

Original Article

Fuzzy Cone Metric Spaces and Common Fixed Point Theorems for Fuzzy type Generalized TK -Contraction with Applications

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Received: 23 February 2026

Revised: 28 March 2026

Accepted: 17 April 2026

Published: 28 April 2026

Abstract - Fixed point methods form a cornerstone of contemporary nonlinear analysis. Their systematic study began with Banach's contraction principle in 1922 [1], which provided a rigorous criterion for the existence and uniqueness of invariant points in metric settings. This landmark idea has since inspired a wide spectrum of investigations, resulting in increasingly generalized frameworks and refined contractive assumptions. In this study, we introduce novel extensions of fuzzy type TK_1 - contraction and TK_2 contraction mapping within the context of fuzzy cone metric spaces and establish new fixed point theorems to support these developments.

Keywords - Fuzzy cone metric space, Fuzzy type TK_1 - contraction and Fuzzy type TK_2 contraction mapping, Fixed point, Common fixed point.

1. Introduction

Fixed point theory, due to its wide applications in mathematics and applied sciences, forms a central framework in nonlinear analysis. It originates from Banach's contraction theorem (1922), which firmly established the existence and uniqueness of fixed points in complete metric spaces. This fundamental result has inspired extensive research, leading to various generalizations and refinements under wider contraction conditions.

In 2007, Huang and Zhang [2] introduced the concept of cone metric spaces and established several fixed point results for contractive mappings within this framework. Their work inspired further research, leading to various extensions and generalizations in cone metric spaces (see [3–14]).

Parallel to these developments, fuzzy set theory, originally introduced by Zadeh [15], was further extended to metric-type structures by Kramosil and Michálek [16] via the concept of fuzzy metric spaces. Within this framework, Gregori and Sapena [17] established several fixed point theorems of contractive type in complete fuzzy metric spaces. Subsequently, numerous researchers have contributed to the advancement of fixed point theory in fuzzy metric spaces (see [18-23]).

Recently, Öner et al. [24] introduced the concept of fuzzy cone metric spaces as a natural generalization of fuzzy metric spaces. They established several fundamental properties of this structure and proved a Banach contraction theorem under the assumption of the existence of a Cauchy sequence.

Furthermore, Bag T. [25] proposed an alternative approach to fuzzy cone metric spaces and, within this framework, obtained fixed point results for fuzzy T-Kannan and fuzzy T-Chatterjea type contraction mappings. Subsequently, a number of researchers have contributed to the development of this theory by extending and refining the fuzzy cone version of the Banach contraction principle, leading to various generalized fixed point theorems in fuzzy cone metric spaces (see [26–30] for further details and related results).

The main objective of the present work is to establish new fixed point theorems for fuzzy-type generalized TK contraction mappings in Fuzzy Cone Metric (FCM) spaces. The results presented herein not only generalize but also improve several existing fixed-point theorems available in the literature, particularly those reported in [25].



2. Basic Fundamental Tools

We now introduce several key definitions and concepts essential for our analysis.

Definition 2.1[15]:

- (i) A function $x: [0,1] \rightarrow \mathbb{R}$ is called a fuzzy real number.
- (ii) A fuzzy real number x is convex if, for all $s \leq t \leq r$,

$$x(t) \geq \cap (x(s), x(r))$$

Definitions 2.2[25]:

The α -level set of fuzzy real number x is defined as

$$[x]_\alpha = \{t \in R: x(t) \geq \alpha\}, \alpha \in [0,1].$$

If there exists $t_0 \in \mathbb{R}$ such that $x(t_0) = 1$, then x is said to be normal. For the $0 < \alpha \leq 1$, α -level set of an upper semi-continuous, convex, normal fuzzy real number η , denoted by $[\eta]_\alpha$, is a closed interval $[a_\alpha, b_\alpha]$, where end points a_α and b_α are admissible in the extended real sense., i.e. $a_\alpha = -\infty$ and $b_\alpha = +\infty$.

A fuzzy real number x is called non-negative if $x(t) = 0, \forall t < 0$.

Definition 2.3 [25]:

- A subset $P \subset E^*(I)$ is called a fuzzy cone if
- (a) P is non-empty, fuzzy closed, and $P \neq \{0\}$.
 - (b) For any $a, b \geq 0$, and $\eta, \delta \in P \Rightarrow a\eta \oplus b\delta \in P$.

Definition 2.4[25]: Let $F \in E^*(I)$ be a fuzzy cone. A partial ordering \leq defined on F by $\eta \leq \delta \Leftrightarrow \delta \oplus \eta \in F$. We write $\eta < \delta$ if $\eta \leq \delta$ but $\eta \neq \delta$, and $\eta \ll \delta$ if $\delta \ominus \eta \in \text{Int } F$, where $\text{Int } F$ denotes the interior of F . A fuzzy cone P is called normal if there exists $k > 0$ such that, for all $x, y \in E$,

$$0 \leq \|x\| \leq \|y\| \Rightarrow \|x\| \leq k \|y\|.$$

The smallest positive number k satisfying this property is the normal constant P .

A fuzzy cone P is regular if every increasing sequence bounded from above converges. Specifically, if $\{x_n\} \subset E$ satisfies

$$\|x\| \leq \|x_2\| \leq \|x_3\| \leq \dots \leq \|x_n\| \leq \|y\| \text{ for some } y \in E,$$

then there exists $x \in E$ such that $\|x_n - x\| \rightarrow 0$ as $n \rightarrow \infty$.

Definition 2.5[25]:

Let X be a non-empty set, and let $d: X \times X \rightarrow E^*(I)$ be a mapping satisfying:

- (Fd_1). $d(x, y) \geq 0$ and $(x, y) = 0$ iff $x = y; \forall x, y \in X$.
- (Fd_2). $d(x, y) = d(y, x), \forall x, y \in X$;
- (Fd_3). $d(x, y) \leq d(x, z) \oplus d(z, y) \forall x, y \in X$.

Then d is called a fuzzy cone metric, and the pair (X, d) is termed a fuzzy cone metric space.

Definition 2.6[25]: Let (X, d) be a fuzzy cone metric space and let $\{x_n\}$ be a sequence in $X, x \in X$, then a sequence $\{x_n\} \subset X$ converges to $x \in X$ if, $c \in E$ with $0 \ll \|c\|$, there exists $N > 0$ such that $d(x_n, x) \ll \|c\|$, for all $n > N$. Denote $\lim_{n \rightarrow \infty} x_n = x$.

Definition 2.7[25]: Let (X, d) be a fuzzy cone metric space and let $\{x_n\}$ be a sequence in X , then a sequence $\{x_n\} \subset X$ is Cauchy if, for every $c \in E$ with $0 \ll \|c\|$, there exists $N > 0$ such that $d(x_n, x_m) \ll \|c\|$, for all $n, m > N$.

Definition 2.8[25]: A fuzzy cone metric space (X, d) is complete if every Cauchy sequence is convergent in X .

Definition 2.9[25]: Let (X, d) have normal fuzzy cone and $\{x_n\}$ be a sequence in X . Then

- (i) $\{x_n\}$ converges to x iff $d(x_n, x) \rightarrow 0$ as $n \rightarrow \infty$.
- (ii) $\{x_n\}$ is a Cauchy sequence iff $d(x_n, x_m) \rightarrow 0$ as $m, n \rightarrow \infty$.

Definition 2.10[30]: A fuzzy mapping $T: X \rightarrow F(X)$ is called fuzzy fixed point at $x \in X$ if $x \in [T_x]_\alpha$, where $\alpha \in (0,1)$.

Definition 2.11[25]: Let (X, d) be a fuzzy cone metric space and $T, R: X \rightarrow X$ be two functions. Then a mapping R is said to be fuzzy TK_1 contraction if there is a constant $b \in [0, \frac{1}{2}]$ such that

$$d(TRx, TRy) \leq \alpha [d(Tx, TRx) \oplus d(Ty, TRy)], \forall x, y \in X.$$

3. Main Results

The following result extends and improves Theorem 3.1 of [28].

Theorem 3.1: Let (X, d) be a complete fuzzy cone metric space, P be a normal fuzzy cone, with fuzzy normal Constant K . Let $T: X \rightarrow X$ be a one-to-one, continuous function, and $Q, R: X \rightarrow X$ be a pair of TK_1 – contraction. Then

(i) For every $x_0 \in X$

$$\lim_{p \rightarrow \infty} d(TQ^{2p+1}x_0, TQ^{2p+2}x_0) = 0$$

and

$$\lim_{p \rightarrow \infty} d(TR^{2p+1}x_0, TR^{2p+3}x_0) = 0$$

(ii) There exists $v \in X$ such that

$$\lim_{p \rightarrow \infty} TQ^{2p+1}x_0 = v = \lim_{p \rightarrow \infty} TR^{2p+2}x_0.$$

(iii) If T is sub-sequentially convergent, then $(Q^{2p+1}x_0)$ and $(R^{2p+2}x_0)$ have a convergent subsequence.

(iv) There is a unique common fixed point $u \in X$ such that

$$Qu = u = Ru.$$

(v) If there is a sequentially convergent sequence, then for each $x_0 \in X$ the iterate sequences $(Q^{2p+1}x_0)$ and $(R^{2p+2}x_0)$ converges to u .

Proof: - Let $x_0 \in X$. Then we define the iterate sequences (x_{2p+1}) and (x_{2p+2}) by

$$\begin{aligned} x_{2p+2} &= Q_{x_{2p+1}} = Q^{2p+1}x_0 \\ &\text{and} \\ x_{2p+3} &= R_{x_{2p+1}} = R^{2p+2}x_0 \end{aligned}$$

Since Q and R are a pair of TK_1 - contractions: we have

$$\begin{aligned} d(Tx_{2p+1}, Tx_{2p+2}) &= d(TQx_{2p}, TQx_{2p+1}) \\ &\leq \alpha [d(Tx_{2p}, TQx_{2p}) \oplus d(Tx_{2p+1}, TQx_{2p+1})] \\ &\leq \alpha [d(Tx_{2p}, Tx_{2p+1}) \oplus d(Tx_{2p+1}, Tx_{2p+2})] \end{aligned} \quad (3.1.1)$$

Similarly

$$\begin{aligned} d(Tx_{2p+2}, Tx_{2p+3}) &= d(TRx_{2p+1}, TRx_{2p+2}) \\ &\leq \alpha [d(Tx_{2p+1}, TRx_{2p+2}) \oplus d(Tx_{2p+2}, TRx_{2p+2})] \\ &\leq \alpha [d(Tx_{2p+1}, Tx_{2p+2}) \oplus d(Tx_{2p+2}, Tx_{2p+3})] \end{aligned} \quad (3.1.2)$$

From (3.1.1) and (3.1.2), it follow that

$$\begin{aligned} d(Tx_{2p+1}, Tx_{2p+2}) &\leq \frac{\alpha}{1-\alpha} d(Tx_{2p}, Tx_{2p+1}) \\ &\text{and} \\ d(Tx_{2p+2}, Tx_{2p+3}) &\leq \frac{\alpha}{1-\alpha} d(Tx_{2p+1}, Tx_{2p+2}) \end{aligned}$$

Now, we conclude by repeating the process, and we get

$$d(TQ^{2p+1}x_0, TQ^{2p+2}x_0) \leq \left(\frac{\alpha}{1-\alpha}\right)^{2p+1} d(Tx_0, TQx_0) \quad (3.1.3)$$

and

$$d(TR^{2p+2}x_0, TR^{2p+3}x_0) \leq \left(\frac{\alpha}{1-\alpha}\right)^{2p+2} d(Tx_0, TRx_0) \quad (3.1.4)$$

Since the cone P is normal with a constant of $E^*(I)$., we obtain

$$\| d(TQ^{2p+1}x_0, TQ^{2p+2}x_0) \| \leq \left(\frac{\alpha}{1-\alpha}\right)^{2p+1} K \| d(Tx_0, TQx_0) \|.$$

Letting $p \rightarrow \infty$ and using $\frac{\alpha}{1-\alpha} < 1$, we conclude that

$$\lim_{p \rightarrow \infty} \| d(TQ^{2p+1}x_0, TQ^{2p+2}x_0) \| = 0.$$

Hence

$$\lim_{p \rightarrow \infty} d(TQ^{2p+1}x_0, TQ^{2p+2}x_0) = 0 \quad (3.1.5)$$

Similarly,

$$\lim_{p \rightarrow \infty} d(TR^{2p+2}x_0, TR^{2p+3}x_0) = 0 \quad (3.1.6)$$

So (i) is proved.

By inequality (3.1.3), for every $p, q \in N$ with $q > p$, we have

$$\begin{aligned} d(Tx_{2p+1}, Tx_{2q+1}) &\leq d(Tx_{2p+1}, Tx_{2p+2}) + \dots + d(Tx_{2q}, Tx_{2q+1}) \\ &\leq \left[\left(\frac{\alpha}{1-\alpha}\right)^{2p+1} + \dots + \left(\frac{\alpha}{1-\alpha}\right)^{2q} \right] d(Tx_0, TQx_0) \\ &= \left(\frac{\alpha}{1-\alpha}\right)^{2p+1} \cdot \frac{1}{1-\left(\frac{\alpha}{1-\alpha}\right)} d(Tx_0, TQx_0). \end{aligned}$$

Therefore,

$$d(Tx_{2p+1}, Tx_{2q+1}) \leq \left(\frac{\alpha}{1-\alpha}\right)^{2p+1} \cdot \frac{1}{1-\left(\frac{\alpha}{1-\alpha}\right)} d(Tx_0, TQx_0) \quad (3.1.7)$$

From (3.1.7), we have

$$\|d(TQ^{2p+1}x_0, TQ^{2q+1}x_0)\| \leq \left(\frac{\alpha}{1-\alpha}\right)^{2p+1} \cdot \frac{K}{1-\left(\frac{\alpha}{1-\alpha}\right)} \|d(Tx_0, TQx_0)\|, \quad (3.1.7)$$

where K is the normal constant of $E^*(I)$.

Taking limit as $q, p \rightarrow \infty$ and by $\frac{\alpha}{1-\alpha} < 1$, we get

$$\lim_{p, q \rightarrow \infty} \|d(TQ^{2p+1}x_0, TQ^{2q+1}x_0)\| = 0.$$

In this way, we have

$$\lim_{p, q \rightarrow \infty} d(TQ^{2p+1}x_0, TQ^{2q+1}x_0) = 0.$$

Hence $(TQ^{2p+1}x_0)$ is a Cauchy sequence in X . Since X is a complete $\exists v \in X$ such that

$$\lim_{p \rightarrow \infty} TQ^{2p+1}x_0 = v. \quad (3.1.8)$$

So (ii) holds.

Now, if T is subsequently convergent, then the sequence $(Q^{2p+1}x_0)$ has a convergent subsequence. So there are $u \in X$ and $(x_{(2p+1)})_i$ such that

$$\lim_{i \rightarrow \infty} Q^{(2p+1)i}x_0 = u \quad (3.1.9)$$

Since T is continuous and by (3.1.7), we obtain

$$\lim_{i \rightarrow \infty} TQ^{(2p+1)i}x_0 = Tu \quad (3.1.10)$$

From (3.1.8) and (3.1.10), we conclude that

$$Tu = v$$

So (iii) holds.

On the other hand

$$\begin{aligned} d(TQu, Tu) &\leq d(TQu, TQ^{(2p+1)i}x_0) \oplus d(TQ^{(2p+1)i}x_0, TQ^{(2p+1)i+1}x_0) \\ &\quad \oplus d(TQ^{(2p+1)i+1}x_0, Tu) \\ &\leq [d(Tu, TQu) \oplus d(TQ^{(2p+1)i}x_0, TQ^{(2p+1)i}x_0)] \\ &\quad \oplus \left(\frac{\alpha}{1-\alpha}\right)^{(2p+1)i} d(Tx_0, TQx_0) \oplus d(TQ^{(2p+1)i+1}x_0, Tu) \end{aligned}$$

Hence,

$$\begin{aligned} d(TQu, Tu) &\leq \left(\frac{\alpha}{1-\alpha}\right) d(TQ^{(2p+1)i+1}x_0, TQ^{(2p+1)i}x_0) \oplus \frac{1}{1-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{(2p+1)i} d(TQx_0, Tx_0) \\ &\quad \oplus \frac{1}{1-\alpha} d(TQ^{(2p+1)i+1}x_0, Tu). \end{aligned}$$

Since K is the normal constant of $E^*(I)$. We have

$$\begin{aligned} \|d(TQu, Tu)\| &\leq \frac{\alpha}{1-\alpha} K \|d(TQ^{(2p+1)i+1}x_0, TQ^{(2p+1)i}x_0)\| \oplus \frac{K}{1-\alpha} \left\| \left(\frac{\alpha}{1-\alpha}\right)^{(2p+1)i} d(TQx_0, Tx_0) \right\| \\ &\quad \oplus \frac{1}{1-\alpha} \|d(TQ^{(2p+1)i+1}x_0, Tu)\|. \rightarrow 0, \text{ as } i \rightarrow \infty. \end{aligned}$$

Using the contractive condition and taking limits, we deduce

$$d(TQu, Tu) = 0.$$

Which implies that the inequality

$$TQu = Tu.$$

Since T is one-one, then $Qu = u$. Thus, u is a fixed point of Q .

Because Q is TK_1 – contraction, we have

$$\begin{aligned} d(TQu, TQv) &\leq \alpha [d(Tu, TQu) \oplus d(Tv, TQv)] \\ &= \alpha [d(Tu, Tu) \oplus d(Tv, Tv)] \end{aligned}$$

If v is another fixed point of Q , then from the injectivity, we get

$$Qu = Qv$$

Or, which is the same, the fixed point is unique. Finally, if T is sequentially convergent, by replacing $(2p + 1)$ for $(2p + 1)i$, we conclude that

$$\lim_{p \rightarrow \infty} Q^{2p+1}x_0 = u$$

This shows that $(Q^{2p+1}x_0)$ converges to the fixed point of Q .

Similarly, we can prove that $(R^{2p+2}x_0)$ converges to the fixed point of R .

$$\lim_{p \rightarrow \infty} Q^{2p+1}x_0 = u = \lim_{p \rightarrow \infty} R^{2p+2}x_0.$$

Hence, u is the unique common fixed point of Q and R . This completes the proof.

Example 3.2: Let $X = C([0,1], \mathbb{R})$ and let define a fuzzy cone metric $d: X \times X \rightarrow E^*(I)$ by $d(x, y) = \|x - y\|_\infty e$, where $\|x - y\|_\infty = \max_{t \in [0,1]} |x(t) - y(t)|$, e is a fixed element of fuzzy cone P , and K is a normal constant P . Then (X, d) is a complete fuzzy cone metric space. Define mappings $T, Q, R: X \rightarrow X$ by

$$\begin{aligned} (Tx)(t) &= x(t), (Qx)(t) = \frac{1}{4} x(t), \text{ and} \\ (Rx)(t) &= \frac{1}{6} x(t), \forall t \in [0,1]. \end{aligned}$$

Since T is one-to-one and continuous. So, for any $x, y \in X$, we have

$$\begin{aligned} d(TQx, TQy) &= d(Qx, Qy) \\ &= \left\| \frac{1}{4} x - \frac{1}{4} y \right\|_\infty e \\ &= \frac{1}{4} d(x, y). \end{aligned}$$

Similarly

$$\begin{aligned} d(TRx, TRy) &= d(Rx, Ry) \\ &= \left\| \frac{1}{6} x - \frac{1}{6} y \right\|_\infty e \\ &= \frac{1}{6} d(x, y). \end{aligned}$$

Hence Q and R satisfy the TK -contraction mappings with constant $\alpha = \frac{1}{4}$ and $\alpha' = \frac{1}{6}$ respectively. Therefore, all hypothesis of Theorem 3.1 are satisfied and $u(t) = 0$ unique common fixed point.

4. Applications

In this section, we apply Theorem 3.1 to establish the existence and uniqueness of solutions of a first-order differential equation and nonlinear boundary problems.

4.1. Application to First-Order Differential Equations

Let us consider

$$\frac{dx(t)}{dt} = T(t, x(t)), t \in [0; 1], \text{ and } x(0) = 0 \quad (4.1.1)$$

where $T: [0,1] \times \mathbb{R} \rightarrow \mathbb{R}$ is continuous and satisfies the Lipschitz condition

$$|T(t, u) - T(t, v)| \leq K|u - v|, 0 < K < 1.$$

Define an operator $Q: X \rightarrow X$ by (4.1.1), can be written as

$$Q(x)(t) = \int_0^t T(s, x(s))ds, \text{ where } x(t) = \int_0^t T(s, x(s))ds.$$

Suppose $X = C([0,1], \mathbb{R})$ with identity mapping $T: X \rightarrow X$. Then

$$\begin{aligned} d(TQx, TQy) &= \|Qx - Qy\|_{\infty} \\ &\leq \sup_{t \in [0,1]} \int_0^t |T(s, x(s)) - T(s, y(s))| ds \\ &\leq K \|x - y\|_{\infty}. \end{aligned}$$

Since $K < 1$ and an operator Q is TK -contraction. So, all the conditions of Theorem 3.1 are satisfied. Hence Q admits a unique fixed point $u \in X$ such that,

$$u(t) = \int_0^t T(s, u(s))ds.$$

Differentiating both sides yields

$$\frac{du(t)}{dt} = T(t, u(t)), t \in [0; 1], \text{ and } u(0) = 0.$$

Therefore, $u(t)$ is the unique solution of the given initial value.

4.2. Application to Nonlinear Boundary value Problems

Let us consider the nonlinear boundary value problem.

$$\begin{cases} x''(t) = T(t, x(t)), t \in [0,1] \\ x(0) = 0, x(1) = 0, \end{cases} \quad (4.2.1)$$

where $T: [0,1] \times \mathbb{R} \rightarrow \mathbb{R}$ is continuous and satisfies the Lipschitz condition

$$|T(t, u) - T(t, v)| \leq K|u - v|, 0 < K < 1.$$

Now, using the Green function.

$$G(t, s) = \begin{cases} t(1-s), 0 \leq t \leq s \leq 1 \\ s(1-t), 0 \leq s \leq t \leq 1. \end{cases} \quad (4.2.2)$$

But by (4.2.1),

$$x(t) = \int_0^1 G(t, s)T(s, x(s))ds. \quad (4.2.3)$$

Then, we define an operator $Q: X \rightarrow X$ by

$$(Qx)(t) = \int_0^1 G(t, s)T(s, x(s))ds.$$

Then for all $x, y \in X$

$$\begin{aligned} d(TQx, TQy) &\leq K \|x - y\|_{\infty} \sup_{t \in [0,1]} |G(t, s)| ds \\ &= \frac{1}{4} K d(x, y). \end{aligned}$$

Since $\frac{1}{4} < 1$, and Q is TK -contraction. So, by Theorem 3.1, Q has a unique fixed point $u \in X$.

5. Conclusion

In this paper, we establish Theorem 3.1, which generalizes and strengthens Theorem 3.1 of [28] within the framework of fuzzy cone metric spaces. The result is obtained under comparatively mild conditions, namely the completeness of the fuzzy cone metric space, the normality of the underlying fuzzy cone, and the existence of a one-to-one continuous mapping along with a pair of TK -contractive mappings. To illustrate the effectiveness of the theorem, several applications have been presented in the framework of fuzzy cone metric spaces, including first-order initial value problems and nonlinear boundary value problems.

Acknowledgements

The Authors would like to thank anonymous reviewers for their comments on the manuscript, which helped us very much in improving and presenting the original version of this paper.

Conflict of Interest

Concerning the title of this object, the originators confirm that there is no contradiction.

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