On Soft πgb-Continuous Functions in Soft Topological Spaces

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ABSTRACT

The aim of this paper is to define and study the concepts of soft πgb-continuous function, soft πgb-irresolute function on soft topological spaces. Also we have introduced the concept soft πgb-closure and soft πgb-interior. Further relationship between soft πgb-continuous function with other soft continuous functions are established.

Keywords: soft set, soft topology, soft πgb-interior, soft πgb closure, soft continuous, soft irresolute,.

1. Introduction

Molodtsov[11,12] initiated the concept of soft set theory as a new mathematical tool and presented the fundamental results of the soft sets. Soft systems provide a general framework with the involvement of parameters. Soft set theory has a wider application and its progress is very rapid in different fields. Levine[9] introduced g-closed sets in general topology. Kannan [7] introduced soft g-closed sets in soft topological spaces. Muhammad Shabir and Munazza Naz [15] introduced soft topological spaces and the notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms. Soft semi-open sets and its properties were introduced and studied by Bin Chen[2]. Kharal et al.[8]introduced soft function over classes of soft sets. Cigdem Gunduz Aras et al., [3] in 2013 studied and discussed the properties of Soft continuous mappings which are defined over an initial universe set with a fixed set of parameters. Mahanta and Das [14] introduced and characterized various forms of soft functions like semi continuous, semi irresolute, semi open soft functions.

In the paper, we have introduced the concept soft πgb-closure and soft πgb-interior. Also the concepts soft πgb-continuity, soft πgb-irresolute functions on soft topological spaces are discussed and some characterizations of these mappings are obtained.

2. Preliminaries

Let U be an initial universe set and E be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U. Let P(U) denote the power set of U, and let $A \subseteq E$.

Definition 2.1 ([11]). A pair (F,A) is called a soft set over U, where F is a mapping given by F : A → P(U). In other words, a soft set over U is a parametrized family of subsets of the universe U. For a particular e ∈ A. F(e) may be considered the set of e-approximate elements of the soft set (F,A).

Definition 2.2. ([4]). For two soft sets (F,A) and (G,B) over a common universe U, we say that (F,A) is a soft subset of (G,B) if

(i) A ⊆ B, and

(ii)∀e ∈ A, F(e) $\tilde{\subseteq}$ G(e).

We write (F,A) ⊆ (G,B). (F,A) is said to be a soft super set of (G,B), if (G,B) is a soft subset of (F,A). We denote it by (F,A) $\tilde{\supseteq}$ (G,B).

Definition 2.3. ([10]). A soft set (F,A) over U is said to be

(i) null soft set denoted by ϕ if ∀e ∈ A, F(e) = ϕ.

(ii) absolute soft set denoted by A, if ∀e ∈ A, F(e) = U.

Definition 2.4. For two soft sets (F,A) and (G,B) over a common universe U,

(i) ([10]) union of two soft sets of (F,A) and (G,B) is the soft set (H,C),where C = A ∪ B, and ∀e ∈ C,

$$H(e) = \begin{cases} F(e) & ,if\ e \in A - B \\ G(e) & ,if\ e \in B - A \\ F(e) \cup G(e) & ,if\ e \in A \cap B \end{cases}$$

We write (F,A) ∪(G,B)=(H,C).

Definition :2.5 ([4])

The Intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe U denoted (F,A) ∩ (G,B) is defined as C= A∩ B and H(e) = F(e) ∩ G(e) for all e∈C.

Definition:2.6 ([15])

For a soft set (F,A) over the universe U, the relative complement of (F,A) is denoted by (F,A)′ and is defined by (F,A)′=(F′,A), where F′: A →P(U) is a mapping defined by F′(e)= U-F(e) for all e∈A.

Definition:2.7 ([15])

Let τ be the collection of soft sets over X, then τ is called a soft topology on X if τ satisfies the following axioms:

1) ϕ, $\tilde{X}$ belong to τ

2) The union of any number of soft sets in τ belongs to τ.
3) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. For simplicity , we can take the soft topological space (X, τ, E) as X throughout the work.

Definition:2.8 ([15])

Let (X, τ, E) be soft space over X. A soft set (F,E) over X is said to be soft closed in X, if its relative complement (F,E)′ belongs to τ. The relative complement is a mapping $F':E\rightarrow P(X)$ defined by $F'(e)= X\text{-}F(e)$ for all $e \in A$.

Definition:2.9 ([15])

Let X be an initial universe set , E be the set of parameters and $\tau=\{ \phi, \tilde{X} \}$. Then τ is called the soft indiscrete topology on X and (X, τ, E) is said to be a soft indiscrete space over X. If $\tau$is the collection of all soft sets which can be defined over X, then τ is called the soft discrete topology on X and (X, τ, E) is said to be a soft discrete space over X.

Definition:2.10 ([15])

Let (X, τ, E) be a soft topological space over X and the soft interior of (F,E) denoted by Int(F,E) is the union of all soft open subsets of (F,E). Clearly, (F,E) is the largest soft open set over X which is contained in (F,E).The soft closure of (F,E) denoted by Cl(F,E) is the intersection of all closed sets containing (F,E). Clearly , (F,E) is smallest soft closed set containing (F,E).

Int (F,E) = $\cup$ { (O,E): (O,E) is soft open and (O,E) $\tilde{\subseteq}$ (F,E)}.

Cl(F,E) = $\cap$\{ (O,E): (O,E) is soft closed and (F,E) $\tilde{\subseteq}$ (O,E)}.

Definition:2.11 ([2],[7],[10])

Let U be the common universe set and E be the set of all parameters. Let (F,A) and (G,B) be soft sets over a common universe set U and A,B $\tilde{\subseteq}$E. Then (F,A) is a subset of (G,B), denoted by (F,A) $\tilde{\subseteq}$ (G,B). (F,A) equals (G,B) , denoted by (F,A)=(G,B) if (F,A) $\tilde{\subseteq}$ (G,B) and (G,B) $\tilde{\subseteq}$ (F,A).

Definition:2.12

A soft subset (A,E) of X is called

(i) a soft generalized closed (Soft g-closed)[7] if Cl(A,E) $\tilde{\subseteq}$ U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in X.

(ii) a soft semi open [2]if (A,E) $\tilde{\subseteq}$ Int(Cl(A,E))

(iii) a soft regular open[6] if (A,E)= Int(Cl(A,E)).

(iv) a soft α-open[6] if (A,E) $\tilde{\subseteq}$ Int(Cl(Int(A,E)))

(v) a soft b-open[6] if (A,E) $\tilde{\subseteq}$ Cl(Int(A,E)) Int(Cl(A,E))

(vi) a soft pre-open[6] set if (A,E) $\tilde{\subseteq}$ Int (Cl(A,E)).

(vii) a soft clopen[6] is (A,E) is both soft open and soft closed.

(viii) a soft β-open[16] set if (A,E) $\tilde{\subseteq}$ Cl(Int (Cl(A,E))).

(ix) a soft generalized β closed (Soft gβ-closed)[1] in a soft topological space (X, τ,E) if βCl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in X.

(x) a soft gsβ closed[1] if βCl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft semi open in X.

(xi) soft πgb-closed in X if sbcl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft π-open in X.

The complement of the soft semi open , soft regular open , soft α-open, soft b-open , soft pre-open sets are their respective soft semi closed , soft regular closed , soft α-closed , soft b-closed and soft pre -closed sets.

The finite union of soft regular open sets is called soft π-open set and its complement is soft π-closed set. The soft regular open set of X is denoted by SRO(X) or SRO(X, τ, E).

Definition:2.13 [7]

A soft topological space X is called a soft $T_{1/2}$-space if every soft g-closed set is soft closed in X.

Definition:2.14[6]

The soft regular closure of (A,E) is the intersection of all soft regular closed sets containing (A,E). (i.e)The smallest soft regular closed set containing (A,E) and is denoted by srcl(A,E).

The soft regular interior of (A,E) is the union of all soft regular open set s contained in (A,E) and is denoted by srint(A,E).

Similarly , we define soft α-closure, soft pre-closure, soft semi closure and soft b-closure of the soft set (A,E) of a topological space X and are denoted by sαcl(A,E), spcl(A,E), sscl(A,E) and sbcl(A,E) respectively.

Definition 2.15. [11] Let (F,E) be a soft set X. The soft set (F,E) is called a soft point , denoted by (x_e,E), if for the element $e \in E$, $F(e) = \{x\}$ and $F(e') = \phi$ for all $e' \in E - \{e\}$.

Definition 2.16. Let (X, τ, E) and (Y, τ', E) be two soft topological spaces.A function $f : (X, \tau, E) \rightarrow (Y, \tau', E)$ is said to be

(i) soft semi-continuous[14] if $f^{-1}((G,E))$ is soft semi-open in (X, τ, E), for every soft open set G,E) of (Y, τ', E).

(ii) soft pre-continuous [16] if $f^{-1}((G,E))$ is soft pre-open in (X, τ, E), for every soft open set (G,E) of (Y, τ', E).

(iii) soft α-continuous [16] if $f^{-1}((G,E))$ is soft α-open in (X, τ, E), for every soft open set (G,E) of (Y, τ', E).

(iv) soft β-continuous [16] if $f^{-1}((G,E))$ is soft β-open in (X, τ, E), for every soft open set (G,E) of (Y, τ', E).

(v) soft β-irresolute [16] if $f^{-1}((G,E))$ is soft β-open in (X, τ, E), for every soft β-open set (G,E) of (Y, τ', E).

(vi) soft πgr-continuous[6] if $f^{-1}((G,E))$ is soft πgr-closed in (X, τ, A) for every soft closed set (G, E) in (Y, τ', E).

(vii)soft πgr-irresolute [6]if $f^{-1}((G, E))$ is soft πgr-closed in (X, τ, A) for every soft πgr- closed set (G, E) in (Y, τ', E).

(viii) soft regular continuous[6] if $f^{-1}((G,E))$ is soft regular closed in (X, τ, A) for every soft closed set (G, E) in (Y, τ', E).

Definition:2.20 [6] Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces and $f_{pu}: SS(X)_A \rightarrow SS(Y)_B$ be a function. Then the function fpu is called soft open mapping if $f_{pu}((G,A)) \in \tau^*$ for all $(G,A) \in \tau$. Similarly, a function $f_{pu}: SS(X)_A \rightarrow SS(Y)_B$ is called a soft closed map if for a closed set (F,A) in τ, the image $f_{pu}((G,B))$ is soft closed in τ^*.

3.Soft πgb-closure and soft πgb-interior

Definition 3.1: Let X be a soft topological space and $(A,E) \tilde{\subseteq} X$. Then s$\pi$gb-cl(A,E) is the intersection of all soft πgb-open sets containing the soft set (A,E) and is denoted by sπgb-cl (A,E).

Lemma 3.2: Let (F,A) and (G,A) be soft subsets of soft topological space X, then we have

a) sπgb-cl(ϕ)=ϕ and sπgb-cl (X)=X

b)If $(F,A) \tilde{\subseteq} (G,A)$, then s$\pi$gb-cl (F,A) $\tilde{\subseteq}$ sπgb-cl (G,A)

c) sπgb-cl (F,A) = sπgb-cl (sπgb-cl (F,A))

d) (F,A) $\tilde{\subseteq}$ sπgb-cl (F,A)

Theorem 3.3: Let (F,A) and (G,A) be soft subsets of soft topological space X, then sπgb-cl ((F,A) $\cap$(G,A)) $\tilde{\subseteq}$ sπgb-cl ((F,A))$\cap$ sπgb-cl ((G,A)).

Proof: Since $(F,A) \cap (G,A) \tilde{\subseteq} (F,A)$ and $(F,A) \cup (G,A) \tilde{\subseteq} (G,A)$, s$\pi$gb-cl ((F,A) $\cap$(G,A)) $\tilde{\subseteq}$ sπgb-cl ((F,A)) and sπgb-cl ((F,A) $\cap$(G,A)) $\tilde{\subseteq}$ sπgb-cl ((G,A)).This implies sπgb-cl ((F,A) $\cap$(G,A)) $\tilde{\subseteq}$ sπgb-cl ((F,A)) $\cap$ sπgb-cl ((G,A)).

Theorem 3.4: Let (F,A) and (G,A) be soft subsets of soft topological space X, then sπgb-cl ((F,A) ∪(G,A)) $\tilde{\supseteq}$ sπgb-cl ((F,A))∪ sπgb-cl ((G,A))

Proof: We know that (F,A) $\tilde{\subseteq}$((F,A) ∪(G,A)) and (G,A) $\tilde{\subseteq}$((F,A) ∪(G,A)), sπgb-cl ((F,A)) $\tilde{\subseteq}$sπgb-cl ((F,A) ∪ (G,A)) and sπgb-cl ((G,A)) $\tilde{\subseteq}$sπgb-cl ((F,A) ∪ (G,A)).This implies sπgb-cl ((F,A)) ∪ sπgb-cl ((G,A)) $\tilde{\subseteq}$sπgb-cl ((F,A) ∪ (G,A)).

Lemma 3.5: Let (F,A) and (G,A) be soft subsets of soft topological space X, then we have

a) sπgb-int(ϕ)=ϕ and sπgb-int (X)=X

b)If (F,A) $\tilde{\subseteq}$(G,A), then sπgb-int (F,A) $\tilde{\subseteq}$ sπgb-int (G,A)

c) sπgb-int (F,A) = sπgb-int (sπgb-int (F,A))

d)(F,A) $\tilde{\subseteq}$ sπgb-int (F,A)

Theorem 3.6: Let (F,A) and (G,A) be soft subsets of soft topological space X, then sπgb-int ((F,A)) ∩ sπgb-int ((G,A)) $\tilde{\supseteq}$ sπgb-int ((F,A) ∩ (G,A)).

Proof: Since (F,A) ∩ (G,A) $\tilde{\subseteq}$(F,A) and (F,A) ∩ (G,A) $\tilde{\subseteq}$ (G,A), sπgb-cl ((F,A) ∩(G,A)) $\tilde{\subseteq}$ sπgb-cl ((F,A)) and sπgb-cl ((F,A) ∩(G,A)) $\tilde{\subseteq}$ sπgb-cl ((G,A)).This implies sπgb-cl ((F,A) ∩(G,A)) $\tilde{\subseteq}$ sπgb-cl ((F,A)) ∩ sπgb-cl ((G,A)).

Theorem 3.7: Let (F,A) and (G,A) be soft subsets of soft topological space X, then sπgb-int ((F,A)) ∪ sπgb-int ((G,A)) $\tilde{\subseteq}$ sπgb-int ((F,A) ∪ (G,A)).

Proof: We know that (F,A) $\tilde{\subseteq}$((F,A) ∪(G,A)) and (G,A) $\tilde{\subseteq}$((F,A) ∪(G,A)), sπgb-int ((F,A)) $\tilde{\subseteq}$sπgb-int ((F,A) ∪ (G,A)) and sπgb-int ((G,A)) $\tilde{\subseteq}$sπgb-int ((F,A) ∪ (G,A)).This implies sπgb-int ((F,A)) ∪ sπgb-int ((G,A)) $\tilde{\subseteq}$sπgb-int ((F,A) ∪ (G,A)).

Definition 3.8 : Let X be a soft topological space and (A,E) $\tilde{\subseteq}$X. Then sπgb-int(A,E) is the union of all soft πgb-open sets contained in (A,E).

Theorem 3.9: Let (A,E) be a soft subset of a soft topological space X. Then X-sπgb-int (A,E) =sπgb-cl(X- (A,E)).

Proof: Let x∈X- sπgb-int(A,E). Then x∉ sπgb-int (A,E). That is every soft πgb-open set (B,E) containing x is such that (B,E) ⊄(A,E). This implies every soft πgb-open set (B,E) containing x intersects X- (A,E). Hence x∈ sπgb-cl(X- (A,E)).Conversely let x∈ sπgb-cl(X- (A,E)).Then every soft πgb-open set (B,E) containing x intersects X- (A,E). That is every soft πgb-open set (B,E) containing x is such that (B,E) ⊄ (A,E). This implies x∉ sπgb-int

(A,E) .Hence sπgb-cl(X- (A,E))⊂X- πgb-int (A,E). Therefore X-sπgb-int (A,E) =sπgb-cl(X-(A,E)).Similarly X-sπgb-cl (A,E) =sπgb-int(X- (A,E)).

4.Soft πgb- continuous functions

In this section, we study the notion of soft πgb-continuous functions and soft πgb-irresolute functions.

Definition 4.1: Let (X, τ , A) and (Y, τ′, B) be two soft topological spaces and $f:(X, \tau, A) \to (Y, \tau', B)$ be a function. Then the function f is

(i) soft πgb- continuous if $f^{-1}(G,B)$ is soft πgb- closed in (X, τ, A) for every soft closed set (G,B) of (Y, τ′, B).

(ii) soft πgb- irresolute if $f^{-1}(G,B)$ is soft πgb- closed in (X, τ, A) for every soft πgb- closed set (G,B) of (Y, τ′, B).

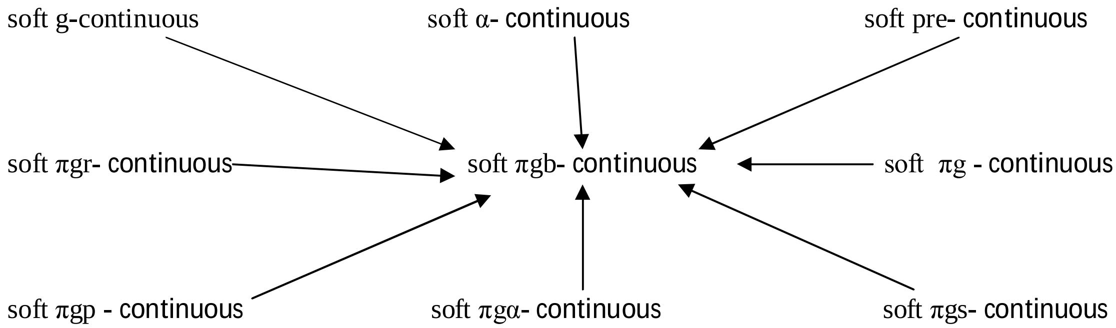
Definition 4.2: Let (X, τ, A) and (Y, τ′, B) be soft topological spaces and $f:(X, \tau, A) \to (Y, \tau', B)$ be a function. Then the function f is

(i) soft πgb- open if f(G,A) is soft πgb-open in (Y, τ′, B) for every soft open set (G,A) of (X, τ, A).

(ii) soft πgb- closed if f(G,A) is soft πgb-closed in (Y, τ′, B) for every soft closed set (G,A) of (X, τ, A).

Theorem 4.3:

a) Every soft continuous function is soft πgb- continuous.
b) Every soft g-continuous function is soft πgb- continuous.
c) Every soft α-continuous function is soft πgb- continuous.
d) Every soft pre-continuous function is soft πgb- continuous.
e) Every soft πgr-continuous function is soft πgb- continuous.
f) Every soft πg-continuous function is soft πgb- continuous.
g) Every soft πgp-continuous function is soft πgb- continuous.
h) Every soft πgα-continuous function is soft πgb- continuous.
i) Every soft πgs-continuous function is soft πgb- continuous.



Remark 4.4: Converse of the above need not be true as seen in the following examples.

Example 4.5: Let X=Y={a,b,c,d}, E={e_1,e_2}.Let F_1,F_2,….,F_6 are functions from E to P(X) and are defined as follows:

$F_1(e_1)$={c},$F_1(e_2)$={a},

$F_2(e_1)$={d},$F_2(e_2)$={b},

$F_3(e_1)$={c,d},$F_3(e_2)$={a,b},

$F_4(e_1)$={a,d},$F_4(e_2)$={b,d},

$F_5(e_1)$={b,c,d},$F_5(e_2)$={a,b,c},

$F_6(e_1)$={a,c,d},$F_6(e_2)$={a,b,d},

Then τ_1 ={Φ,X,(F_1,E),.....(F_6,E)} is a soft topology and elements in τ are soft open sets.

Let G_1,G_2,G_3,G_4 are functions from E to P(Y) and are defined as follows:

$G_1(e_1)$={a},$G_1(e_2)$={d}

$G_2(e_1)$={b},$G_2(e_2)$={c}

$G_3(e_1)$={a,b},$G_3(e_2)$={c,d}

$G_4(e_1)$={b,c,d},$G_4(e_2)$={a,b,c}

Then τ_2 ={Φ,X,(G_1,E),(G_2,E), (G_3,E), (G_4,E)} be a soft topology on Y.Let f:X→Y be an identity map.Here the inverse image of the soft closed set (A,E)={{c,d},{a,b}} in Y is not soft closed, soft α-closed , soft pre closed, soft πgα-closed, soft πgp-closed, soft πg-closed, soft πgs-closed in X. Hence not soft continuous, soft α-continuous, soft pre-continuous soft πgα-continuous, soft πgp-continuous, soft πg-continuous, soft πgs-continuous. Also soft closed set (B,E)={{a},{d}} in Y is not soft πgr-closed in X. Hence not soft πgr-continuous. Soft closed set (C,E)={{b,c,d},{a,b,c}} in Y is not soft g-closed in X.Hence not soft g-continuous.

Theorem 4.6: (a) Let f: (X, τ, A) → (Y, τ′, B) be a soft function, then the following statements are equivalent.

(i) f is soft πgb-continuous

(ii) The inverse image of every soft open set in Y is also soft πgb-open in X.

(b) If f: (X, τ, A) → (Y, τ′, B) is soft πgb-continuous, then f(sπgb-cl(A,E)) $\tilde{\subseteq}$ s-cl(f(A,E) for every subset (A,E) of X.

Proof : (a) (i)⇒(ii)Let (G,B)∈SO(Y), then (Y-(G,B))∈SC(Y). Since f is soft πgb-continuous ,f^{-1} (Y-(G,B))∈ SπGBC(X).Hence [X-(f^{-1}(G,B))∈ SπGBC(X).Then f^{-1}(G,E)∈ SπGBO(X).

(ii)⇒(i)Follows from definition.

(b) Let (A,E) $\tilde{\subseteq}$X. Since f is soft πgb-continuous and (A,E) $\tilde{\subseteq}$ f^{-1}(s-cl(f (A,E))),we obtain sπgb-cl((A,E)) $\tilde{\subseteq}$ f^{-1}(s-cl(f((A,E))) and then f(sπgb-cl((A,E))) $\tilde{\subseteq}$s-cl(f((A,E))).

Theorem 4.7: If a soft function f: (X, τ, A) → (Y, τ′, B) is soft πgb-closed, then sπgb-cl(f(A,E)) $\tilde{\subseteq}$ f(s-cl((A,E)) for every subset (A,E) of (X, τ, E).

Proof : Let (G,A) $\tilde{\subseteq}$X. Since f is soft πgb-closed and f((G,A) $\tilde{\subseteq}$ f(s-cl(G,A)),we obtain sπgb-cl(f(G,A)) $\tilde{\subseteq}$ sπgb-cl(f(s-cl(G,A))).Since f(s-cl(G,A)) is soft πgb-closed in (Y, τ′, B), sπgb-cl(f(s-cl(G,A)))= f(s-cl(G,A)) for every soft set (A,E) of (X, τ, E).Hence sπgb-cl(f(G,A)) $\tilde{\subseteq}$ f(s-cl(G,A)) for every soft set (G,A) of a soft topological space (X, τ, E).

Remark 4.8: Composition of two soft πgb-continuous functions need not be soft πgb-continuous.

Example 4.9: Let X=Y=Z={a,b,c,d}, E={e_1,e_2}.Let F_1,F_2,….,F_6 are functions from E to P(X) and are defined as follows:

$F_1(e_1)=\{c\}$,$F_1(e_2)=\{a\}$,

$F_2(e_1)=\{d\}$,$F_2(e_2)=\{b\}$,

$F_3(e_1)=\{c,d\}$,$F_3(e_2)=\{a,b\}$,

$F_4(e_1)=\{a,d\}$,$F_4(e_2)=\{b,d\}$,

$F_5(e_1)=\{b,c,d\}$,$F_5(e_2)=\{a,b,c\}$,

$F_6(e_1)=\{a,c,d\}$,$F_6(e_2)=\{a,b,d\}$,

Then τ′={Φ,X,(F_1,E),.....(F_6,E)} is a soft topology and elements in τ are soft open sets.

Let G_1,G_2 are functions from E to P(Y) and are defined as follows:

$G_1(e_1)=\{a\}$,$G_1(e_2)=\{d\}$

$G_2(e_1)=\{b,c,d\}$,$G_2(e_2)=\{a,b,c\}$

Then τ″={Φ,X,(G_1,E),(G_2,E),} be a soft topology on Y.

Let H_1 be function from E to P(Z) and is defined as follows:

$H_1(e_1)=\{b\}$,$H_1(e_2)=\{c\}$

Let f: (X, τ, E) → (Y, τ′, E) and g: (Y, τ′, E) → (Z, τ″, E) be identity mapping. Then f and g are soft πgb-continuous but g o f is not soft πgb-continuous.

Proposition 4.10: Every soft πgb- irresolute function is soft πgb- continuous.

Remark 4.11: Converse of the above need not be true as seen in the following example.

Example 4.12: Let X=Y={a,b,c,d}, E={e_1,e_2}.Let F_1,F_2,….,F_6 are functions from E to P(X) and are defined as follows:

$F_1(e_1)=\{c\},F_1(e_2)=\{a\}$,

$F_2(e_1)=\{d\},F_2(e_2)=\{b\}$,

$F_3(e_1)=\{c,d\},F_3(e_2)=\{a,b\}$,

$F_4(e_1)=\{a,d\},F_4(e_2)=\{b,d\}$,

$F_5(e_1)=\{b,c,d\},F_5(e_2)=\{a,b,c\}$,

$F_6(e_1)=\{a,c,d\},F_6(e_2)=\{a,b,d\}$,

Then $\tau_1=\{\Phi,X,(F_1,E),.....(F_6,E)\}$ is a soft topology and elements in τ are soft open sets.

Let G_1,G_2 are functions from E to P(Y) and are defined as follows:

$G_1(e_1)=\{a\},G_1(e_2)=\{d\}$

$G_2(e_1)=\{b,c,d\},G_2(e_2)=\{a,b,c\}$

Then $\tau_2=\{\Phi,X,(G_1,E),(G_2,E),\}$ be a soft topology on Y.Let f:X⟶Y be an identity map. Here for every soft closed set in Y, the inverse image is soft πgb-closed set in X. Hence soft πgb-continuous. But for soft πgb-closed set (A,E)={{a,c,d},{a,b,d}}in Y , its inverse image is not soft πgb-closed set in X. Hence not soft πgb-irresolute.

Theorem 4.13 : Let f: (X, τ, E) → (Y, τ′, E) be soft πgb-continuous and g: (Y, τ′, E) → (Z, τ″, E) is soft continuous, then g∘f: (X, τ, E) ⟶ (Z, τ″, E) is soft πgb- continuous.

Proof: Let (A, E) be soft closed in (Z, τ″, E). Since g is soft continuous, g^{-1} ((A,E)) is soft closed in (Y, τ′, E) .Since f is soft πgb-continuous ,$f^{-1}(g^{-1}((A,E)))$ is soft πgb -closed in (X, τ, E). $(g\circ f)^{-1}((A,E))$ is soft πgb-closed in (X, τ, E) .Hence g∘f is soft πgb- continuous.

Theorem 4.14 : If f: (X, τ, E) → (Y, τ′, E) and g: (Y, τ′, E) → (Z, τ″, E) be soft functions. Then

(i) Let f be soft πgb-irresolute and g is soft πgb- continuous. Then g∘f is soft πgb- continuous.

(ii) If f be soft πgb-irresolute and g is soft πgb-irresolute, then g∘f is soft πgb- irresolute.

(iii)Let (Y, τ′, E) be a soft πgb-space. If f is soft πgb-continuous and g is soft πgb- continuous then g∘f is soft continuous.

Proof: Let (A, E) be a soft closed subset of (Z, τ″, E).Since g is soft πgb- continuous, then g^{-1}((A, E)) is soft πgb-closed subset of (Y, τ′, E).Since f is soft πgb-irresolute, then $(g\circ f)^{-1}((A, E))= f^{-1} (g^{-1}((A, E)))$ is soft πgb-closed subset of (X, τ, E) .Hence g∘f is soft πgb-continuous.

(ii) Let (A, E) be a soft πgb-closed subset of (Z, τ″, E).Since g is soft πgb- irresolute, then g^{-1}((A, E)) is soft πgb-closed subset of (Y, τ′, E) .Since f is soft πgb-irresolute, then $(g∘f)^{-1}$((A, E))= f^{-1} (g^{-1}((A, E))) is soft πgb -closed subset of (X, τ, E) .Hence g∘f is soft πgb- irresolute.

(iii) Let (A, E) be a soft closed subset of (Z, τ″, E).Since g is soft πgb-continuous, then g^{-1}((A, E)) is soft πgb-closed subset of (Y, τ′, E) .Since f is a soft πgb-space, then $(g∘f)^{-1}$((A, E))= f^{-1} (g^{-1}((A, E))) is soft closed subset of (X, τ, E) .Hence g∘f is soft continuous.

Theorem 4.15: Let (X, τ, E) be a soft πgb-space. If (X, τ, E) → (Y, τ′, E) is surjective, soft closed and soft πgb-irresolute, then (Y, τ′, E) is a soft πgb-space.

Proof: Let (A, E) be a soft πgb- closed subset of (Y, τ′, E) Since f is soft πgb- irresolute,f^{-1}((A, E)) is soft πgb-closed subset of (X, τ, E).Since (X, τ, E) is a soft πgb- space,f^{-1}((A, E)) is a soft closed subset of (X, τ, E).By hypothesis, it follows that (A, E) is soft closed subset of (Y, τ′, E).Hence (Y, τ′, E) is a soft πgb-space.

Theorem 4.16 : Let (X, τ, E) be a soft topological space, (Y, τ′, E) be a soft πgb-space and f: (X, τ, E) → (Y, τ′, E) be soft πgb-continuous ,then f is soft πgb-irresolute.

Proof: Let (A, E) be soft πgb-closed in (Y, τ′, E) Since (Y, τ′, E) is a soft πgb-space, (A, E) is a soft closed set in (Y, τ′, E).By hypothesis,f^{-1}((A, E)) is soft πgb-closed in (X, τ, E).Hence f is soft πgb-irresolute.

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