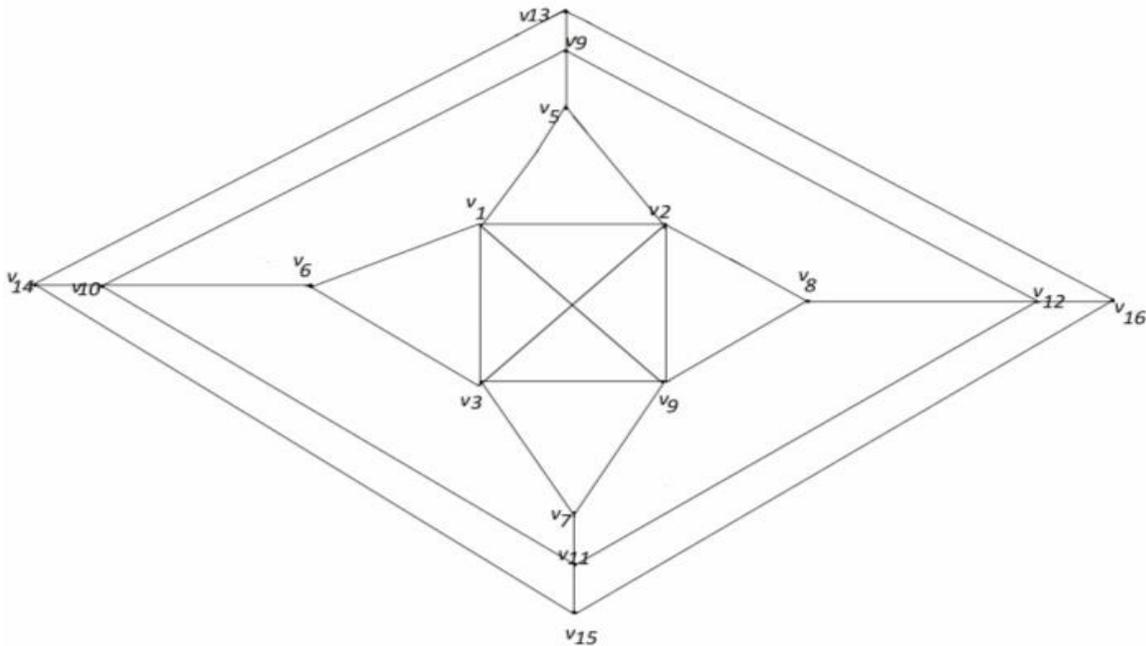


Fig10: irregular

In fig 10: $W_G(v_1) = 33, W_G(v_2) = 33, W_G(v_3) = 33, W_G(v_4) = 33, W_G(v_5) = 33, W_G(v_6) = 33, W_G(v_7) = 33, W_G(v_8) = 33, W_G(v_9) = 30, W_G(v_{10}) = 30, W_G(v_{11}) = 30, W_G(v_{12}) = 30, W_G(v_{13}) = 38, W_G(v_{14}) = 38, W_G(v_{15}) = 38, W_G(v_{16}) = 38$ then $D_1 = \{v_9, v_{10}, v_{11}, v_{12}\}, D_2 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}, D_3 = \{v_{13}, v_{14}, v_{15}, v_{16}\}, D_4 = \{v_5, v_{10}\}$ we get $\gamma(G) = |D| = 4, D(G) = \{v_9, v_{10}, v_{11}, v_{12}\}$

Table – 1: distance matrix of fig 10 irregular

c/r	v ₁	v ₂	v ₃	v ₄	v ₅	v ₆	v ₇	v ₈	v ₉	v ₁₀	v ₁₁	v ₁₂	v ₁₃	v ₁₄	v ₁₅	v ₁₆	$\sum r_j$
v ₁	0	1	1	1	1	2	2	2	2	3	3	3	3	3	4	4	33
v ₂	1	0	1	1	1	2	2	1	2	3	3	2	3	4	4	3	33
v ₃	1	1	0	1	2	1	1	2	3	2	2	3	4	3	3	4	33
v ₄	1	1	1	0	2	2	1	1	3	3	2	2	4	4	3	3	33
v ₅	1	1	2	2	0	2	3	2	1	2	3	2	2	3	4	3	33
v ₆	1	2	1	2	2	0	2	3	2	1	2	3	3	2	3	4	33
v ₇	2	2	1	1	3	2	0	2	3	2	1	2	4	3	2	3	33
v ₈	2	1	2	1	2	3	2	0	2	3	2	1	3	4	3	2	33
v ₉	2	2	3	3	1	2	3	2	0	1	2	1	1	2	3	2	30
v ₁₀	2	3	2	3	2	1	2	3	1	0	1	2	2	1	2	3	30
v ₁₁	3	3	2	2	3	2	1	2	2	1	0	1	3	2	1	2	30
v ₁₂	3	2	3	2	2	3	2	1	1	2	1	0	2	3	2	1	30
v ₁₃	3	3	4	4	2	3	4	3	1	2	3	2	0	1	2	1	38
v ₁₄	3	4	3	4	3	2	3	4	2	1	2	3	1	0	1	2	38
v ₁₅	4	4	3	3	4	3	2	3	3	2	1	2	2	1	0	1	38
v ₁₆	4	3	4	3	3	4	3	2	2	3	2	1	1	2	1	0	38
$\sum c_j$	33	33	33	33	33	33	33	33	30	30	30	30	38	38	38	38	536

From the above diagrams, it is explained that the method of finding domination sets and domination number and the wiener index is calculated for each and every graph. Table 1 explained the method of finding distance matrix for the irregular graph.

IV. CONCLUSION

In this paper, it is explained that the concept of domination sets and domination number using wiener index, distance matrix are discussed. Moreover, we have analysed the relations between distance and domination. It has numerous applications in modern science and engineering.

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