Near Meanness on Special Types of Graphs

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Abstract

Let G = (V, E) be a graph with p vertices and q edges and let f: V(G) → { 0, 1, 2, …., q-1, q+1} be an injection. The graph G is said to have a near mean labeling if for each edge, there exist an induced injective map f^**: E(G) → {1, 2, …., q} defined by*

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

The graph that admits a near mean labeling is called a near mean graph (NMG). In this paper, we proved that the graphs Special types of graphs Triangular snake, Quadrilateral snake, C_n^+, $S_{m,3}$, $S_{m,4}$ *and Parachutes are near mean graphs.*

Key Words : *Near mean labeling, Near mean graph. 2000 Mathematics Subject Classification 05C78.*

1. Introduction

By a graph, we mean a finite simple and undirected graph. The vertex set and edge set of a graph G denoted are by V(G) and E(G) respectively. A triangular snake is obtained from the path ($v_1v_2…v_n$) by replacing every edge by a triangle C_3. A quadrilateral snake Q_n is obtained from a path (u_1,u_2,u_3, …, u_n) by replacing every edge by a cycle C_4. Crown C_n^+ is a graph obtained from a cycle C_n of length n, in which each vertex v_i of the cycle is joined with a vertex u_i, $1 \le i \le n$. $S_{m,n}$ is a graph obtained by fusing one end vertex of n copies of path of length m-1. Parachute $P_{2,n-2}$ is a graph obtained from wheel W_n by removing n-2 consecutive spokes. Terms and notations not used here are as in [2].

2. Preliminaries

The mean labeling was introduced in [3]. Let G be a (p, q) graph and we define the concept of near mean labeling as follows.

Let f : V(G) → { 0, 1, 2, …., q-1, q+1} be an injection and also for each edge e = uv, it induces a map f^*: E(G) → {1, 2, …., q} defined by

$$f^*(uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

A graph that admits a near mean labeling is called a near mean graph. We have proved in [4], P_n, C_n, $K_{2,n}$ are near mean graphs and K_n (n> 4) and $K_{1,n}$ (n >4) are not near mean graphs. In [5], we proved that the graphs Book B_n, Ladder L_n, Grid $P_n \times P_n$, Prism $P_m \times C_3$ and $L_n \odot K_1$ are near mean graphs. In [6], we proved that *Join* of graphs, K_2 + mK_1, K_n^1 + $2K_2$, $S_m + K_1$ P_n+$2K_1$ and double fan are near mean graphs. In [7], we proved Family of trees, Bi-star, Sub-division Bi-star $P_m \odot 2K_1$, $P_m \odot 3K_1$, $P_m \odot K_{1,4}$ and $P_m \odot K_{1,3}$ are near mean graphs. In this paper, Special class of graphs Triangular snake, Quadrilateral snake, C_n^+, $S_{m,3}$, $S_{m,4}$, and Parachutes are proved as near mean graphs.

3. Near Meanness On Special Types Of Graphs

Theorem 3.1 : Triangular snake T_n is a near mean graph.

Proof:

Let $V(T_n) = \{ [u_i : 1 \le i \le n\text{-}1], [v_j : 1 \le j \le n] \}$

$E(T_n) = \{ [(u_iv_i) \cup (v_iv_{i+1}) : 1 \le i \le n\text{-}1] \cup [(u_iv_{i+1}) : 1 \le i \le n\text{-}1] \}$

Define $f : V(T_n) \rightarrow \{0, 1, 2, \ldots, q\text{-}1 = 3n\text{-}4, 3n\text{-}2\}$

by $f(u_i) = 3(i\text{-}1) \quad 1 \le i \le n\text{-}1$

$f(v_i) = 3i\text{-}2 \quad 1 \le i \le n$

The induced label of the edges are $f^*(u_iv_i) = 3i\text{-}2 \quad 1 \le i \le n\text{-}1$

$f^*(u_iv_{i+1}) = 3i\text{-}1 \quad 1 \le i \le n\text{-}1$

$f^*(v_iv_{i+1}) = 3i \quad 1 \le i \le n\text{-}1$

Hence, T_n is a near mean graph. For example the near mean labeling of T_4 is shown in figure 1.

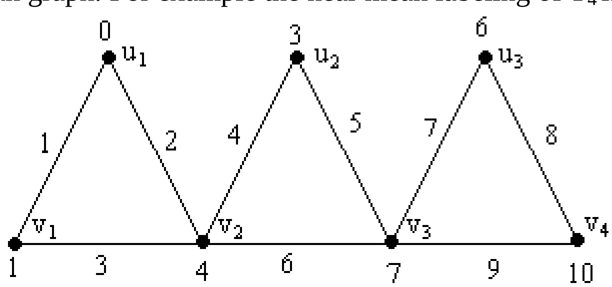

Figure 1

Theorem 3.2 : Quadrilateral snake Q_n is a near mean graph.

Proof: Let $V(Q_n) = \{ [u_i : 1 \le i \le n], [v_i, w_i : 1 \le i \le n\text{-}1] \}$

$E(Q_n) = \{[(u_iv_i) \cup (u_iw_i) : 1 \le i \le n\text{-}1] \cup [(u_{i+1}v_i) \cup (u_{i+1}w_i) : 1 \le i \le n\text{-}1]\}$

Define $f : V(Q_n) \rightarrow \{0, 1, 2, \ldots, 4n\text{-}5, 4n\text{-}3\}$ by

$f(u_1) = 0$

$f(u_i) = 4i - 5 \quad 2 \le i \le n$
$f(v_i) = 4i - 2 \quad 1 \le i \le n - 1$
$f(w_i) = 4i \quad 1 \le i \le n - 2$
$f(w_{n-1}) = 4n - 3$

The induced edge labeling are

$f^*(u_iv_i) = 4i - 3 \quad 1 \le i \le n - 1$
$f^*(u_{i+1}v_i) = 4i - 1 \quad 1 \le i \le n - 1$
$f^*(u_iw_i) = 4i - 2 \quad 1 \le i \le n - 1$
$f^*(u_{i+1}w_i) = 4i \quad 1 \le i \le n - 1$

Hence, Q_n is a near mean graph. For example the near mean labeling of Q_5 is shown in figure 2.

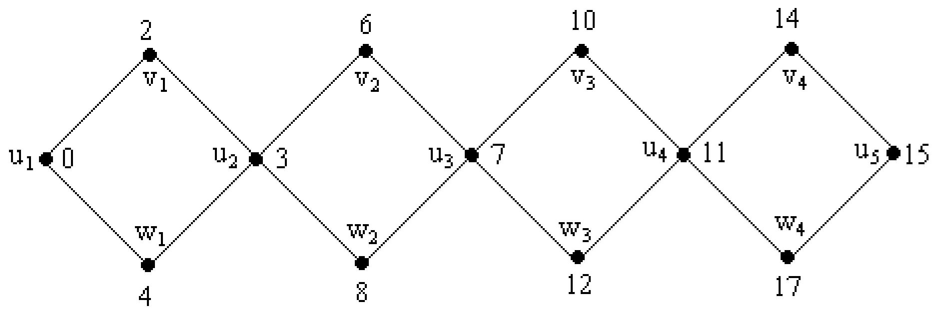

Figure 2

Theorem 3.3 : Crown C_n^+ is a near mean graph

Proof: Let $V(C_n^+) = \{ u_i, v_i : 1 \le i \le n \}$

$E(C_n^+) = \{ [(v_iv_{i+1}) : 1 \le i \le n\text{-}1] \cup (v_1v_n) \cup [(u_iv_i) : 1 \le i \le n] \}$

Case i : Let n be odd, n = 2m+1

Define $f : V(C_n^+) \to \{0, 1, 2, \ldots, 2n\text{-}1, 2n+1\}$ by

$f(v_1) = 2$
$f(v_i) = 2i - 1 \quad 2 \le i \le m+1$
$f(v_i) = 2i \quad m+2 \le i < n$
$f(v_n) = 2n - 1$
$f(u_1) = 0$
$f(u_2) = 1$
$f(u_i) = f(v_i) - 1 \quad 3 \le i < n$
$f(u_n) = 2n + 1$

The induced edge labelings are

$f^*(v_1v_2) = 3$
$f^*(v_iv_{i+1}) = 2i \quad 2 \le i \le m$
$f^*(v_iv_{i+1}) = 2i+1 \quad m+1 \le i \le 2m$
$f^*(v_1v_n) = n+1$
$f^*(u_iv_i) = i \quad 1 \le i \le 2$

$f^*(u_iv_i) = f(v_i) \qquad 3 \le i \le n-1$

$f^*(u_nv_n) = f(v_n) + 1$

Case ii : Let n be even.

When $n = 4$, the near mean labeling of C_4^+ is in figure 3.1

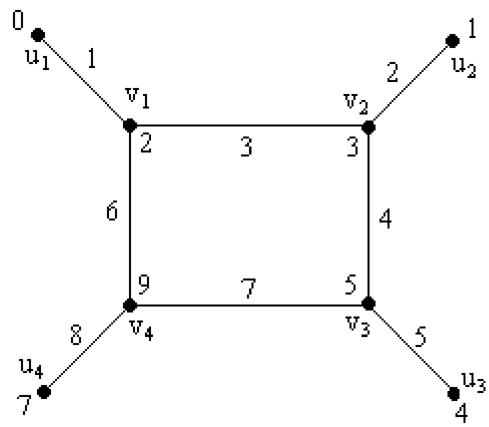

Figure 3.1

When $n > 4$ and even, let $n = 2m$, $m \ge 3$

Define $f : V(C_n^+) \to \{0, 1, 2, \ldots, 2n-1, 2n+1\}$ by

$f(v_1) = 2$

$f(v_i) = 2i - 1 \qquad 2 \le i \le m+1$

$f(v_i) = 2i \qquad m+2 \le i < n$

$f(v_n) = 2n - 1$

$f(u_1) = 0$

$f(u_2) = 1$

$f(u_i) = f(v_i) - 1 \qquad 3 \le i < n, \text{ if } i \ne \frac{n+2}{2}$

$f(u_i) = f(v_i) + 1 \qquad \text{if } i = \frac{n+2}{2}$

$f(u_n) = 2n + 1$

The induced edge labelings are

$f^*(v_1v_2) = 3$

$f^*(v_iv_{i+1}) = 2i \qquad 2 \le i \le m$

$f^*(v_iv_{i+1}) = 2i+1 \qquad m+1 \le i < n$

$f^*(v_1v_n) = n+1$

$f^*(u_iv_i) = f(v_i) - 1 \qquad 1 \le i \le 2$

$f^*(u_iv_i) = f(v_i) \qquad 3 \le i \le n-1, i \ne \frac{n+2}{2}$

$f^*(u_iv_i) = f(u_i)$ if $i = \frac{n+2}{2}$

$f^*(u_nv_n) = 2n$

Hence, C_n^+ is a near mean graph. For example the near mean labeling of (C_7^+ : n - odd) and (C_6^+ : n - even) are shown in figure 3.2 and 3.3 respectively.

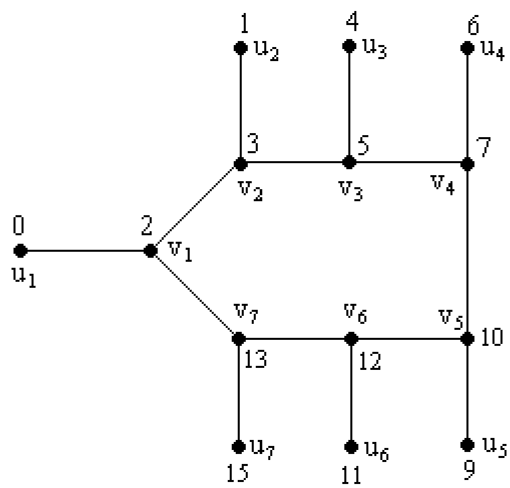

Figure 3.2

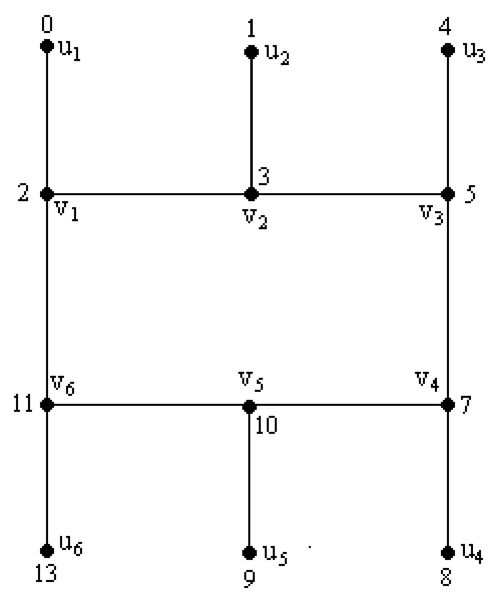

Figure 3.3

Theorem 3.4 : The graph $S_{m,3}$ is a near mean graph

Proof: Let $V(S_{m,3}) = \{ u_i, v_i, w_i : 1 \le i \le m \}$

$E(S_{m,3}) = \{ [(u_iu_{i+1}) \cup (v_iv_{i+1}) \cup (w_iw_{i+1}) : 1 \le i \le m\text{-}1] \}$

Define $f : V(S_{m,3}) \rightarrow \{0, 1, 2, \ldots, 3m–4, 3m –2\}$ by

$f(u_i) = i - 1 \qquad 1 \le i < m$

$f(v_i) = 2(m-1) - (i - 1) \qquad 1 \le i < m$

$f(w_i) = 2(m-1) + i \qquad 1 \le i < m - 2$

$f(u_m) = f(v_m) = f(w_m) = m - 1$

$f(w_{m-1}) = 3m - 2$

The induced edge labeling of $f^*(u_iu_{i+1}) = i \qquad 1 \le i < m$

$f^*(v_iv_{i+1}) = 2(m-1) - (i-1) \qquad 1 \le i < m$

$f^*(w_iw_{i+1}) = 2(m-1) + i + 1 \qquad 1 \le i < m - 1$

$f^*(w_{m-1}w_m) = 2m-1$

Hence, $S_{m,3}$ is a near mean graph. For example the near mean labeling of $S_{5,4}$ is shown in figure 4.

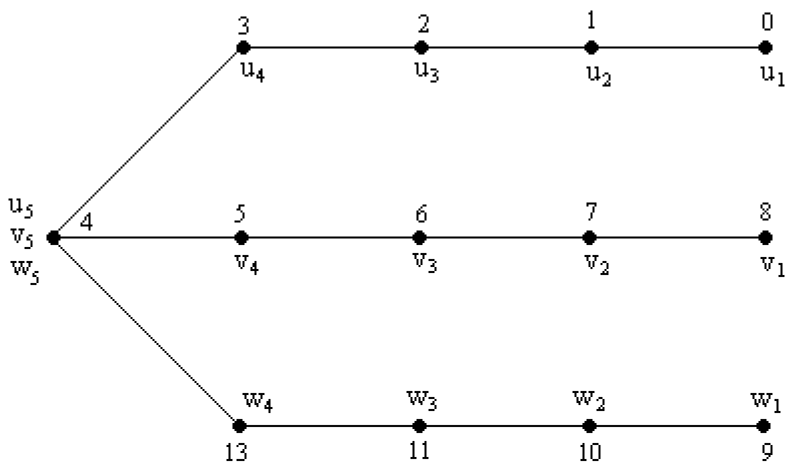

Figure 4

Theorem 3.5 : The graph $S_{m,4}$ is a near mean graph

Proof:

Let $V(S_{m,4}) = \{ u_{ij} : 1 \le i \le 4, 1 \le j \le m\}$

$E(S_{m,4}) = \{ (u_{ij}u_{i\,j+1}) : 1 \le i \le 4, 1 \le j \le m - 1 \}$

Let $V = u_{im}$, $1 \le i \le 4$

Define $f : V(S_{m,4}) \rightarrow \{0, 1, 2, \ldots, 4m-5, 4m-3\}$ by

$f(v) = 2(m-1)$

$f(u_{1j}) = m - 1 - j \qquad 1 \le j < m$

$f(u_{2j}) = m + j - 2 \qquad 1 \le j < m$

$f(u_{3j}) = 3m - 2 - j \qquad 1 \le j < m$

$f(u_{4j}) = 3(m-1) + j \qquad 1 \le j < m - 1$

$f(u_{4m}) = 4m - 3$

The induced edge labeling is

$f^*(vu_{im-1}) = i\,(m-1) \qquad i = 1, 2$

$f^*(vu_{im-1}) = (i-1)m + 1 \qquad i = 3, 4$

$f^*(u_{1j}u_{1j+1}) = m-1-j \qquad 1 \le j \le m-2$

$f^*(u_{2j}u_{2j+1}) = m-1+j \quad 1 \le j \le m-2$
$f^*(u_{3j}u_{3j+1}) = 3(m-1)-j+1 \quad 1 \le j \le m-2$
$f^*(u_{4j}u_{4j+1}) = 3(m-1)+1+j \quad 1 \le j \le m-2$

It gives the edges labeling from {1, 2, …., q }

Hence, $S_{m,4}$ is a near mean graph. For example the near mean labeling of $S_{5,4}$ is shown in figure 5.

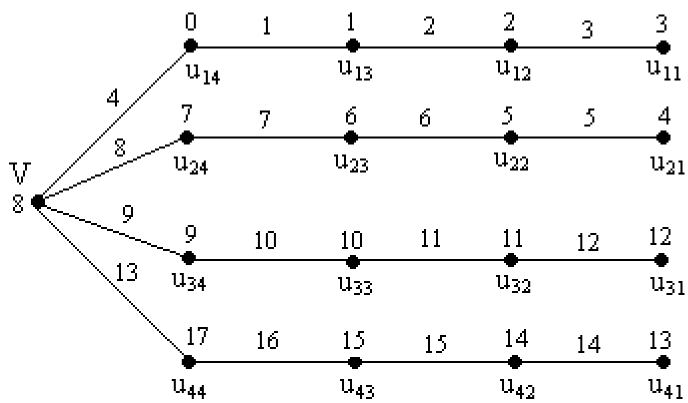

Figure 5

Theorem 3.6 : Parachute $P_{2, n-2}$ is a near mean graph.

Proof : Let $P_{2, n-2} = \{V, E\}$ such that

$V = \{ v, u_i : 1 \le i \le n \}$

$E = \{ [(u_iu_{i+1}) : 1 \le i \le n-1] \cup (u_1u_n) \cup (vv_1) \cup (vu_n) \}$

We define $f : V \to \{0, 1, 2, \ldots, n+1, n+3\}$ by

$f(v) = 0$

$f(u_1) = 2$

$f(u_n) = 3$

$f(u_2) = n+3$

$$f(u_{i+2}) = n+2-i, \begin{cases} 1 \le i \le \frac{n-1}{2}, & \text{if } n \text{ is odd} \\ 1 \le i \le \frac{n}{2}-1, & \text{if } n \text{ is even} \end{cases}$$

$$f(u_{n-i}) = 3+i, \begin{cases} 0 \le i < \frac{n-3}{2}, & \text{if } n \text{ is odd} \\ 1 \le i < \frac{n}{2}-1, & \text{if } n \text{ is even} \end{cases}$$

The induced edge labelings are

$f^*(vu_1) = 1$

$f^*(vu_n) = 2$

$f^*(u_1u_n) = 3$

$$f^*(u_1u_2) = \begin{cases} \dfrac{n+5}{2}, & \text{if } n \text{ is odd} \\ \dfrac{n+6}{2}, & \text{if } n \text{ is even} \end{cases}$$

$$f^*(u_{i+1}u_{i+2}) = n+3-i, \begin{cases} 1 \le i \le \dfrac{n-1}{2}, & \text{if } n \text{ is odd} \\ 1 \le i \le \dfrac{n}{2}-1, & \text{if } n \text{ is even} \end{cases}$$

$$f^*(u_{n+1-i}u_{n-i}) = 3+i, \begin{cases} 1 \le i \le \dfrac{n-3}{2}, & \text{if } n \text{ is odd} \\ 1 \le i \le \dfrac{n}{2}-1, & \text{if } n \text{ is even} \end{cases}$$

It is clear that edges get distinguished label from {1, 2, … , q}
Hence, the parachute is a near mean graph. For example the near mean labeling of ($P_{2,5}$: n - odd) and ($P_{2,6}$: n - even) are shown in figure 6.1 and 6.2 respectively.

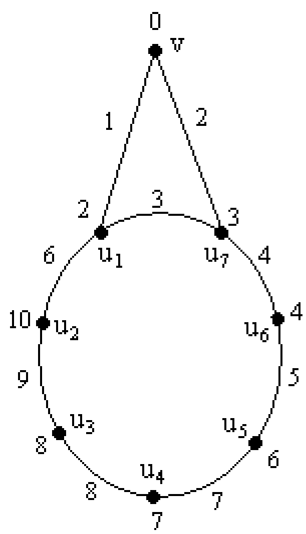

Figure 6.1

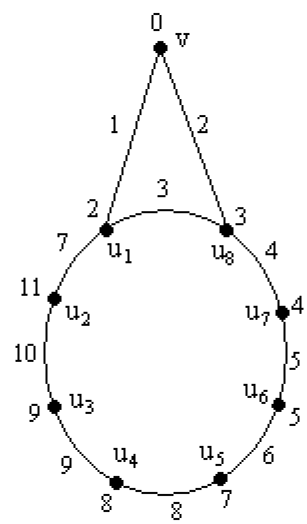

Figure 6.2

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