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Super Inverse Domination in Graphs

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Abstract - *Let $G=(V(G),E(G))$ be a connected simple graph. A subset S of $V(G)$ is a dominating set of G if for every $u\in V(G)\setminus S$, there exists $v\in S$ such that $uv\in E(G)$. A dominating set S is an inverse dominating set with respect to a minimum dominating set D of G if $S\subseteq V(G)\setminus D$. An inverse dominating set S is called a super inverse dominating set of G if for every vertex $u\in V(G) S$, there exists $v\in S$ such that $N_G(v)\cap(V(G)\setminus S)=\{u\}$. In this paper, we investigate the concept of super inverse dominating set and give the domination number of some special graphs.*

Keywords: *dominating set, inverse dominating set, super dominating set, super inverse dominating set*

I. INTRODUCTION

Suppose that $G=(V(G),E(G))$ is a simple graph with vertex set $V(G)$ and edge set $E(G)$. In simple graph, we mean, finite and undirected graph with neither loops nor multiple edges. For the general graph theoretic terminology, the readers may refer to [1].

A vertex v is said to dominate a vertex u if uv is an edge of G or $v=u$. A set of vertices $S\subseteq V(G)$ is called a dominating set of G if every vertex not in S is dominated by at least one member of S. The size of a set of least cardinality among all dominating sets for G is called the domination number of G and is denoted by $\gamma(G)$. A dominating set of cardinality $\gamma(G)$ is called $\gamma\text{-}set\ of\ G$. Domination in a graph has been a huge area of research in graph theory. It was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [2]. Domination in graphs has been studied in [3-14].

A dominating set S is called a super dominating set if for every vertex $u\in V(G)\setminus S$, there exists $v\in S$ such that $N_G(v)\cap (V(G) S)=\{u\}$. The super domination number of G, is the minimum cardinality of a super dominating set of G and is denoted by $\gamma_{sup}(G)$ [15]. A super dominating set of cardinality $\gamma_{sup}(G)$ is called $\gamma_{sup}\text{-}set\ of\ G$. Super domination has been studied in [16-23].

Let D be a minimum dominating set in G. The dominating set $S\subseteq V(G)\setminus D$ is called an inverse dominating set with respect to D. The inverse domination number of G, is the minimum cardinality of an inverse dominating set of G and is denoted by $\gamma^{-1}(G)$. A inverse dominating set of cardinality $\gamma^{-1}(G)$ is called $\gamma^{-1}\text{-}set\ of\ G$. The inverse domination has been studied in [24-34].

Motivated by the idea of super [15] and inverse [24] domination in graphs, we initiate the study of super inverse dominating set. An inverse dominating set S is a super inverse dominating set if for every vertex $u\in V(G)\setminus S$, there exists $v\in S$ such that $N_G(v)\cap (V(G)\setminus S)=\{u\}$. The super inverse domination number of G, is the minimum cardinality of a super inverse dominating set of G and is denoted by $\gamma^{-1}_{sup}(G)$. A inverse dominating set of cardinality $\gamma^{-1}_{sup}(G)$ is called $\gamma^{-1}_{sup}\text{-}set\ of\ G$. In this paper, we investigate the concept and give the domination number of some special graphs.

II. RESULTS

Remark 2.1. *The set $S=V(G)$ is a super dominating set and an inverse dominating set.*

Proof : If $S=V(G)$, then every vertex in $V(G)\setminus S=\emptyset$ vacuously satisfies the definitions of a super dominating set and an inverse dominating set. ■

Remark 2.2. *Every graph G has a super dominating set and an inverse dominating set.*

Proof: By Remark 2.1. $S=V(G)$ is a super dominating set and an inverse dominating set. ■

From the definitions of super inverse dominating set and Remark 2.2 the following is immediate.

Remark 2.3. *Let G be a nontrivial graph. Then* $1\le\gamma(G)\le\gamma^{-1}_{sup}(G)\le n$.

The following results says that $\gamma^{-1}_{sup}(G)$ ranges over all integers from 1 to n.

Theorem 2.4. *Given positive integers k, m and n such that $1 \leq k \leq m \leq n-1$, where $n \geq 2$, there exists a connected graph G with $|V(G)| = n$, $\gamma(G) = k$, and $\gamma^{-1}_{sup}(G = m$.*

Proof: Consider the following cases.

Case 1. Suppose that $1 = k = m = n-1$.
Then $G = K_2 = [v_1, v_2]$ with $D = \{v_1\}$ a γ*-set* of G and $S = \{v_2\}$ a γ^{-1}_{sup}-set of G. (see Figure 1).

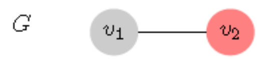

Figure 1: A graph G with $1 = k = m = n-1$.

Thus, $|V(G)| = 2 = n$, $\gamma(G) = 1 = k$, and $\gamma^{-1}_{sup}(G) = 1$.

Case 2. Suppose that $1 \leq k = m \leq n-1$.
Then $G \cong P_r \circ \overline{K}_r$ with $n = 2k$ for all positive integer k (see Figure 2).

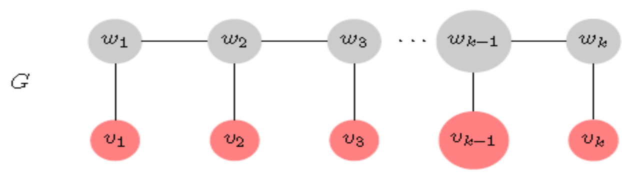

Figure 2: A graph G with $1 \leq k = m \leq n-1$.

The set $D = \{w_1, w_2, \ldots, w_k\}$ is a γ-set of G and $S = \{v_1, v_2, \ldots, v_k\}$ is a γ^{-1}_{sup}-set of G. Thus, $|V(G)| = 2k = n$, $\gamma(G) = |D| = k$, and $\gamma^{-1}_{sup}(G) = |S| = k = m$.

Case 3. Suppose that $1 < k < m < n-1$.
Then $G \cong P_2 \square P_p$ where $p \equiv 1(mod4)$ and $p \neq 1$. (see Figure 3).

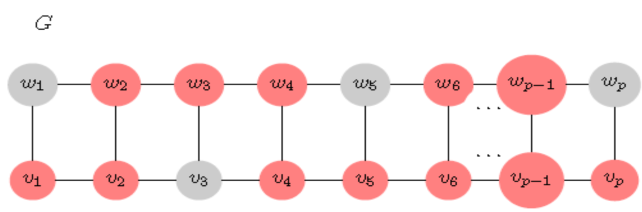

Figure 3: A graph G with $1 < k < m < n-1$.

Let $n = 2p$, $k = \frac{p+1}{2}$, and $m = 2p - k$. The set $D = \left\{w_{4r-3} : r = 1,2,\ldots,\frac{p+3}{4}\right\} \cup \left\{v_{4r-1} : r = 1,2,\ldots,\frac{p-1}{4}\right\}$ is a γ-set of G and $S = V(G) \setminus D$ is a γ^{-1}_{sup}*-set of* G. Thus, $|V(G)| = 2p = n$, $\gamma(G) = |D| = \frac{p+3}{4} + \frac{p-1}{4} = \frac{p+1}{2} = k$, and $\gamma^{-1}_{sup}(G) = |S| = |V(G) \setminus D| = n - k = 2p - k = m$.

Case 4. Suppose that $1 = k < m = n-1$.
Then $G \cong K_1 + \overline{K}_m$ where $m \geq 2$ (see Figure 4).

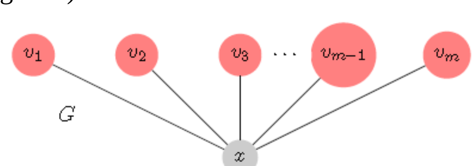

Figure 4: A graph G with $1 = k < m = n-1$.

The set $D = \{x\}$ *is a* γ-set of G and $S = \{v_i : i = 1,2,\dots,m\}$ is a γ_{sup}^{-1}-set of G. Thus, $|V(G)| = m + 1 = n$, $\gamma(G) = |D| = 1 = k$, and $\gamma_{sup}^{-1}(G) = |S| = m$. ■

Corollary 2.5. *The difference between* $\gamma_{sup}^{-1}(G) - \gamma(G)$ *can be made arbitrarily large.*

Proof: By Theorem 2.4, there exists a connected graph G such that $\gamma(G) = 1$ and $\gamma_{sup}^{-1}(G) = n + 1$. Then $\gamma_{sup}^{-1}(G) - \gamma(G) = (n + 1) - 1 = n$. Hence, the difference between $\gamma_{sup}^{-1}(G) - \gamma(G)$ can be made arbitrarily large. ■

Let $P_n = [v_1, v_2, \dots, v_n]$ such that $V(P_n) = \{v_1, v_2, \dots, v_n\}$ and $E(P_n) = \{v_1v_2, v_2v_3, \dots, v_{n-1}v_n\}$. The next result shows the super inverse domination number of a path graph P_n.

Theorem 2.6. *Let* $G = P_n$ *of order* $n \geq 2$. *Then*

$$\gamma_{sup}^{-1}(G) = \begin{cases} \frac{2n}{3}, & if\ n \equiv 0 (\mod 3) \\ \frac{2(n-1)}{3}, & if\ n \equiv 0 (\mod 3) \\ \frac{2n-1}{3}, & if\ n \equiv 0 (\mod 3). \end{cases}$$

Proof : Let $G = P_n$ of order $n \geq 2$.
Case 1. $n \equiv 0 (mod\ 3)$. Consider the graph G below (see Figure 5).

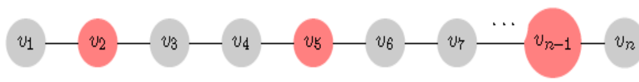

Figure 5: A graph G with $\gamma_{sup}^{-1}(G) = \frac{2n}{3}$.

The set $D = \{v_{3i-1} : i = 1,2,\dots,\frac{n}{3}\}$ is the minimum dominating set of G and the set $S = V(G) \setminus D$ is the γ_{sup}^{-1}-set of G. Thus, $\gamma_{sup}^{-1}(G) = |S| = |V(G) \setminus D| = |V(G)| - |D| = n - \frac{n}{3} = \frac{2n}{3}$.

Case 2. $n \equiv 1 (mod\ 3)$. Consider the graph G below (see Figure 6).

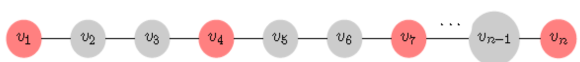

Figure 6: A graph G with $\gamma_{sup}^{-1}(G) = \frac{2(n-1)}{3}$.

The set $D = \{v_{3i-2} : i = 1,2,\dots, \frac{n+2}{3}\}$ is the minimum dominating set of G and the set $S = V(G) \setminus D$ is the γ_{sup}^{-1}-set of G. Thus, $\gamma_{sup}^{-1}(G) = |S| = |V(G) \setminus D| = |V(G)| - |D| = n - \frac{n+2}{3} = \frac{2n-2}{3} = \frac{2(n-1)}{3}$.

Case 3. $n \equiv 2 (mod\ 3)$. Consider the graph G below (see Figure 7).

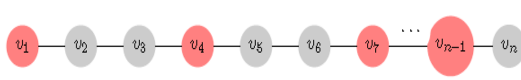

Figure 7: A graph G with $\gamma_{sup}^{-1}(G) = \frac{2n-1}{3}$.

The set $D = \{v_{3i-2} : i = 1,2,\dots,\frac{n+1}{3}\}$ is the minimum dominating set of G and the set $S = V(G) \setminus D$ is the γ_{sup}^{-1}-set of$G$. Thus, $\gamma_{sup}^{-1}(G) = |S| = |V(G) \setminus D| = |V(G)| - |D| = n - \frac{n+1}{3} = \frac{3n-n-1}{3} = \frac{2n-1}{3}$. ■

Let $C_n = [v_1, v_2, \dots, v_n]$ such that $V(C_n) = \{v_1, v_2, \dots, v_n\}$ and $E(C_n) = \{v_1v_2, v_2v_3, \dots, v_{n-1}v_n, v_nv_1\}$. The next result shows the super inverse domination number of a cycle graph C_n.

Theorem 2.7. *Let* $G = C_n$ *of order* $n \geq 3$. *Then*

$$\gamma_{sup}^{-1}(G) = \begin{cases} \frac{2n}{3}, if\ n \equiv 0(\ mod\ 3) \\ \frac{2(n-1)}{3}, if\ n \equiv 0(\ mod\ 3) \\ \frac{2n-1}{3}, if\ n \equiv 0\ (\ mod\ 3). \end{cases}$$

Proof : Let $G = C_n$ of order $n \geq 2$.
Case 1. $n \equiv 0(mod\ 3)$. Consider the graph G below (see Figure 8).

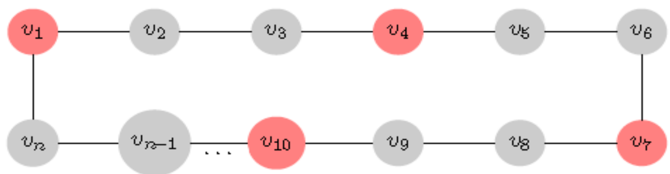

Figure 8: A graph G with $\gamma_{sup}^{-1}(G) = \frac{2n}{3}$.

The set $D = \{v_{3i-2}: i = 1,2,\ldots, \frac{n}{3}\}$ is the minimum dominating set of G and the set $S = V(G) \setminus D$ is the γ_{sup}^{-1}-set of G. Thus, $\gamma_{sup}^{-1}(G) = |S| = |V(G) \setminus D| = |V(G)| - |D| = n - \frac{n}{3} = \frac{2n}{3}$.

Case 2. $n \equiv 1(mod\ 3)$. Consider the graph G below (see Figure 9).

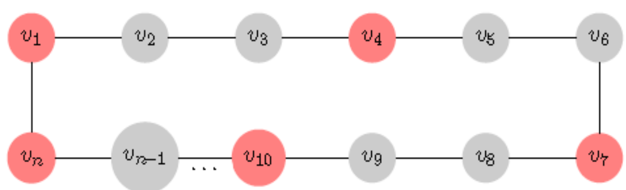

Figure 9: A graph G with $\gamma_{sup}^{-1}(G) = \frac{2(n-1)}{3}$.

The set $D = \{v_{3i-2} : i = 1,2,\ldots, \frac{n+2}{3}\}$ is the minimum dominating set of G and the set $S = V(G) \setminus D$ is the γ_{sup}^{-1}- set of G. Thus, $\gamma_{sup}^{-1}(G) = |S| = |V(G) \setminus D| = |V(G)| - |D| = n - \frac{n+2}{3} = \frac{2n-2}{3} = \frac{2(n-1)}{3}\}$.

Case 3. $n \equiv 2(mod\ 3)$. Consider the graph G below (see Figure 10).

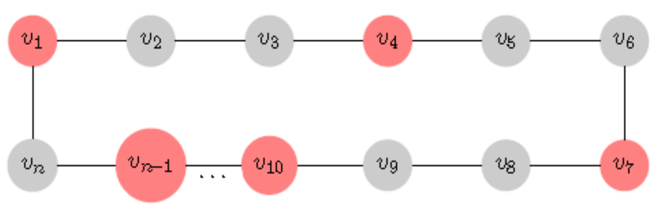

Figure 10: A graph G with $\gamma_{sup}^{-1}(G) = \frac{2n}{3}$.

The set $D = \{v_{3i-2}: i = 1,2,\ldots, \frac{n+1}{3}\}$ is the minimum dominating set of G and the set $S = V(G) \setminus D$ is the γ_{sup}^{-1}- set of G. Thus, $\gamma_{sup}^{-1}(G) = |S| = |V(G) \setminus D| = |V(G)| - |D| = n - \frac{n+1}{3} = \frac{3n-n-1}{3} = \frac{2n-1}{3}$. ■

The following result is an immediate consequence of Theorem 2.6 and Theorem 2.7.

Corollary 2.8. *Let* $G = P_n$ *of order* $n \geq 2$ *or* $G = C_n$ *of order* $n \geq 3$. *Then* $\gamma_{sup}^{-1}(G) = n - \gamma(G)$.

Theorem 2.9. *Let $G = K_1 + H$ be a connected graph of order $n \geq 2$. Then $\gamma_{sup}^{-1}(G) = n - 1$.*
Proof : Let $G = K_1 + H$ be a connected graph of order $n \geq 2$. The $V(K_1)$is a γ-$set\ of\ G$ and $S = V(G) \setminus V(K_1)$ is a γ_{sup}^{-1}-set of G. Thus, $\gamma_{sup}^{-1}(G) = |V(G) \setminus V(K_1)| = |V(G)| - |V(K_1)| = n - 1$. ■

Corollary 2.10. *If a graph G is a wheel $W_n = K_1 + C_{n-1}$, or a star $S_n = K_1 + \overline{K}_{n-1}$, or a fan $F_n = K_1 + P_{n-1}$, or a complete graph K_n, then $\gamma_{sup}^{-1}(G) = n - 1$.*

Proof : Clearly, $K_n = K_1 + K_{n-1}$. Thus, if G is W_n, S_n, F_n, or K_n, then $\gamma_{sup}^{-1}(G) = n - 1$ by Theorem 2.9. ■
A complete bipartite graph $K_{m,n}$ is a graph that has its vertex set partitioned into two subsets of m and n vertices, respectively with an edge between every pair of vertices if and only if one vertex in the pair is in the first subset and the other vertex is in the second subset.

Remark 2.11. *If a graph G is a complete bipartite $K_{m,n}$ with $m \geq 2$ and $n \geq 2$, then $\gamma_{sup}^{-1}(G) = n - 2$.*

VI. CONCLUSIONS

In this paper, we introduced the concept of super inverse domination in graphs and prove that given positive integers k, m and n such that $1 \leq k \leq m \leq n - 1$, where $n \geq 2$, there exists a connected graph G with $|V(G)| = n$, $\gamma(G) = k$, and $\gamma_{sup}^{-1}(G) = m$. Further, we prove the domination number of a path graph P_n, a cycle C_n, a wheel graph W_n, a fan graph F_n, a star graph S_n, a complete graph K_n, and a complete bipartite $K_{m,n}$.

Some related problems on super inverse domination in graphs are still open for research.
1. Characterize the super inverse dominating sets of the join, corona, Cartesian product, and lexicographic product of two graphs.
2. Find the inverse domination number of the join, corona, Cartesian product, and lexicographic product of two graphs.

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