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Original Article

An Application of Markov Chains in Optimizing Stock Portfolios in the Vietnamese Stock Market

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Abstract *- Markov chains are widely used as statistical models in various fields. In this paper, the author introduces the application of Markov chains to optimize stock portfolios. Markov chains are used to forecast the future returns of stocks. From this, investors can decide which stocks to invest in to optimize profits. Empirical analysis was conducted on some stock codes listed on the Vietnamese Stock Exchange.*

Keywords *- Markov chains, Investment portfolio, Return on investment.*

1. Introduction

The stock market has been a vibrant, potentially most discussed topic in recent years. There are no large enterprises in the world that exist without the stock market. There have been many annual conferences and research on this topic. There are many different research directions to approach the stock market, such as research direction from the Markov chain perspective [1,2], [5], [6], [7,8,9], [11], [16,17], [21], [24], research direction from the optimization perspective [3], [12], [13], [18,19,20], [26,27], research direction from the machine learning perspective [4], [10], [14,15], [22,23], [25], [28], ... All of these research directions focus on the goal of forecasting stock fluctuations, finding the most beneficial way for investors.

The goal of investors is to earn as much profit as possible through buying, selling, or owning specific investment portfolios. Any hasty decision can lead to losses. Understanding the expected rate of return will help investors build appropriate investment strategies to maximize profits and minimize risks. Investors often mitigate risks by investing money in various companies, trading different stocks, and receiving different dividends. Harry Markowitz, an American economist, laid the foundation for modern portfolio theory and addressed the issue of generating the highest expected returns while showing investors how to minimize risks. His method is also frequently used in portfolio analysis both in our country and around the world. Recently, the Markov chain method has become appealing to investors.

This non-parametric method is less complex and yields results similar to the Markowitz model, but it is faster and easier. In the field of finance, the Markov chain method was first used by McQueen and Thorley in their essay (McQueen & Thorley [8]) and later by Doubleday and Esunge in the U.S. stock market (Doubleday & Esunge [11]). There are also related articles on the Nigerian stock market (Agwuegbo, Adewole, & Maduegbuna [21]) and the Prague Stock Exchange (Svoboda & Lukas [16]). These authors analyzed different stock markets at various times, considering the number of different company stocks. However, they share a common analytical point: stock return rates can be classified into several classes with common characteristics, primarily divided into three classes: increasing, decreasing, or stable.

The Vietnamese stock market has been established and developed for over 20 years. Although there have been significant advancements in market capitalization and the number of listed assets, the Vietnamese stock market is still considered a small, emerging market that is very attractive but carries many risks. Therefore, optimizing the investment portfolio to maximize profits and minimize risks is very important. In this paper, we apply the Markov chain model to analyze the changes in daily return rates based on current stock prices to predict future return rates. This can help investors make decisions on which company stocks to invest in. The empirical analysis is conducted on ten Ho Chi Minh City Stock Exchange stocks.

2. Basic Definitions and Knowledge

Consider a system that is observed at discrete times 0,1,2, ... Suppose those observations are $X_0, X_1, \dots, X_n, \dots$. Then, we have a sequence of random variables (X_n) in which X_n is the state of the system at time n. Assume that each X_n $(n = 0,1,2,\dots)$ is a

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discrete random variable. Let E be the set of values of (X_n). Then, E is a finite or countable set, and its elements are denoted by $i, j, k, \ldots$. We call E is the state space of the system.

We say that the sequence of random variables (X_n) is a Markov chain if for all $n_1 < \cdots < n_k < n_{k+1}$ and for all $i_1, i_2, \ldots, i_k, i_{k+1} \in E$, we have

$$P\{X_{n_{k+1}} = i_{k+1} | X_{n_1} = i_1, X_{n_2} = i_2, \ldots, X_{n_k} = i_k\} = P\{X_{n_{k+1}} = i_{k+1} | X_{n_k} = i_k\}.$$

Thus, if the system's current state is known, then the past and future states are independent.

The transition probability after a step p_{ij} is the conditional probability that the system at time n at state i will move to state j at time $n+1$, which means

$$p_{ij} = P\{X_{n+1} = j | X_n = i\}.$$

The matrix $\mathbb{P}(n) = (p_{ij})$, where $0 \le p_{ij} \le 1, \forall i, j \in E, \sum_{j \in E} p_{ij} = 1$ is called the transition probability matrix after one step; to be short, one says it is a transition probability matrix.

The transition probability after n steps is the probability that the system at state i will move to state j after n steps, which means

$$p_{ij}(n) = P\{X_{m+n} = j | X_m = i\}.$$

The matrix $\mathbb{P}(n) = (p_{ij}(n))$, is the transition probability matrix after n step. The elements of this matrix have the property

$$0 \le p_{ij}\ (n) \le 1, \forall i, j \in E; \sum_{j \in E} p_{ij}(n) = 1.$$

Suppose the state space E has d elements, i.e. $E = \{1,2, \ldots, d\}$. Then, $\mathbb{P}$ and $\mathbb{P}(n)$ are square matrices of order d. And for all $n, m = 0,1,2, \ldots$ we have

$$\mathbb{P}(n+m) = \mathbb{P}(n).\mathbb{P}(m).$$

Let $u_i(n) = P(X_n = i)$ be the probability that the system at time n in state i. Denote the vector

$$U(n) = \left(u_1(n), u_2(n), \ldots, u_d(n)\right),$$

Which describes the distribution of the system at time n. We call $U = U(0) = (u_1, u_2, \ldots, u_d)$ where $u_i = u_i(0) = P(X_0 = i)$ is the initial distribution of the system.

It is not difficult to see that

$$U(m+n) = U(m).\mathbb{P}^n, \tag{1}$$

$$U(n) = U.\mathbb{P}^n. \tag{2}$$

The initial distribution U is called stationary if $U(n)$ does not depend on n, which means $U(n) = U, \forall n$ or $U = U.\mathbb{P}$. We say that a Markov chain has a limiting distribution if for all $i, j\ \in E =\ \{1,2, \ldots, d\}$ there exists. $\lim_{n \to \infty} p_{ij}(n) = \pi_j$ that does not depend on i and satisfies the conditions: $\pi_j \ge 0, \sum_{j \in E} \pi_j = 1$. In that case, we call $\pi = (\pi_1, \pi_2, \ldots, \pi_d)$ is a limiting distribution.

One of the important problems in the study of Markov chains is to find conditions to ensure the existence of a limiting distribution and the existence of a stationary distribution. In the following, one has two important results about Markov chains:

Theorem 2.1. (see [17], chapter 5) If a limiting distribution exists, then it is a uniquely stationary distribution.

Theorem 2.2. (see [7], chapter 6) A Markov chain has a limiting distribution $\pi = (\pi_1, \pi_2, \ldots, \pi_d)$ where $\pi_j \ge 0, \sum_{j \in E} \pi_j = 1$ if and only if the chain is regular, that means there exists n_0 such that $p_{ij}(n_0) > 0, \forall i, j \in E$.

Therefore, if the Markov chain is regular, then a limiting distribution exists, and this distribution is the uniquely stationary distribution, and it is found by solving the system of equations:

$$\pi = \pi.\mathbb{P} \Leftrightarrow \pi.(1 - \mathbb{P}) = 0. \tag{3}$$

Using Markov chains can help us estimate the probability of events occurring in the future by analyzing the known probabilities in the present. Because this technique is relatively simple, it has been widely applied in various fields such as economics, medicine, psychology, etc. In the rest of the article, the author presents the application of Markov chains in optimizing stock portfolios.

3. Applying Markov Chains to Optimize Portfolios

3.1. Model Building

The process of analyzing and optimizing portfolios based on Markov chains is carried out in the following steps:

- Collect data on stock prices of companies.
- Build the state space.
- Find the state transition probability matrix.
- Calculate to obtain the initial distribution.
- Calculate to obtain the limit distribution.
- Analyze and make decisions.

To build the state space and transition probability matrix from the data on closing prices of stocks, we calculate the daily return rate sequence $\{r_n\}$ for each company's stock according to the following formula

$$r_n = \ln \frac{S_n}{S_{n-1}}$$

Where S_n is the closing price on the n^{th} day of the company's stock under consideration.

The closing price of stocks on two consecutive days does not change much, so the daily return rate also has little difference. With the obtained values of $\{r_n\}$, we divide the values into three classes corresponding to the following states:

State	S_1 (low return)	S_1 (medium return)	S_1 (high return)
Value of r_n	$< -0.5\%$	$[-0.5\%, 0.5\%]$	$> 0.5\%$

We consider the sequence of returns as a Markov chain with state space $E = \{S_1, S_2, S_3\}$. For each stock, from calculating the number of return values in the state $S_i, i = 1,2,3$, we find the initial distribution

$$U = U(0) = (u_1, u_2, u_3)$$

Where $u_i = \frac{f_i}{\sum_{i=1}^{3} f_i}$, f_i is the number of times the return value in the state $S_i, i = 1,2,3$ and $\sum_{i=1}^{3} f_i$ is the number of elements of the sequence $\{r_n\}$.

Based on calculating the number of times f_{ij} transitions from state i to state j after one step, we construct the state transition frequency matrix $F = (f_{ij}), i, j \in E$. From this, we can find the transition probability matrix as follows:

$$\mathbb{P} = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}$$

Where $p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{3} f_{ij}}$.

Applying equation (3), we find the limit distribution $\pi = (\pi_1, \pi_2, \pi_3)$. From the limit distribution, we analyze and select the stocks to include in the portfolio, which are stocks with the probability that the return rate in state S_3 is higher than the probability that the return rate in the state S_1.

3.2. Application of the Model to Real Data

The data processed in this paper are the closing prices of 10 stocks listed on the Ho Chi Minh City Stock Exchange (HOSE). Closing price data is collected from April 5, 2023, to April 5, 2024, including 252 values for each stock (source: https://s.cafef.vn). Data processing is performed using R and Excel software.

The list of company stocks considered in this paper is given in Table 1.

Table 1. Stocks and companies mentioned

No.	Stock code	Company
1	FPT	FPT Joint Stock Company
2	GVR	Vietnam Rubber Industry Group
3	HPG	Hoa Phat Group Joint Stock Company
4	PLX	Vietnam National Petroleum Group
5	SAB	Saigon Beer - Alcohol - Beverage Corporation
6	SSI	SSI Securities Corporation
7	VIC	Vingroup Corporation - Joint Stock Company
8	VJC	Vietjet Aviation Joint Stock Company
9	VNM	Vietnam Dairy Products Joint Stock Company
10	VPB	Vietnam Prosperity Joint Stock Commercial Bank

With the collected data, a total of 2520 closing price values of 10 stocks, calculate the daily return series for each stock. Then, classify the return values into three states. S_1, S_2, S_3 as presented in section 3.1. The number of times the return value falls into states S_1, S_2, S_3 and the initial distribution $U(0)$ is shown in Table 2.

Table 2. Number of times the return value falls into states and initial distribution

Stock Code	S_1	S_2	S_3	$U(0)$
FPT	63	102	86	$(0.2510 \quad 0.4064 \quad 0.3426)$
GVR	84	49	118	$(0.3347 \quad 0.1952 \quad 0.4701)$
HPG	81	74	96	$(0.3227 \quad 0.2948 \quad 0.3825)$
PLX	77	99	75	$(0.3068 \quad 0.3944 \quad 0.2988)$
SAB	96	89	66	$(0.3825 \quad 0.3546 \quad 0.2629)$
SSI	76	70	105	$(0.3028 \quad 0.2789 \quad 0.4183)$
VIC	83	107	61	$(0.3307 \quad 0.4263 \quad 0.2430)$
VJC	80	106	65	$(0.3187 \quad 0.4223 \quad 0.2590)$
VNM	86	91	74	$(0.3426 \quad 0.3626 \quad 0.2948)$
VPB	85	81	85	$(0.3387 \quad 0.3227 \quad 0.3386)$

The daily return state transition frequency matrix and the transition probability matrix are given in Table 3.

Table 3. State transition frequency matrix and transition probability matrix

Stock	State transition frequency matrix	Transition probability matrix
FPT	$F_{FPT} = \begin{pmatrix} 16 & 27 & 20 \\ 27 & 38 & 36 \\ 20 & 36 & 30 \end{pmatrix}$	$\mathbb{P}_{FPT} = \begin{pmatrix} 0.2539 & 0.4286 & 0.3175 \\ 0.2673 & 0.3762 & 0.3565 \\ 0.2326 & 0.4186 & 0.3488 \end{pmatrix}$
GVR	$F_{GVR} = \begin{pmatrix} 29 & 10 & 44 \\ 15 & 14 & 20 \\ 39 & 25 & 54 \end{pmatrix}$	$\mathbb{P}_{GVR} = \begin{pmatrix} 0.3494 & 0.1205 & 0.5301 \\ 0.3061 & 0.2857 & 0.4082 \\ 0.3305 & 0.2119 & 0.4576 \end{pmatrix}$
HPG	$F_{HPG} = \begin{pmatrix} 22 & 26 & 32 \\ 24 & 24 & 26 \\ 35 & 23 & 38 \end{pmatrix}$	$\mathbb{P}_{HPG} = \begin{pmatrix} 0.2750 & 0.3250 & 0.4000 \\ 0.3243 & 0.3243 & 0.3514 \\ 0.3646 & 0.2396 & 0.3958 \end{pmatrix}$
PLX	$F_{PLX} = \begin{pmatrix} 22 & 35 & 20 \\ 24 & 39 & 35 \\ 30 & 25 & 20 \end{pmatrix}$	$\mathbb{P}_{PLX} = \begin{pmatrix} 0.2857 & 0.4546 & 0.2597 \\ 0.2449 & 0.3980 & 0.3571 \\ 0.4000 & 0.3333 & 0.3636 \end{pmatrix}$
SAB	$F_{SAB} = \begin{pmatrix} 43 & 32 & 20 \\ 28 & 39 & 22 \\ 25 & 17 & 24 \end{pmatrix}$	$\mathbb{P}_{SAB} = \begin{pmatrix} 0.4526 & 0.3369 & 0.2105 \\ 0.3146 & 0.4382 & 0.2472 \\ 0.3788 & 0.2576 & 0.3636 \end{pmatrix}$

SSI	$F_{SSI} = \begin{pmatrix} 21 & 12 & 42 \\ 20 & 24 & 26 \\ 34 & 34 & 37 \end{pmatrix}$	$\mathbb{P}_{SSI} = \begin{pmatrix} 0.2800 & 0.1600 & 0.5600 \\ 0.2857 & 0.3429 & 0.3714 \\ 0.3238 & 0.3238 & 0.3524 \end{pmatrix}$
VIC	$F_{VIC} = \begin{pmatrix} 32 & 32 & 19 \\ 25 & 56 & 25 \\ 25 & 19 & 17 \end{pmatrix}$	$\mathbb{P}_{VIC} = \begin{pmatrix} 0.3855 & 0.3855 & 0.2290 \\ 0.2358 & 0.5284 & 0.2358 \\ 0.4098 & 0.3115 & 0.2787 \end{pmatrix}$
VJC	$F_{VJC} = \begin{pmatrix} 16 & 37 & 27 \\ 33 & 47 & 25 \\ 30 & 22 & 13 \end{pmatrix}$	$\mathbb{P}_{VJC} = \begin{pmatrix} 0.2000 & 0.4625 & 0.3375 \\ 0.3143 & 0.4476 & 0.2381 \\ 0.4615 & 0.3385 & 0.2000 \end{pmatrix}$
VNM	$F_{VNM} = \begin{pmatrix} 25 & 34 & 27 \\ 33 & 30 & 27 \\ 28 & 26 & 20 \end{pmatrix}$	$\mathbb{P}_{VNM} = \begin{pmatrix} 0.2907 & 0.3953 & 0.3140 \\ 0.3667 & 0.3333 & 0.3000 \\ 0.3784 & 0.3513 & 0.2703 \end{pmatrix}$
VPB	$F_{VPB} = \begin{pmatrix} 32 & 26 & 27 \\ 24 & 28 & 29 \\ 28 & 27 & 29 \end{pmatrix}$	$\mathbb{P}_{VPB} = \begin{pmatrix} 0.3765 & 0.3059 & 0.3176 \\ 0.2963 & 0.3457 & 0.3580 \\ 0.3333 & 0.3214 & 0.3453 \end{pmatrix}$

From the initial distribution $U = U(0)$ and the transition probability matrix, we can find the distribution in the following periods based on equations (1) (2). This Markov chain is regular, so there is a limiting distribution. This is a stationary distribution found in equation (3). For example, to find the limiting distribution of the rate of return of FPT stock, we solve the system of equations

$$(\pi_1, \pi_2, \pi_3)\mathbb{P}_{FPT} = (\pi_1, \pi_2, \pi_3).$$

Solving this system, we find: $\pi_1 \approx 0.252, \pi_2 \approx 0.404, \pi_3 \approx 0.344$.

Thus, the limiting distribution of the rate of return of FPT stock is $(0.252, 0.404, 0.344)$. This means, in the future, the return on FPT shares will be less than 0.5% with a probability of 0.252, the return will be in the range $[-0.5\%;\ 0.5\%]$ with a probability of 0.404 and the return will be higher than 0.5% with probability 0.344.

Similarly, the limit distribution of the returns on the remaining shares is given in Table 4.

Table 4. Limit distribution of returns

Stock	Limit distribution
FPT	(0.2520, 0.4040, 0.3440)
GVR	(0.3320, 0.1960, 0.4720)
HPG	(0.3238, 0.2920, 0.3842)
PLX	(0.3039, 0.3958, 0.3003)
SAB	(0.3846, 0.3516, 0.2638)
SSI	(0.3000, 0.2800, 0.4200)
VIC	(0.3273, 0.4287, 0.2440)
VJC	(0.3164, 0.4240, 0.2596)
VNM	(0.3440, 0.3600, 0.2960)
VPB	(0.3358, 0.3241, 0.3401)

With the results found in the table above, we can choose the optimal portfolio of 5 stocks with the probability that the return rate is in state. S_3 is higher than the probability that the return rate in the state S_1. These are the following stocks: FPT, GVR, HPG, SSI, and VPB.

4. Conclusion

This paper has presented a method to optimize the stock portfolio by using Markov chains, analyzing the daily return rate of stocks, and considering the return rate series as a Markov chain with three states. The empirical analysis was conducted on ten Ho Chi Minh City Stock Exchange stocks. In addition to the empirical analysis, the paper also presents theoretical issues for applying this method. With the issues presented, it can be seen that this method has the advantage of being quite simple. This method provides preliminary results on the rate of return, which can help investors choose which securities to invest in to get high profits. The results obtained from the method are used for forecasting and are not necessarily true to what will happen in reality. When using this method, the transition probability matrix is assumed to be constant over time, which is difficult to ensure in practice.

On the other hand, this method also does not mention the risk factor, which always exists when investing. However, many econometric methods that consider both profit and risk will be complicated, take much time to analyze and evaluate, and the results are not significantly different. Therefore, the Markov chain method can still be considered by investors to analyze, evaluate and make appropriate securities investment decisions.

References

[1] Abdelmoula Dmouj, “*Stock Price Modelling: Theory and Practice*,” Vrije Universiteit Faculty of Sciences Amsterdam, The Netherlands, pp. 1-40, 2006. [Google Scholar] [Publisher Link]

[2] Siti Nazifah Zainol Abidin, and Maheran Mohd Jaffar, “Forecasting Share Price of Small Size Companies in Bursa Malaysia using Geometric Brownian Motion,” *Applied Mathematics & Information Sciences*, vol. 8, no. 1, pp. 107-112, 2014. [CrossRef] [Google Scholar] [Publisher Link]

[3] Jashua Achiam et al., “Constrained Policy Optimization,” *Proceedings of the 34th International Conference on Machine Learning*, pp. 22-31, 2017. [Google Scholar] [Publisher Link]

[4] Andrea Lonza, *Reinforcement Learning Algorithms with Python: Learn, Understand, and Develop Smart Algorithms for Addressing AI Challenges*, Packt Publishing, pp. 1-366, 2019. [Google Scholar] [Publisher Link]

[5] Imre Csiszar, and Janos Korner, *Information Theory: Coding Theorems for Discrete Memoryless Systems*, Cambridge University Press, 2011. [Google Scholar] [Publisher Link]

[6] Divya Aggarwal, “Random Walk Model and Asymmetric Effect in Korean Composite Stock Price Index,” *Afro-Asian Journal of Finance and Accounting*, vol. 8, no. 1, pp. 85-104, 2018. [CrossRef] [Google Scholar] [Publisher Link]

[7] Dang Hung Thang, *Stochastic Processes and Stochastic Computation*, Hanoi National University Publishing House, 2006.

[8] Grant McQueen, and Steven Thorley, “Are Stock Returns Predictable? A Test Using Markov Chains,” *The Journal of Finance*, vol. 46, no. 1, pp. 239-263, 1991. [CrossRef] [Google Scholar] [Publisher Link]

[9] G. Bormetti et al., “A Backward Monte Carlo Approach to Exotic Option Pricing,” *European Journal of Applied Mathematics*, vol. 29, no. 1, pp. 146-187, 2018. [CrossRef] [Google Scholar] [Publisher Link]

[10] Hongyang Yang et al., “Deep Reinforcement Learning for Automated Stock Trading: An Ensemble Strategy,” *Proceedings of the First ACM International Conference on AI in Finance*, pp. 1-8, 2021. [CrossRef] [Google Scholar] [Publisher Link]

[11] Kevin J. Doubleday, and Julius N. Esunge, “Application of Markov Chains to Stock Trends,” *Journal of Mathematics and Statistics*, vol. 7, no. 2, pp. 103-106, 2011. [Google Scholar] [Publisher Link]

[12] Sham M. Kakade, “A Natural Policy Gradient,” *Advances in Neural Information Processing Systems*, pp. 1-8, 2001. [Google Scholar] [Publisher Link]

[13] Karl W. Cobbe et al., “Phasic Policy Gradient,” *Proceedings of the 38th International Conference on Machine Learning*, pp. 1-8, 2021. [Google Scholar] [Publisher Link]

[14] Laura Graesser, and Wah Loon Keng, *Foundations of Deep Reinforcement Learning: Theory and Practice in Python*, Addison-Wesley Data & Analytics Series, 2019. [Google Scholar] [Publisher Link]

[15] Maxim Lapan, *Deep Reinforcement Learning Hands-On: Apply Modern RL Methods to Practical Problems of Chatbots, Robotics, Discrete Optimization, Web Automation, and More*, Second Edition, Packt Publishing, 2020. [Google Scholar] [Publisher Link]

[16] Milan Svoboda, and Ladislav Lukas, “Application of Markov Chain Analysis to Trend Prediction of Stock Indices,” *Proceedings of 30th International Conference Mathematical Methods in Economics*, pp. 1-6, 2012. [Google Scholar] [Publisher Link]

[17] Nguyen Duy Tien, *Probability Models and Applications. Part I: Markov Chains and Applications*, Hanoi National University Publishing House, 2000.

[18] Qing Wang et al., “Divergence-Augmented Policy Optimization,” *Advances in Neural Information Processing Systems*, 2019. [Google Scholar] [Publisher Link]

[19] John Schulman et al., “Trust Region Policy Optimization,” *Proceedings of the 32nd International Conference on Machine Learning*, pp. 1889-1897, 2015. [Google Scholar] [Publisher Link]

[20] John Schulman et al., “Proximal Policy Optimization Algorithms,” *arxiv*, pp.1-12, 2017. [CrossRef] [Google Scholar] [Publisher Link]

[21] S.O.N. Agwuegbo, A.P. Adewole, and A.N. Maduegbuna, “A Random Walk Model for Stock Market Prices,” *Journal of Mathematics and Statistics*, vol. 6, no. 3, pp. 342-346, 2010. [Google Scholar] [Publisher Link]

[22] Richard S. Sutton, and Andrew G. Barto, *Reinforcement Learning: An Introduction*, Second Edition, The MIT Press, 1998. [Google Scholar] [Publisher Link]

[23] Wenhang Bao, and Xiao-yang Liu, “Multi-Agent Deep Reinforcement Learning for Liquidation Strategy Analysis,” *arXiv*, pp. 1-9, 2019. [CrossRef] [Google Scholar] [Publisher Link]

[24] W. Farida Agustini, Ika Restu Affianti, and Endah RM Putri, "Stock Price Prediction using Geometric Brownian Motion," *Journal of Physics: Conference Series*, pp. 1-12, 2018. [CrossRef] [Google Scholar] [Publisher Link]

[25] Xiao-Yang Liu et al., "FinRL: A Deep Reinforcement Learning Library for Automated Stock Trading in Quantitative Finance," *arXiv*, pp. 1-12, 2020. [CrossRef] [Google Scholar] [Publisher Link]

[26] Yuhui Wang et al., "Trust Region-Guided Proximal Policy Optimization," *Advances in Neural Information Processing Systems*, 2019. [Google Scholar] [Publisher Link]

[27] Yuhui Wang, Hao He, and Xiaoyang Tan, "Truly Proximal Policy Optimization," *Proceedings of the 35th Uncertainly in Artificial Intelligence Conference*, vol. 115, pp. 113-122, 2020. [Google Scholar] [Publisher Link]