

Original Article

# The Solution

Dilip Kumar Bhowmik

Independent Researcher, Kolkata, India.

Corresponding Author : [bhowmikdilip54@gmail.com](mailto:bhowmikdilip54@gmail.com)

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**Abstract** - Why it is so was a matter of great question. So many have long been trying to find it  $a^2 + b^2 = c^2$  but  $a^3 + b^3 \neq c^3$  and accordingly have been trying and also presented so many reasons acceptable or not. Here, I present a very simple, understandable, and obvious reason. It will help the mathematical world solve and prepare different mathematical propositions to solve daily and scientific problems.

**Keywords** - The created numbers, Two-dimensional space, Three-dimensional space.

## 1. Introduction

As a lad, when I faced Mathematics, I left embarrassed with  $-1 < 0$ ,  $(+) \times (-) = -/(-) \times (-) = + \sqrt{-1} = i$  but not  $-1$  and  $i$  like.

One such is the diameter and circumference relation. It is said that the relation is fixed. So many for years had given different values. In fine, the relation stood to be that the circumference is  $22/7$  times the length of the diameter, which is not even correct. However, day-to-day activities are going on with this measure with a pinch of salt for practical purposes, that is, with accepted deviation and variances.

This measure is very much required for making rings to make utensils, from car rings to huge turbines, and from the measures of a circle to measure the circumference of a galaxy or its cluster.

Still, the relation between the diameter and the circumference is not determined to be accepted as perfect.

In this respect, the academically and institutionally paid ones behave in such a manner that the ones are the sole right holder and the authority to do and talk about the field only. They neglect and disregard the personal efforts as an amateur, using the word to abuse and call names.

This is purely a modern-day phenomenon. In ancient times, sages and scholars used to cultivate knowledge on their own single-handedly. I started following the age-old honorable as the cultivation of knowledge cannot be anyone's sole property, whatever authority and self one possesses. However, some activities yield better and quicker results with collective wealth and effort.

Let me tell you what I have found in my endeavor. I may fail, yet it is an effort to see if it can help human beings.

**A simple solution why  $a^3 + b^3 \neq c^3$  .....**

**The Fact**

We find a rhythmic move of numbers created by the exponential numbers.

Exponent 1		$1^1$	$2^1$	$3^1$	$4^1$	$5^1$	$6^1$	$7^1$	$8^1$	$9^1$	$10^1$
Numbers Created		1	2	3	4	5	6	7	8	9	10
Difference of numbers	←	← 1	← 1	← 1	← 1	← 1	← 1	← 1	← 1	← 1	1
Exponent 2		$1^2$	$2^2$	$3^2$	$4^2$	$5^2$	$6^2$	$7^2$	$8^2$	$9^2$	$10^2$
Numbers Created		1	4	9	16	25	36	49	64	81	100



Difference of numbers		3	5	7	9	11	13	15	17	19	
Exponent 3		1 <sup>3</sup>	2 <sup>3</sup>	3 <sup>3</sup>	4 <sup>3</sup>	5 <sup>3</sup>	6 <sup>3</sup>	7 <sup>3</sup>	8 <sup>3</sup>	9 <sup>3</sup>	10 <sup>3</sup>
Numbers Created	1	8	27	64	125	216	343	512	729	1000	

Created difference of numbers  $\rightarrow 7 \rightarrow 19 \rightarrow 37 \rightarrow 61 \rightarrow 91 \rightarrow 127 \rightarrow 169 \rightarrow 217 \rightarrow 271$

As the power of numbers increases, the difference of created numbers also increases and that it also in a systematic way.

### ***Relation between exponent and incremental difference of created numbers***

Power	Difference between created numbers
1	$1 \times 1 = 1$
2	$2 \times 1 = 2$
3	$3 \times 2 = 6$

When the power is 1, the difference between the created numbers is 1 as a result, any number with power 1 is equal to the next created number – when added with 1. Thus, any number with power 1 equals the next created number when added with 1. Thus, any number with power 1 + any number with power 1 is equal to the created number by adding the original numbers.

Thus  $1^1 + 2^1 = 3^1$        $4^1 + 5^1 = 9^1$        $100^1 + 3^1 = 103^1$  and so on.

$$a^1 + b^1 = c^1 \quad a^1 + b^1 = c^1 \quad a^1 + b^1 = c^1$$

However, when the power is 2, the difference created by the created numbers increases by 2 consecutively.

Like  $2, 2+2, 2+2+2, 2+2+2+2, \dots$  and so on.

As a result, even numbers, including squares, are created.

But the first difference is always 1 as it starts from  $0^1 \ 1^1 = 1$

$$0^1 \ 1^1 \text{ or } 0^2 \ 1^2 \text{ or } 0^3 \ 1^3 \text{ or } 0^4 \ 1^4 \dots$$

Accordingly, the actual difference is always +1

It is true for all the differences created by all the powers

Hence, the difference of created numbers by exponent two is like

$$1-3-5-7-9-11-13-15-17-19$$

And as the series moves on, we find some odd numbers, which are squares of other odd numbers, like

$$9=3^2 \quad 25=5^2 \quad 49=7^2 \quad 81=9^2 \dots$$

That leads to number creation like  $a^2 + b^2 = c^2$

$$4^2 + 9 = 5^2 \quad 12^2 + 25 = 13^2 \dots$$

$$3^2 \quad 5^2$$

### The Problem

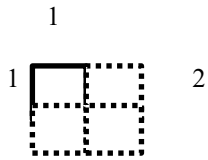
Now, in the case of exponent 3 all the differences consecutively increase by 6, but as the first difference is 1 ( $0^3 - 1^3$ ), the difference grows by 1 and becomes 7 instead of 6, 19 for 18, 37 for 36, etc.

But 7-19-37-61-91-127-169-217.....are none cubes of any number nor even the consecutive sums.

As a result no such relation  $a^3 + b^3 = c^3$  is found through  $0^3 + 1^3 = 1^3$

Now the question why is  $a^1 + b^1 = c^1$  and  $a^2 + b^2 = c^2$  but  $a^3 + b^3 \neq c^3$ ?

In the case of power 2, it is a measure of the square, and the dimensional field is like this:



In the case of power 2, that is a square, the relation of increasing a square to the next square is:-

Origin  $+(2 \times \text{side say } 1 + \text{corner that is } 1^2)$

If the side is 5, then the value is  $5^2$

If the side is 6 then the value is  $6^2$

This means  $6^2 = 5^2 + 5 \times 2 + 1^2$

If the relation  $2 \times \text{side} + 1^2$  shows a result that equals a square, then

$a^2 + b^2 = c^2$  is obvious.

### The Problem

But in the case of a cube, there are 3 sides. Hence, by increasing a cube to the next higher cube, the rule is quite different.

Present side<sup>3</sup> + Present side<sup>2</sup> + present side  $\times$  next side + next side<sup>2</sup>

Or  $a^3 + (a^2 + ab + b^2)$

This is like :-  $1^3 \longrightarrow 2^3 = 1^3 + (1^2 + 1 \times 2 + 2^2) = 8$

$2^3 \longrightarrow 3^3 = 2^3 + (2^2 + 2 \times 3 + 3^2) = 27$

$3^3 \longrightarrow 4^3 = 3^3 + (3^2 + 3 \times 4 + 4^2) = 64$

$4^3 \longrightarrow 5^3 = 4^3 + (4^2 + 4 \times 5 + 5^2) = 125$

Here, the incremental part  $a^2 + ab + b^2$  is even less than  $(a+b)^2$  by  $ab$ . As a result, it never is a cube of a number, though at times, the square of a number, such as  $7^2 + 2 \times 8 + 8^2 = 169 = 13^2$

Hence, it is clear that as the incremental part is not a cube, it is clear that  $a^3 + b^3 \neq c^3$

Because the incremental part is always  $<$  cube of any number.

Even the multiple of  $a^2 + ab + b^2$  cannot be a cube of any number. Only a cube multiplied by a cube can create a number that is the cube of a number.

More so as  $1^3 + 2^3 + 3^3 = 6^2$ .....that is when two or more cube numbers are added, it creates a square number by the addition of base numbers, and that is too

$1^3 + 2^3 + 3^3 = 6^2$ , etc., not  $2^3 + 3^3 = 5^2$ , cubes of consecutive numbers starting from 1.

Now, in the case of higher exponents,

If we observe, we will find that there are no more dimensions or sides than 3, and the higher exponents are multiple of exponents of 3 only. Let us see how.

$$\begin{array}{ll} 2^3 \rightarrow 3^3 = 2^3 + (2^3 + 2 \times 3 + 3^2) = 3^3 & 3^3 \rightarrow 4^3 = 3^3 + (3^2 + 3 \times 4 + 4^2) = 4^3 \\ 2^4 \rightarrow 3^4 = \{2^3 + (2^2 + 2 \times 3 + 3^2)\} 3 = 3^4 & 3^4 \rightarrow 4^4 = \{3^3 + (3^2 + 3 \times 4 + 4^2)\} 4 = 4^4 \\ 2^5 \rightarrow 3^5 = \{2^3 + 2^2 + 2 \times 3 + 3^2\} 3^2 = 3^5 & 3^5 \rightarrow 4^5 = \{3^3 + (3^2 + 3 \times 4 + 4^2)\} 4^2 = 4^5 \\ 2^6 \rightarrow 3^6 = \{2^3 + (2^2 + 2 \times 3 + 3^2)\} 3^3 = 3^6 & 3^6 \rightarrow 4^6 = \{3^3 + (3^2 + 3 \times 4 + 4^2)\} 4^3 = 4^6 \end{array}$$

As a consequence the  $a^4 + \dots + b^4 \dots \neq c^4 \dots$

Result and decision  $a^1 + b^1 = c^1$ ,  $a^2 + b^2 = c^2$  but  $a^3 + \dots + b^3 \dots \neq c^3 \dots$

## 2. Conclusion

I am sure the proof will close the search for the reason of  $a^2 + b^2 = c^2$  but  $a^3 + \dots + b^3 \dots \neq c^3 \dots$ . It would help the mathematical world to present different formulas and theorems to solve problems according to their needs.

## References

I have to be clear that I had never consulted any book, research work, paper, or article that was published. Whatever I did was on my own.