International Journal of Mathematics Trends and Technology
ISSN: 2231-5373/ https://doi.org/10.14445/22315373/IJMTT-V71I4P105

Volume 71 Issue 4, 37-54, April 2025
© 2025 Seventh Sense Research Group®

Original Article

Fraction Battles: Software for Rational Numbers using Representational Tools

Roza Vlachou

Mathematics Education and Multimedia Laboratory, Department of Primary Education, University of the Aegean, Rhode, Greece.

[1]*Corresponding Author : r.vlachou@aegean.gr*

Received: 28 February 2025 Revised: 30 March 2025 Accepted: 17 April 2025 Published: 30 April 2025

Abstract *- This paper presents a software application focused on the concepts of equal parts of the unit whole, improper fractions, and the ordering of rational numbers using the geometric model of the number line. The software aims to familiarize students with positive rational numbers through various activities in a dynamic multimedia environment and to help them overcome the difficulties they often encounter with fractions. This is achieved using representational tools and the 10+1 foundational mathematical elements of the applied theoretical framework "RhodeScript," on which the added value of the software is based. Findings from long-standing educational Research determined the content and activities of the software. This digital game aims to address possible gaps in existing teaching practices related to fractions, to support school textbooks and the Mathematics Curriculum, and to assist elementary and lower secondary school students in overcoming the mentioned conceptual difficulties. When implemented as supplementary material during the instruction of rational numbers, the software was found to help mitigate students' difficulties with fractional concepts.*

Keywords *- Educational software, representational tools, positive rational numbers, 10+1 points theory (RhodeScript), primary education.*

1. Introduction

The concept of fractions constitutes one of the most significant cognitive domains in Mathematics. It is introduced to students from the early years of primary education across various educational systems worldwide. In the Greek educational system, fractions are taught from the first grade of primary school, according to the new curricula, reflecting the importance and necessity of the concept for the comprehension of other mathematical ideas. Research has demonstrated a strong correlation between students' understanding of fractions and their performance in problem construction and solving. [31] Furthermore, the use of fractional quantities can foster a more explicit application of structures and relationships in algebraic contexts. [32] In this regard, fractional knowledge significantly influences the manner in which students formulate equations to represent multiplicative relationships between two unknown quantities. [33]

The interconnection between rational numbers and broader mathematical concepts has prompted considerable interest among educational researchers, establishing fractions as a challenging and compelling area for investigation. Despite their importance, however, fractions present substantial difficulties in comprehension for students across all educational stages. These learning difficulties often become ingrained within students' cognitive frameworks and can persist into adulthood. [3] Consequently, misconceptions are formed, impeding not only the educational process but also the students' long-term learning trajectories.

In view of the pivotal role that fractions play in the understanding of other mathematical concepts and considering the persistent difficulties faced by primary school students internationally, the development of an educational digital game was initiated. This game aims to mitigate the challenges students encounter with rational numbers and to enhance their conceptual understanding in this critical area.

The Fraction Battles (see Figure 1) is a game that targets concepts such as equal parts of a fractional unit, improper fractions, and ordering rational numbers using the number line as a geometric model. It was developed using various software programs and programming tools such as Scratch, PowerPoint, Kidspiration, Pinnacle Studio, Smooth Board, and Hotpotatoes, among others. The objective of the game is to help students become familiar with positive rational numbers and reduce the challenges they face with fractions by utilizing representational tools and the 10+1 core mathematical components of the RhodeScript theory [5, 4], which form the foundation of the software's added pedagogical value.

A key characteristic of this educational software is that its content and activities were not arbitrarily chosen or designed but were meticulously selected based on extensive findings from long-term Research into students' difficulties with fractions. Each activity in the game is aimed at addressing a specific learning difficulty. The language used in the activities was finalized after pilot testing. It is intended for students in the final years of primary school (5th and 6th grade), where it was piloted, as well as for students in the early grades of middle school.

Fig. 1 The homepage of the Fraction Battles software

2. Theoretical Framework

2.1. Representational Tools in Mathematics

The [6] highlights the critical role of representations in teaching and the significant influence they have had on mathematics educators, as they offer a means of addressing students' difficulties with fractions. A mathematical idea or concept can be represented in three ways: manipulative models/objects, visual (illustrated representation), and symbolic (written notation). This notion of multiple representations is grounded in the understanding that children's conceptual development progresses from concrete experiences to more abstract thinking. Representations serve as reference points for helping students learn mathematics more effectively.

According to some researchers [34-36], representing mathematical objects in multiple ways is important in mathematical understanding and brings value to teaching processes. In addition, recent trends in curriculum standards, including standards developed by the National Council of Teachers of Mathematics [37], have highlighted the productive role that drawn models and other representational tools can play in teaching and learning mathematics. [38] Although the representations add complexity, using a range of representational tools is necessary for developing children's understanding of fractions because each provides links to the underlying fractions concepts, and children require support to make active connections within and between the various representations. [39]

However, [14] called attention to a cognitive paradox hidden within various representations. Handling these representational tools and choosing the distinguishing features of the concept we must treat and convert is not learnt automatically. This learning results from explicit teaching in which the teacher must render the student co-responsible. Teachers often underestimate this aspect and pass from one register to another, believing the student follows. The teacher can jump from one register to another without problems because he has already conceptualized it, while the student does not. [11]

Building on this perspective, the designed software/game leverages technology to present representational tools to improve students' performance in rational numbers. The added value of the software lies in its ability to present a wide range of representational tools and facilitate translation between different systems of representation—something not feasible without appropriate software. It also incorporates the 10+1 structural elements of the RhodeScript Theory.

2.2. The Applied Theoretical Framework: RhodeScript

The Laboratory of Mathematics Didactics and Multimedia at the University of the Aegean has been organizing and overseeing the theory and practice of Applied (and essentially Applicable) Mathematics Didactics since its founding in 2000. This is done through Practical Training for final-year students of the Education Department, placing them in real classroom settings in elementary schools across Rhodes. Based on years of experience, the research team proposed a teaching framework for managing mathematics content in real classrooms. This framework draws on mathematical tools from the RhodeScript Theory, as researched and implemented by [5, 4]. The RhodeScript framework is built around eleven fundamental mathematical practices—10+1 tools—forming its pedagogical backbone. The name "RhodeScript" comes from the initials of these mathematical tools in English:

1. Representations
2. History of mathematics
3. Open problems
4. breach of Didactical contract
5. Estimation and mental Computation
6. Spatial ability and geometric transformations
7. Counterexamples
8. Realistic Mathematics Education
9. Interdisciplinary
10. Posing problem
11. Technology

This framework aims to enhance mathematical literacy through various practices, methods, and tools that encourage students to grasp mathematical concepts in meaningful contexts. It promotes knowledge discovery through multiple problem-solving strategies, externalizing and exchanging diverse approaches.

2.3. Student Difficulties with Fractions

The concept of rational numbers is one of the most important in mathematics. It is introduced to students from the early elementary school years, both in the Greek education system and internationally. However, despite its significance, rational numbers present numerous conceptual challenges to students across all grade levels. These difficulties persist in students' cognitive structures for extended periods-even into adulthood. [17] As a result, misconceptions hinder the learning process and students' academic progression. [23] One key challenge is the symbolism of rational numbers, which often leads to misunderstandings. Many students struggle to perceive a fraction a/b as a single number, instead treating it as two separate whole numbers. [24] A typical example of this misunderstanding appears when adding fractions, e.g. $\frac{5}{6}+\frac{2}{3}$ students tend to add both the numerators and denominators, arriving at $\frac{7}{9}$as a result. This stems from viewing the numerator and denominator as independent rather than interconnected entities. [28]

Another example involves comparing fractions with the same numerator, such as $\frac{1}{5}$ and$\frac{1}{3}$. Many students choose the one with the larger denominator (e.g., $\frac{1}{5}$) because they mistakenly think that five is greater than 3 in this context. Additionally, due to students' familiarity with the sequence of whole numbers—where no numbers exist between two consecutive integers—they often extend this logic to fractions, failing to grasp the concept of density. [21] They assume that no number can exist between two fractions, missing the idea that there are infinitely many fractions in between. Students also find it challenging to order fractions from smallest to largest (and vice versa) and struggle with the fundamental concept of dividing a whole into equal parts, even after receiving instruction on fractions. [30, 29] Another critical challenge is their ability to translate between different representational systems of fractions, a skill essential for solving problems and understanding mathematical concepts. [7, 9, 10]

3. Methodology

1,892 participants participated in the research studies that shaped the software's goals, purpose, and content. The sample was selected using a combination of census, stratified, and convenience sampling methods, depending on the objectives and needs of each research phase. The Research followed qualitative and quantitative approaches, incorporating content analysis and case studies. This created a triangulation of methodology, time, location, and theory to ensure the validity of findings. [8] Data collection tools included questionnaires and tests designed by the researchers and finalized after pilot studies. Additional methods included semi-structured interviews, video recordings, observations, and literature reviews. Data analysis involved descriptive analysis and the use of the Statistical Implicative Analysis (SIA) model [15], which was applied using the CHIC software

(Cohesive Hierarchical Implicative Classification) [13] and Microsoft Excel. The implicative analysis is visually presented in similarity diagrams that group variables based on how similarly participants responded. Variables that elicited similar responses are grouped. Software evaluation was based on criteria established by [26]—these methodological procedures aimed to produce reliable findings that could be generalized.

4. Software Rules

The "Fraction Battles" educational game can be played in groups or individually, depending on the number of students in the class. In this study, it was implemented using a group format. A magnetic whiteboard, video projector, magnetic pawns, and die are required to play the game. Students/players place their pawns at the start of the digital board (see Figure 2).

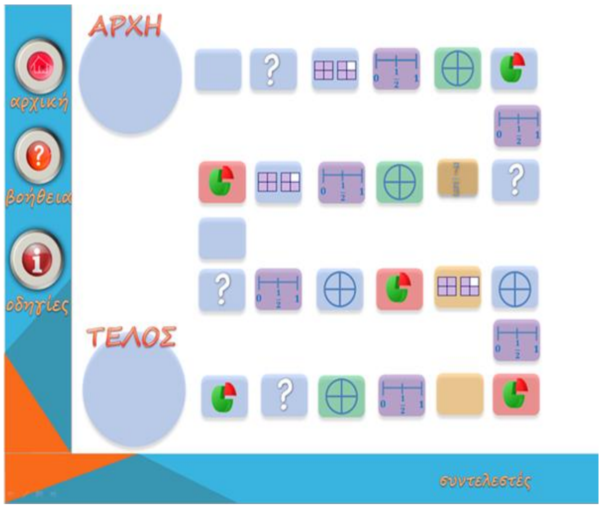

Fig. 2 The digital dashboard of Fraction Battles Software

To win, players must reach the finish line by rolling the die and following the path shown in Diagram 2. The path includes 27 spaces. When students land on one of these spaces, they must click on the corresponding position to answer a question/task. If they answer correctly, they continue; otherwise, they must wait for their next turn. Spaces that represent improper fractions contain related activities. Those that represent the unit fraction or the number line contain corresponding tasks. Spaces displaying the software's symbol combine two or even all three core concepts.

The spaces with a question mark (?) contain player instructions, while blank spaces serve as reflection points. The software includes a help section for students (see Figure 3) and instructions for teachers and students. This guidance is available in both written and audio formats and is provided for each activity.

Fig. 3 Plan w ith the help of the software

Each activity addresses specific student difficulties with fractions, as identified in long-term Research and international literature. [3] Thus, each task is aligned with a particular concept, has a defined goal, and is intended to bridge specific learning gaps. Additionally, the tasks are organized by difficulty level. In their presentation, which follows, the cognitive domain within the area of fractions they are addressing is mentioned, along with the difficulties that students face in this area, as identified by research findings [2, 1], as well as the objective and description of the activities.

5. Software Activities

5.1. Activity 1

Conceptual Axis: Partitioning of the whole unit.

Detected Difficulty: Students are unaware of the need to divide the whole unit into equal parts.

Objective: Understanding the necessity of dividing the whole unit into equal parts.

Description: Created using Hotpotatoes (JQui), this activity allows students to select shaded parts representing unit fractions. It includes counterexamples to challenge textbook norms and features answer checking and time limits (see Figure 4).

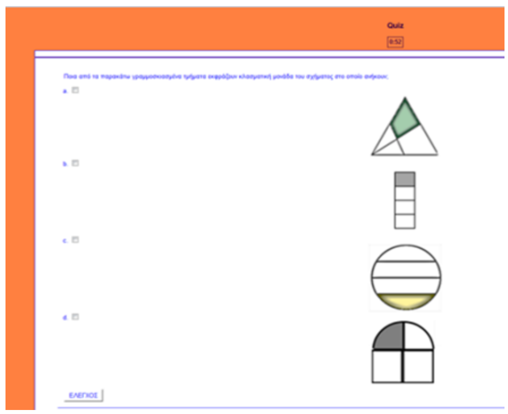

Fig. 4 Activity to familiarize with the equal division of the integer unit

5.2. Activity 2

Conceptual Axis: Partitioning of the fractional unit.

Detected Difficulty: Students struggle to shift between different representations of the fractional unit.

Objective: Understanding the necessity of partitioning the unit into equal parts using multiple representations.

Description: Using Hotpotatoes (JMatch), students match visual representations from two columns. The activity addresses continuous/discrete quantities, number line models, and symbolic forms with feedback and time constraints (see Figure 5).

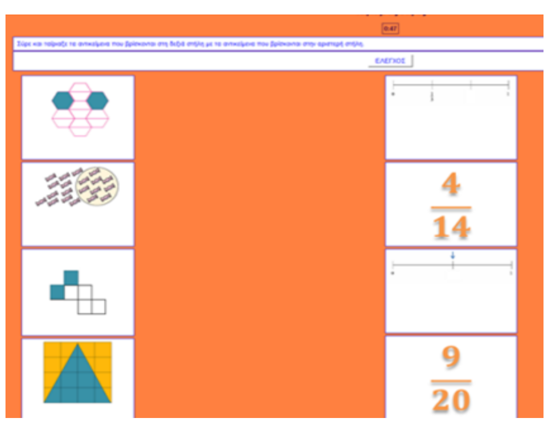

Fig. 5 Activity involving the translation from one form of representation to hk in relation to the equal partitioning of the fractional unit

5.3. Activity 3

Conceptual Axis: Improper fractions.

Detected Difficulty: Students struggle to represent and identify improper fractions across various forms.

Objective: Understand and manage improper fractions in different representations.

Description: Designed in Scratch, students control a unit fraction to 'eat' only the correct improper fraction representations. Correct actions trigger positive feedback; incorrect ones produce a warning sound (see Figure 6).

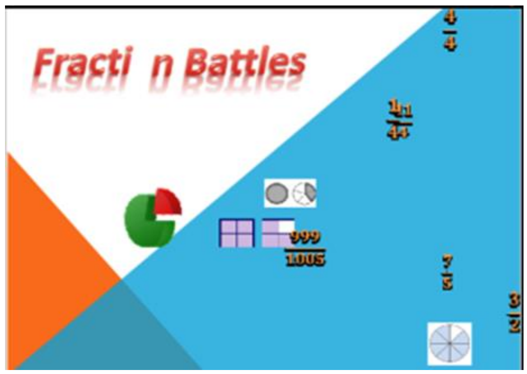

Fig. 6 Activity involving the translation from one form of repre sentation to another in relation to improper fractions

5.4. Activity 4

Conceptual Axis: Ordering fractions on a number line.

Detected Difficulty: Students count all given points on the number line, including zero.

Objective: Practice identifying the number of segments on a number line.

Description: Students choose from three options indicating how many equal parts divide a given number line. Audio feedback helps confirm or correct responses (see Figure 7).

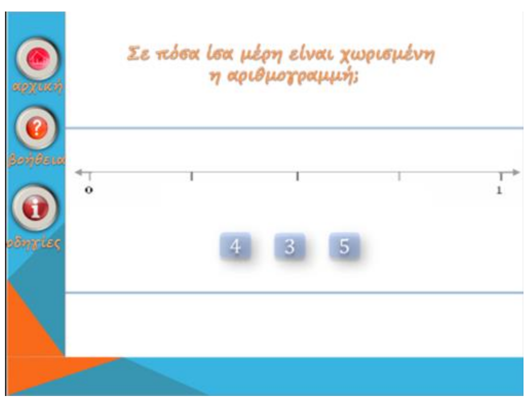

Fig.7 Activity related to the geometric model of the number line

5.5. Activity 5

Conceptual Axis: Ordering fractions on a number line.

Detected Difficulty: Students struggle with ordering unlike fractions.

Objective: Understand how to order rational numbers.

Description: Using Kidspiration, students place fractions on the number line and justify their choices. Other students may correct the sequence collaboratively (see Figure 8).

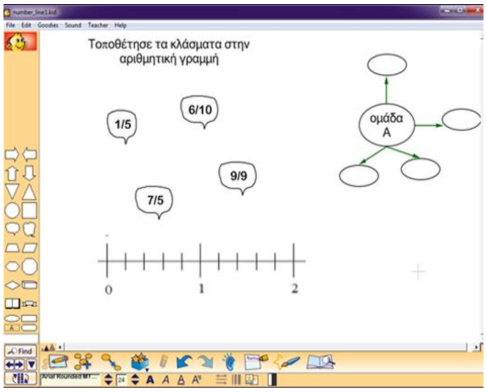

Fig. 8 Activity of ordering fractions as represent tation on the geometric model of the number line

5.6. Activity 6

Conceptual Axis: Ordering fractions on a number line.

Detected Difficulty: Students believe no fraction exists between two others.

Objective: Develop an understanding of the density of rational numbers.

Description: Students are asked to name a fraction between two others. Clicking on the space reveals more fractions, reinforcing that infinite fractions exist between any two (see Figure 9, 10).

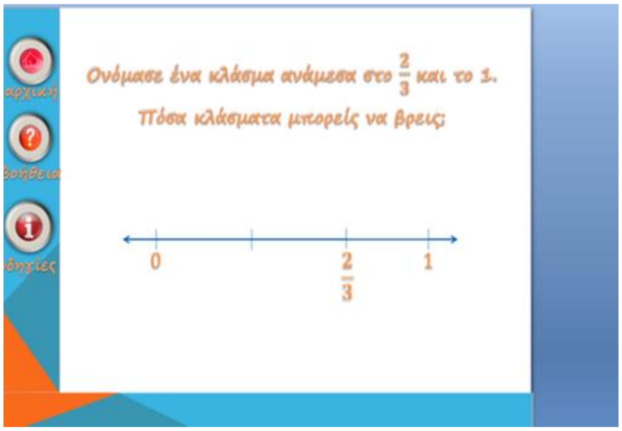

Fig. 9 Activity on the density of rational numbers

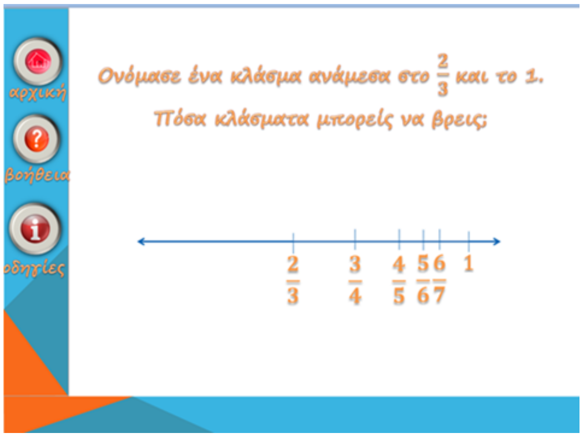

Fig. 10 Displaying fractions after selecting the given interval

5.7. Activity 7

Conceptual Axis: Improper fractions.

Detected Difficulty: Students struggle to represent improper fractions visually.

Objective: Understand improper fractions through diagrammatic representation.

Description: Using Smoothboard 2, students shade parts of a diagram to represent 10/8, correcting errors using tools from the interface (see Figure 11).

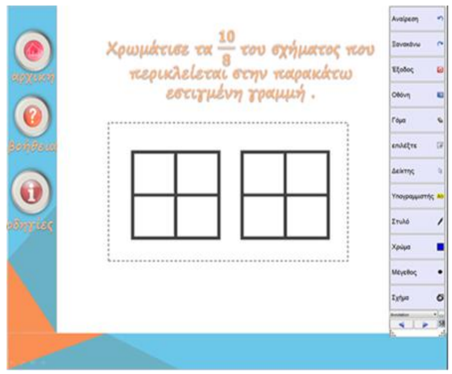

Fig. 11 Activity for familiarizing with the concept of improper fractions

5.8. Activity 8

Conceptual Axis: Partitioning the fractional unit and improper fractions.

Detected Difficulty: a) Students do not grasp the need for equal parts; b) students misidentify improper fractions.

Objective: Develop awareness of partitioning and recognizing improper fractions.

Description: A Hotpotatoes (JQui) exercise with feedback and time limits challenges students to identify 1/4, using examples and non-examples, including improper fractions (see Figure 12).

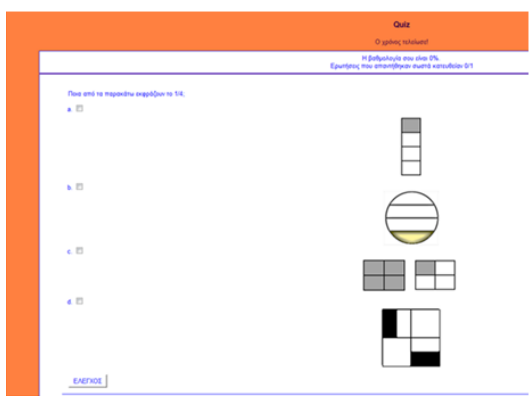

Fig. 12 Activity for equal partitioning and recognizing the fractional unit and improper fractions

5.9. Activity 9

Conceptual Axis: Improper fractions and partitioning the fractional unit.

Detected Difficulty: Students fail to recognize improper fractions and the need for equal partitioning.

Objective: Familiarize students with improper fractions and partitioning necessity.

Description: Students match different representations (e.g., visual, symbolic) using drag-and-drop in a Hotpotatoes (JMatch) activity (see Figure 13).

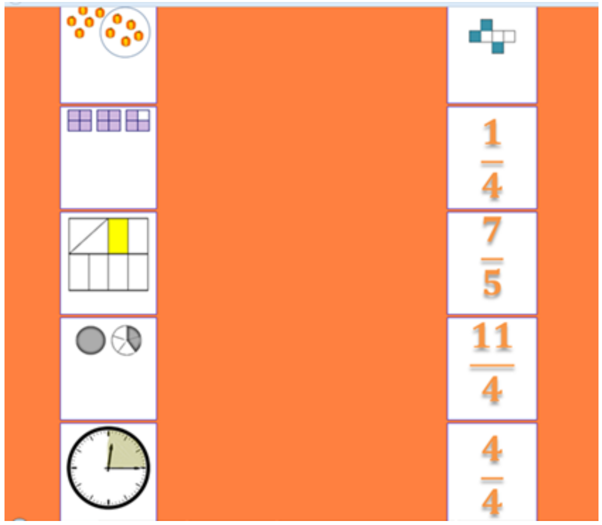

Fig. 13 Activity for familiarizing with improper fractions and the equal partitioning of the fractional unit

5.10. Activity 10

Conceptual Axis: Combined concepts.

Detected Difficulty: Students struggle with translating between representations of improper fractions, partitioning, and ordering.

Objective: Understand and connect these core concepts through diverse representations.

Description: Students complete a table with various representations (e.g., diagrams, words, decimals) using tools provided on-screen (see Figure 14).

Fig. 14 Combinatory activity of transferring from one field of representation to another

5.11. Activity 11

Conceptual Axis: Ordering fractions on a number line.

Detected Difficulty: Students struggle to locate fractions on the number line.

Objective: Elicit rules and strategies for placing fractions.

Description: Students identify which fraction corresponds to a marked point. Justifications are shared and discussed with peers (see Figure 15).

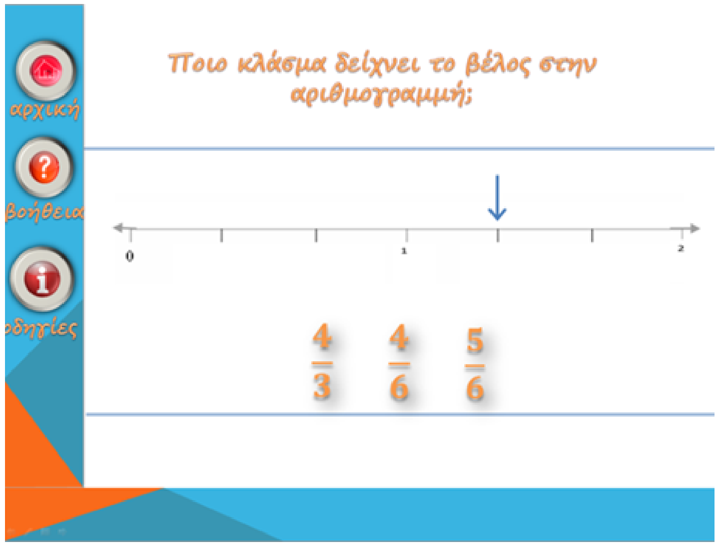

Fig. 15 Activity for recognizing a fraction on the geometric model of the number line

5.12. Activity 12

Conceptual Axis: Ordering fractions on a number line.

Detected Difficulty: Students struggle with misconceptions about denominators and density.

Objective: Develop strategies for ordering fractions.

Description: Students evaluate given statements and support their reasoning using counterexamples when appropriate (see Figure 16).

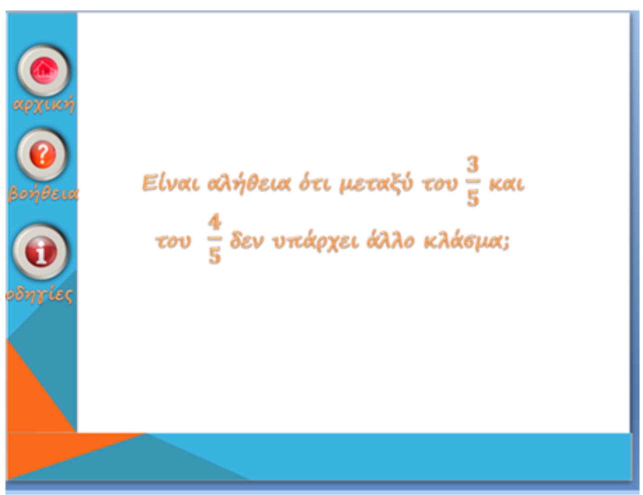

Fig. 16 An indicative question for activity 12

5.13. Activity 13

Conceptual Axis: Ordering fractions, improper fractions, and partitioning.

Detected Difficulty: Students have trouble ordering, unlike fractions in varied representations.

Objective: Develop familiarity with ordering and inter-representation translation.

Description: Each student holds a card showing a fraction in a unique form. They must order themselves according to value (see Figure 17).

Fig. 17 Example of cards indicated by the software

5.14. Activity 14

Conceptual Axis: Ordering fractions on a number line.

Detected Difficulty: Students struggle with ordering unlike fractions.

Objective: Enhance ordering skills via historical context.

Description: The students attempt to place the given prehistoric fractions on the number line (see Figure 18). Along with the prehistoric symbols, the fraction representing each symbol in its modern form is also provided. Using authentic texts from the history of Mathematics [12, 18], the students construct a modern number line with representations from the past.

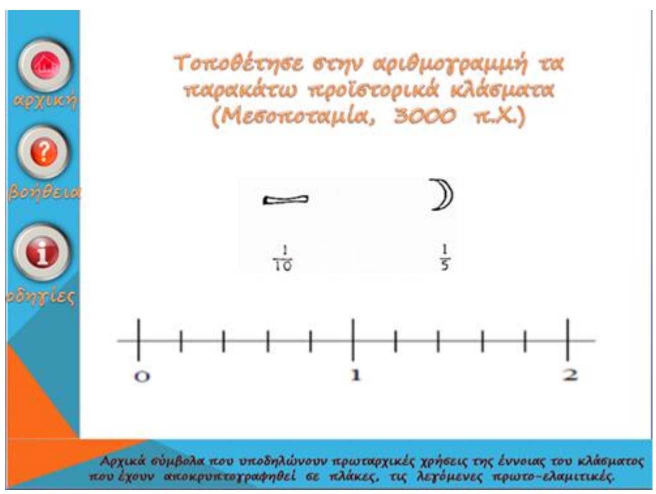

Fig. 18 Prehistoric fraction symbols for ordering on the number line

3.15. Activity 15

Conceptual Axis: Partitioning the fractional unit.

Detected Difficulty: Students have trouble identifying the 'whole'.

Objective: Understand unit partitioning and defining the whole.

Description: Students identify which shaded sections represent a unit fraction using a time-limited Hotpotatoes (JQui) activity with feedback (see Figure 19).

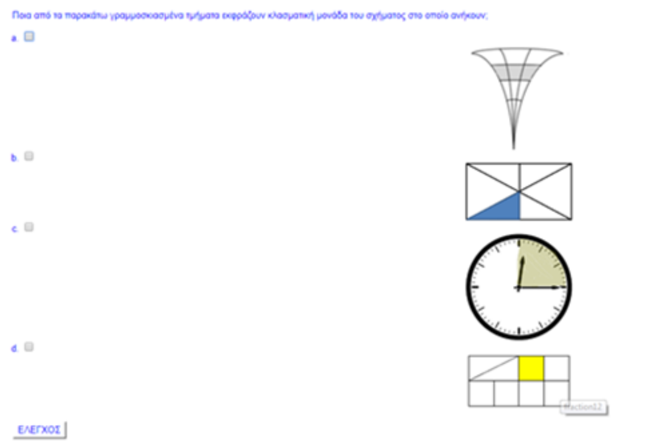

Fig. 19 Exercise for recognizing the fractional unit

3.16. Activity 16 & 17

Conceptual Axis: Partitioning the fractional unit.

Detected Difficulty: Students find reconstructing the whole from a fraction hard. [11]

Objective: Develop the skill of forming wholes from parts using multiple representations.

Description: The students attempt to represent the given archaic fractions using various forms of representation (see Figure 20). The fraction that each symbol represents in its modern form is also provided along with the archaic symbols. Through authentic texts from the history of Mathematics [12, 18], the students construct representations by comparing them with past ones.

Fig. 20 Activity with archaic representations of fractions

3.17. Activity 18

Conceptual Axis: Improper fractions.

Detected Difficulty: Students struggle to recognize and form improper fractions.

Objective: Familiarize students with constructing improper fractions.

Description: Students receive four cards to form an improper fraction of their choice, then represent it visually (see Figure 21).

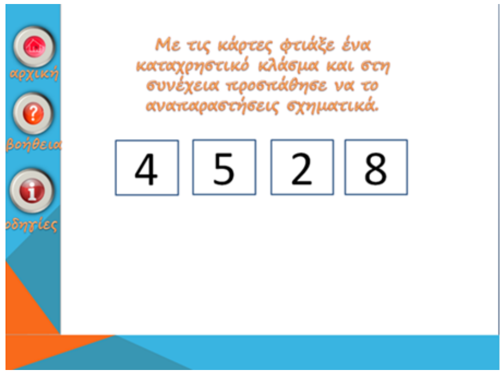

Fig. 21 Digital cards for the formation and design of improper fractions

3.18. Activity 19

Conceptual Axis: Improper fractions and unit partitioning.

Detected Difficulty: Students struggle to define the whole and identify improper fractions in diagrams.

Objective: Develop skills to identify wholes and improper fractions.

Description: Students answer questions using a digital Tangram, moving shapes around to support their answers (see Figure 22).

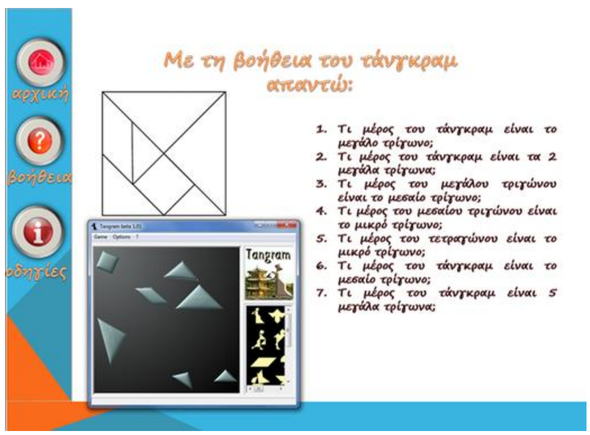

Fig. 22 The digital environment of Activity 19

3.19. Activity 20

Conceptual Axis: Improper fractions.

Detected Difficulty: Students have Difficulty recognizing and constructing improper fractions.

Objective: Develop skills in constructing improper fractions.

Description: Given three elements, students create a short word problem and solve it. For example: 'Gerasimos ate $\frac{5}{4}$ of a pizza. Draw the pieces he ate (see Figure 23).

Beyond the activities related to the three concepts under examination-namely, the equal partitioning of the fractional unit, the ordering of fractions on the geometric model of the number line, and improper fractions-there are also four spaces on the software's game board marked with the symbol "?". If a player lands on one of these spaces, they must follow the instructions provided. These instructions are formulated within the context of rational numbers. More specifically, when a player must move a certain number of spaces backwards, the instruction is phrased as follows: "Move back as many spaces as correspond to $\frac{1}{9}$ of the 27 squares on the board." Similarly, when a player misses a turn, the instruction states: "You lose $\frac{3}{3}$ of a turn." Additionally, three spaces do not display any symbol. These are the reflection spaces. More specifically, when a team lands on one of these spaces, they may ask for clarifications, help, or anything else they believe will support their successful progression in the game.

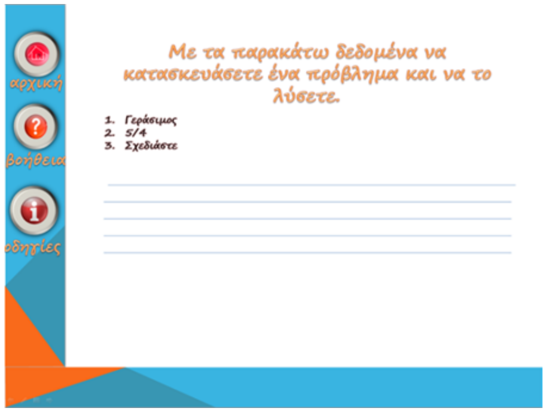

Fig. 23 The data provided to the students for problem-posing

6. Software Evaluation

The evaluation criteria used to assess the educational game Fraction Battles were based on the two criteria developed by [26].

6.1. Group A

This group includes general pedagogical criteria and addresses the following questions:

- *Is the curriculum adequately covered?*
- The game's content is thoroughly covered, as defined by the present Research. It addresses the three core topics: partitioning of the fractional unit, improper fractions, and number line ordering.
- *Is the required time available?*
- The implementation of the game fits within a two-hour teaching session, which is included in the overall instructional planning of this study.
- *Is the instructional content appropriate for the student's level?*
- All activities were designed based on questionnaires given to students in grades 5, 6, and 7 to assess their difficulty levels.
- *Are the activities interesting?*
- Based on implementation, students found the activities engaging and enjoyable.
- *Is the software pedagogically sound and free of undesirable side effects?*
- Care was taken to avoid negative side effects. No such issues were observed during its application.
- *Does it support collaborative/complex tasks?*
- The game was primarily designed for group-based learning.
- *What skills, attitudes, and values does it foster?*
- The software is designed to reduce students' difficulties with fractions. Evidence from its application shows improved attitudes toward fractions and mathematics in general.
- *Does it come with sufficient guidance and support material for teachers?*
- While teacher guides and support materials are available, their adequacy has not yet been formally evaluated, as the software was used solely within the scope of this study and has not been tested independently in classroom settings.

6.2. Group B

This group includes technological aspects concerning the use of the computer as a learning tool and responds to the following:

- *Are the interactive capabilities of the computer effectively utilized?*
- Several of the activities use interactive tools.
- *Does the software allow for monitoring of the learning process by both student and teacher?*
- It includes open-ended features that allow for teacher oversight and modification.

- *Does the program handle user input effectively, avoiding technical issues?*
- The application ran smoothly without functional issues.
- *Are the graphics, sound, and animation used constructively or just for show?*
- An effort was made to balance aesthetic appeal with educational value, avoiding superficial design.
- *Is error correction pedagogically sound, or does it rely on traditional practices?*
- A mix of behaviorist and more student-centred correction methods are used, allowing for group or teacher intervention.
- *Is student feedback and reinforcement handled effectively and appropriately?*
- The teacher plays the primary role in feedback and reinforcement, with the software supporting this process.
- *Is the program user-friendly?*
- Students did not encounter usage difficulties. However, the software still requires additional programs (e.g., Scratch) to function independently.
- *Does the software incorporate current technological trends?*
- While the primary goal was pedagogical rather than technological innovation, relevant advancements were considered where possible.

7. Results and Discussion

7.1. Observations from Similarity Diagrams

The analysis of the research results was conducted using similarity diagrams. These diagrams (see Figure 24, 25) present groupings of variables based on participant behavior during task resolution and reflect the potential similarity relationships among them. More specifically, the comparison was made based on the Diagram of Figure 24, which shows the outcomes of teaching interventions on fractions without using the present software, as implemented by [29].

Analysis of the diagrams yields several significant insights. In Group A (see Figure 24), encompassing tasks related to the partitioning of the fractional unit and the representation of improper fractions, a robust association is evidenced with variables BelB2 and BelB1, which pertain to students' beliefs that their enjoyment of mathematics would increase if it were perceived as less difficult.

In Group C (see Figure 24), comprehension of improper fractions appears to be strongly influenced by students' attitudes towards mathematics, as indicated by the associations between variables (ImpD2z BelA3) and (BelA2 BelA1.1). Notably, students who expressed a positive disposition toward mathematics and acknowledged its significance in everyday life exhibited superior performance in tasks involving improper fractions.

Moreover, in Group D (see Figure 24), students' ability to accurately order fractions on a number line demonstrates a significant correlation with variables BelA1, BelD1, and BelE2, corresponding respectively to the enjoyment of learning mathematics, the perceived relevance of real-world applications during instruction, and satisfaction with personal academic performance. Students endorsing these beliefs attained higher levels of success in tasks requiring the ordering of fractions on a number line.

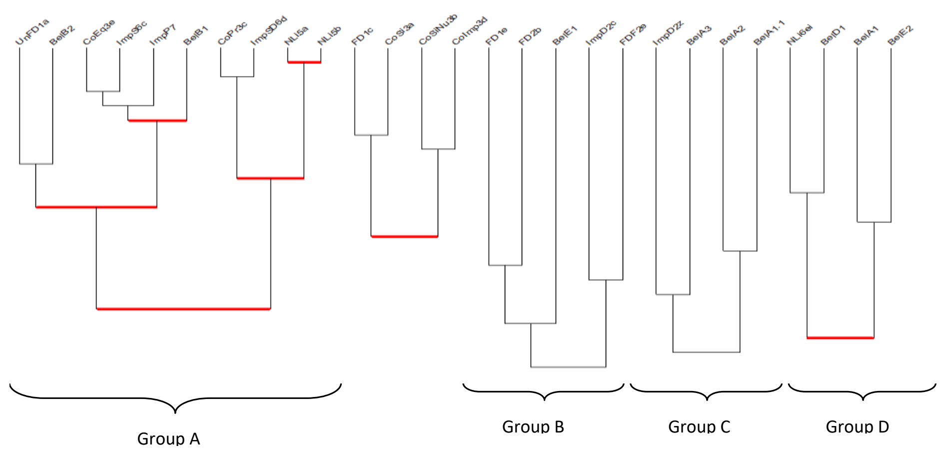

Fig. 24 Similarity Diagram of pre-test variables (Vlachou & Avgerinos, 2018) [29]

A comparison of the findings from the Diagram in Figure 24 with those from the Diagram in Figure 25, which reflect post-test results following the implementation of Fraction Battles, reveals that the enhanced performance in fractions positively impacted students' attitudes toward mathematics. In particular, within Group A (see Figure 25), improved performance on tasks involving improper fractions notably fostered a sense of increased satisfaction with their mathematical abilities, as reflected in the variables (ImpD1d BelD1) and (BelA2 BelD2).

In Group B (see Figure 25), modifications in instructional approach through integrating the software and its 10+1 structural elements resulted in significant gains in student performance, particularly on tasks related to improper fractions and identifying fractions between two others. This shift was accompanied by a change in students' perceptions of mathematics, which they had previously deemed overly challenging, as evidenced by the associations (((FD1e CoSiNu3b) (CoEq3e ImpS6c)) ((ImpSD6d ImpP7J) (BelB1 BelB2))).

Furthermore, in Group C (see Figure 25), improvements in performance on more complex tasks, such as articulating reasoning during the resolution of improper fraction problems, led to a shift in students' attitudes, steering them toward a more positive outlook. These students expressed greater enthusiasm for learning mathematics and recognized the importance of mathematical proficiency, as indicated by the variables ((NLiImp6eii NLiImp6eiii) (ImpP7 BelE2)).

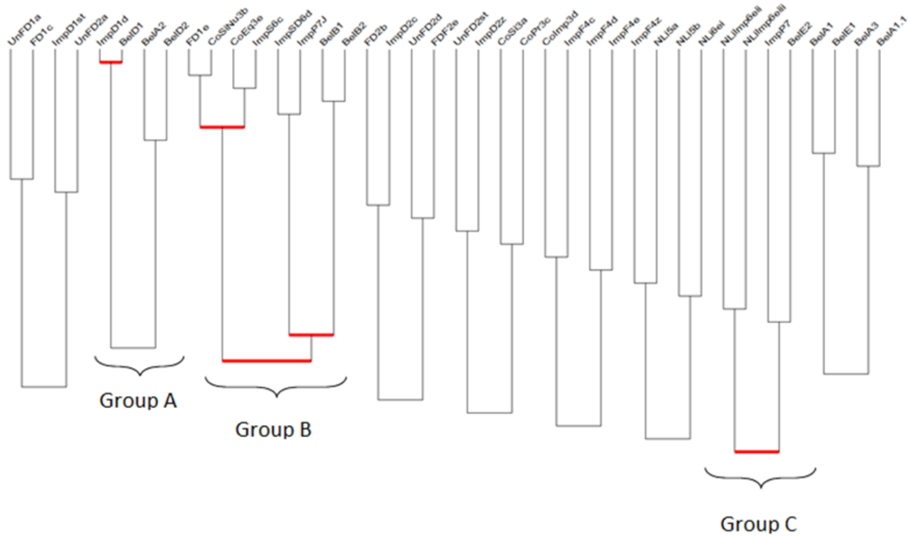

Fig. 25 Similarity diagram of post-test variables for the focus group

8. Conclusions

The domain of rational numbers is crucial to students' mathematical education, as mastering it facilitates understanding many other mathematical concepts. This explains why many researchers focus on exploring student difficulties in this area. A recurring theme in much of this Research is the idea that the teaching method is a key factor influencing how students come to understand fractions over time. [19, 22, 25, 27] Other researchers argue that frequent exposure to a particular concept representation increases familiarity and improves learning outcomes. [16, 20]

Therefore, it becomes evident that each researcher attributes students' difficulties with rational numbers to different causes. Despite continuous research efforts, textbook changes, and numerous instructional suggestions, students struggle with the same issues, and discouraging performance patterns persist across all education levels. Our research team believes these difficulties stem from a combination of factors, including those highlighted in previous Research and possibly new ones that emerge during ongoing studies. Through extensive investigation, we aim to identify these factors and propose effective, evidence-based solutions to address them within the Greek educational system.

One such proposal is the development of the educational software Fraction Battles, which was designed based on the findings of long-term Research. Its goal is to help students grasp the core concepts of rational numbers playfully and engagingly. It uses meaningful learning situations that combine New Technologies with representational tools, critical thinking with creativity, knowledge with play and school with enjoyment. The software focuses on three key conceptual areas: partitioning of the whole unit, improper fractions and ordering of fractions on the number line. Its activities are based on the 10+1 structural components of the RhodeScript Theory. The findings from the similarity diagram analyses indicate that the representational tools, in combination with the RhodeScript framework, significantly supported students in overcoming conceptual difficulties. More specifically, using multiple representations helped students improve their ability to order fractions on the number line, and using counterexamples contributed to a deeper understanding of why equal partitioning of the unit is necessary. Overall, the quantity and quality of representational tools available to students play a critical role in understanding fundamental fraction concepts, such as equal partitioning and improper fractions. As a result, improved performance in these areas led to a positive shift in student attitudes toward mathematics, boosted their confidence, and made them feel more satisfied with their learning outcomes.

References

[1] E. Avgerinos, and R. Vlachou, "The Consistency Between the Concepts of Equal Parts of the Unit, Improper Fractions and Problem Solving at Candidate Teachers of Education Departments," *Proceedings of the 30th Hellenic Conference on Mathematical Education*, pp. 135-147, 2013. [Google Scholar]

[2] Roza Vlachou, and Evgenios Avgerinos, "Current Trend and Studies on Representation of Fractions," *7th Mediterranean Conference on Mathematics Education,* pp.135-159, 2012.

[3] E., Avgerinos, R., Vlachou, andK., Kantas, "Comparing Different Age Student Abilities on the Concept and Manipulation of Fractions," *Research on Mathematical Education and Mathematics Applications*, pp. 159-169, 2012. [Google Scholar]

[4] E. Avgerinos, R. Vlachou, and D. Remoundou, *Mathematical Tools in Education: Applied Mathematics Teaching and Practical Application of Rhode Script Theory in the Elementary School*, Rhodes, Greece: University of the Aegean, 2023.

[5] Evgenios P. Avgerinos, Roza G. Vlachou, and Dimitra I. Remoundou, "Development and Implementation of a Didactical Framework of 10+1 Elements for the Reinforcement of Students' Mathematical Ability and Attitude Towards Mathematics: Part I," *Proceedings of International Conference on Educational Research: Confronting Contemporary Educational Challenges through Research,* pp. 17-29, 2018. [Google Scholar]

[6] Jerome S. Bruner, *The Process of Education*, Cambridge, MA: Harvard University Press, 1960. [Google Scholar]

[7] Xi Chen, and Yeping Li, "Instructional Coherence in Chinese Mathematics Classroom—A Case Study of Lessons on Fraction Division," *International Journal of Science and Mathematics Education,* vol. 8, no. 4, pp. 711-735, 2010. [CrossRef] [Google Scholar] [Publisher Link]

[8] L. Cohen, and L. Manion, *Methodology of Educational Research*, *Trans.*, Athens: Ekfrassi, 1997. [Google Scholar]

[9] Anika Dreher, and Sebastian Kuntze, "Teachers' Professional Knowledge and Noticing: The Case of Multiple Representations in the Mathematics Classroom," *Educational Studies in Mathematics*, vol. 88, no. 1, pp. 89-114, 2015. [CrossRef] [Google Scholar] [Publisher Link]

[10] Anika Dreher, Sebastian Kuntze, and Stephen Lerman, "Why Use Multiple Representations in the Mathematics Classroom? Views of English and German Preservice Teachers," *International Journal of Science and Mathematics Education*, vol. 14, no. 2, pp. 363-382, 2016. [CrossRef] [Google Scholar] [Publisher Link]

[11] Martha Isabel Fandiño Pinilla, "Fractions: Conceptual and Didactic Aspects," *Didactic Acta Comenius University,* vol. 7, pp. 23-45, 2007. [Google Scholar]

[12] Michael N. Fried, "Can Mathematics Education and History of Mathematics Coexist?," *Science & Education*, vol. 10, no. 4, pp. 391-408, 2001. [CrossRef] [Google Scholar] [Publisher Link]

[13] R. Gras et al., "Implicative Statistical Analysis," *Data Science, Classification, and Related Methods*, pp. 412-419, 1998. [CrossRef] [Google Scholar] [Publisher Link]

[14] R. Duval, "Registers of Semiotic Representations and Cognitive Functioning of Thought," *Annals of Didactics And Cognitive Sciences*, ULP, IREM Strasbourg. vol. 5, pp. 37-65, 1993. [Google Scholar] [Publisher Link]

[15] R. Gras et al., "Statistical Implicative Analysis, Theory and Applications," *Studies in Computational Intelligence*, Springer-Verlag, Berlin-Heidelberg Consultable, vol. 127, 2008. [Google Scholar]

[16] Jeremy Hodgen et al., "Lower Secondary School Students' Knowledge of Fractions," *Research in Mathematics Education*, vol. 12, no. 1, pp. 75-76, 2010. [CrossRef] [Google Scholar] [Publisher Link]

[17] Christine Howe et al., "Rational Number and Proportional Reasoning in Early Secondary School: Towards Principled Improvement in Mathematics," *Research in Mathematics Education,* vol. 17, no. 1, pp. 38-56, 2015. [CrossRef] [Google Scholar] [Publisher Link]

[18] Uffe Thomas Jankvist, "A Categorization of the "Whys" and "Hows" of Using History in Mathematics Education," *Educational Studies in Mathematics,* vol. 71, no. 3, pp. 235-261, 2009. [CrossRef] [Google Scholar] [Publisher Link]

[19] C. Janvier, "Problems of Representation in the Teaching and Learning of Mathematics," *American Psychological Association*, pp. 27-32, 1987. [Google Scholar] [Publisher Link]

[20] Chunlian Jiang, and Boon Liang Chua, "Strategies for Solving Three Fraction-Related Word Problems on Speed: A Comparative Study between Chinese and Singaporean Students," *International Journal of Science and Mathematics Education*, vol. 8, no. 1, pp. 73-96, 2010. [CrossRef] [Google Scholar] [Publisher Link]

[21] Susan J. Lamon, "The Development of Unitizing: Its Role in Children's Portioning Strategies," *Journal for Research in Mathematics Education*, vol. 27, no. 2, pp.170-193, 1996. [CrossRef] [Google Scholar] [Publisher Link]

[22] J.J. Lo, "Conceptual Bases of Young Children's Solution Strategies of Missingvalue Proportional Tasks," *Proceedings of the Seventeenth International Conference of Psychology of Mathematics Education: PME XVII*, Tsukuba, Japan: University of Tsukuba, pp. 162-177, 1993. [Google Scholar]

[23] Yujing Ni, "Semantic Domains of Rational Number and the Acquisition of Number Equivalence," *Contemporary Educational Psychology*, vol. 26, no. 3, pp. 400-417, 2001. [CrossRef] [Google Scholar] [Publisher Link]

[24] Donald M. Peck, and Stan Ley M. Jencks, "Conceptual Issues in the Teaching and Learning of Fractions," *Journal for Research in Mathematics Education*, vol. 12, no. 5, pp. 339-348, 1981. [CrossRef] [Google Scholar] [Publisher Link]

[25] Anna Sfard, "On the Dual Nature of Mathematical Conceptions: Reflections on Processes and Objects as Different Sides of the Same Coin," *Educational Studies in Mathematics*, vol. 22, pp. 1-36, 1991. [CrossRef] [Google Scholar] [Publisher Link]

[26] David Squires, and Anne McDougall, *Choosing and Using Educational Software: A Teacher's Guide*, London: The Falmer Press, 1994. [Google Scholar] [Publisher Link]

[27] Leen Streefland, *Fractions in Realistic Mathematics Education: A Paradigm of Developmental Research,* Dordrecht, The Netherlands: Kl, 1991. [Google Scholar] [Publisher Link]

[28] Xenia Vamvakoussi, and Stella Vosniadou, "Understanding the Structure of the Set of Rational Numbers: A Conceptual Change Approach," *Learning and Instruction*, vol. 14, no. 5, pp.453-467, 2004. [CrossRef] [Google Scholar] [Publisher Link]

[29] Roza Vlachou, and Evgenios Avgerinos, "Multiple Representatıons and Development of Students' Self-Confidence on Rational Number," *Experiences of Teaching with Mathematics, Sciences and Technology*, vol. 4, pp. 567-586, 2018. [Google Scholar] [Publisher Link]

[30] Hajime Yoshida, and Kooji Sawano, "Overcoming Cognitive Obstacles in Learning Fractions: Equal-Partitioning and Equal-Whole," *Japanese Psychological Research*, vol. 44, no. 4, pp. 183-195, 2002. [CrossRef] [Google Scholar] [Publisher Link]

[31] Soo Jin Lee, and Jaehong Shin, "Distributive Partitioning Operation in Mathematical Situations Involving Fractional Quantities," *International Journal of Science and Mathematics Education*, vol. 13, no. 2, pp. 329-355, 2015. [CrossRef] [Google Scholar] [Publisher Link]

[32] Susan B. Empson, Linda Levi, and Thomas P. Carpenter, "The Algebraic Nature of Fractions: Developing Relational Thinking in Elementary School," *Early Algebraization*, pp. 409-428, 2011. [CrossRef] [Google Scholar] [Publisher Link]

[33] Mi Yeon Lee, and Amy J. Hackenberg, "Relationships between Fractional Knowledge and Algebraic Reasoning: The Case of Willa," *International Journal of Science and Mathematics Education*, vol. 12, no. 4, pp. 975-1000, 2014. [CrossRef] [Google Scholar] [Publisher Link]

[34] Ben Shneiderman, Jock Mackinlay, and Stuart K. Card, *Readings in Information Visualization: Using Vision to Think*, San Francisco: Morgan Kaufmann Publishers, 1999. [Google Scholar] [Publisher Link]

[35] A.A. Cuoco, and F.R. Curcio, *The Roles of Representation in School Mathematics: 2001 Yearbook*, Reston, VA: National Council of Teachers of Mathematics, 2001. [Google Scholar]

[36] Anika Dreher, and Sebastian Kuntze, "Teachers' Professional Knowledge and Noticing: The Case of Multiple Representations in the Mathematics Classroom," *Educational Studies in Mathematics*, vol. 88, no. 1, pp. 89-114, 2015. [CrossRef] [Google Scholar] [Publisher Link]

[37] *Principles and Standards for School Mathematics*, National Council of Teachers of Mathematics, pp. 1-20, 2000. [Publisher Link]

[38] Erik Jacobson, and Andrew Izsák, "Knowledge and Motivation as Mediators in Mathematics Teaching Practice: The Case of Drawn Models for Fraction Arithmetic," *Journal of Mathematics Teacher Education*, vol. 18, no. 5, pp. 467-488, 2015. [CrossRef] [Google Scholar] [Publisher Link]

[39] Alice Hansen, Manolis Mavrikis, and Eirini Geraniou, "Supporting Teachers' Technological Pedagogical Content Knowledge of Fractions through Co-Designing a Virtual Manipulative," *Journal of Mathematics Teacher Education*, vol. 19, no. 2-3, pp. 205-226, 2016. [CrossRef] [Google Scholar] [Publisher Link]