TIME- COST TRADE- OFF PROBLEM USING TYPE-2 TRAPEZOIDAL FUZZY NUMBERS

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Abstract : In this paper, we proposed to find the optimal duration of fully fuzzy time- cost trade -off problem using type-2 Trapezoidal fuzzy number. An illustration is provided to demonstrate the efficiency of the proposed method.

Key words: Type-2 Trapezoidal fuzzy number, Fuzzy project network, Critical path, Crashing.

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1. INTRODUCTION:

Time -cost trade- off problem is one of the main aspects of project scheduling the method of solving these kinds of problems require a scheduling with more stability against environmental variations. The process of reducing the original project duration is called crashing PERT/CPM networks. In many studies aiming at accomplishing a desired deadline with the lowest amount of cost is one of the most important and useful tools for project managers. Since there is a need to allocate extra resources in PERT/CPM crashing networks and the project managers are intendcd to spend the lowest possible amount of money to achieve the maximum crashing time, in which both direct and indirect cost will be influenced in the project, therefore in some researchers the term ‘ time-cost trade -off is also used for this purpose.

Project management is a very important field employed for scheduling activities and monitoring the progress, in competitive and fluctuating environments. The feasible duration time required to perform a specific project is determined using critical path method. However, because of competitive priorities, time is important and the completion time of a project determined using critical path method should be reduced to meet a deadline requested. In this situation, project crashing problem arises. Project crashing analysis is concerned with shortening the project duration time by accelerating some of its activities at an additional cost. In general, the parameters of the problem are accepted as certain and the project crashing problems are solved using deterministic solution techniques. In reality, because of uncertain environment conditions, incomplete or unobtainable information, there can be ambiguity in the parameters of the problem. The uncertainty in the parameters can be modeled via fuzzy set theory. Using fuzzy models gives the chance of better project management decisions with more stability under uncertain environmental factors.

The concept of a type-2 fuzzy set, which is an extension of the concept of an arbitrary fuzzy set, was introduced by Zadeh[11]. A type-2 fuzzy set is characterized by a membership function (ie) the membership value for each element of this set is a fuzzy set in [0,1], unlike an ordinary fuzzy set where the membership value is a crisp number in [0,1]. Hisdal[2] discussed the IF THEN ELSE statement and interval valued fuzzy sets of higher type. Jhon [3] studied an appraisal of theory and application on type-2 fuzzy sets. Stephan Dinagar and Anbalagan [8] prevented a new ranking function and arithmetic operations on generalized type-2 trapezoidal fuzzy numbers. Younes Saeinia et.al.,[9] discussed time -cost trade -off in a fuzzy project when the needed resource for executing of project is consumable and constraints. Shakelia Sathish et.al., [6] proposed a new approach to fuzzy network crashing in a project network whose activity times are uncertain. Feng-Tse Lin [1] introduces a fuzzy time cost trade off problem based on statistical confidence interval estimates. Shin-Pin Chen et.al., [7] proposed a novel approach for time-cost trade-off analysis of a project network in fuzzy environments.

The paper is organized as follows. In section-2, Some basic concepts are given. In section-3, Arithmetic operations on Type-2 Trapezoidal fuzzy numbers and algorithm are discussed. In section-4, to illustrate the proposed time –cost trade-off problem a numerical example is solved.

2.PRELIMINARIES:

2.1 Fuzzy Set: A fuzzy set is characterized by a membership function mapping the elements of domain, space or universe of discourse X to the unit interval [0,1].

A fuzzy set $\tilde{A}$ is set of ordered pairs $\{x, \mu_A(x)/x \in R\}$ where $\mu_A(x) : R \to [0,1]$ is upper semi continuous. Function $\mu_A(x)$ is called membership function of the fuzzy set.

2.2. Fuzzy Number: A fuzzy number f in the real line R is a fuzzy set f : $R \to [0,1]$ that satisfies the following properties.

(i) f is piecewise continous

(ii) There exist an $x \in R$ such that f(x) = 1

(iii) f is convex (ie), if $x_1, x_2 \in R$, and $\alpha \in [0,1]$,then

$$f(\lambda x_1 + (1-\lambda)x_2) \geq f(x_1) \wedge f(x_2)$$

2.3 Type-2 Fuzzy Set: A Type-2 fuzzy set denoted $\tilde{A}$, is characterized by a Type-2 membership function $\mu_A(x,.u)$ where $x \in X$ and $u \in J_x \subseteq$ [0,1].

ie, $\tilde{A} = \{ ((x,u), \mu_A(x.u)) / \forall x \in X, \forall u \in J_x \subseteq [0,1] \}$ in which $0 \leq \mu_A(x.u) \leq 1$. $\tilde{A}$ can be expressed as $\tilde{A} = \int_{x \in X} \int_{u \in J} \mu_A(x.u)/(x,u)$ $J_x \subseteq [0,1]$, where $\int\int$ denotes union over all admissible x and u. For discrete universe of discourse $\int$ is replaced by $\sum$.

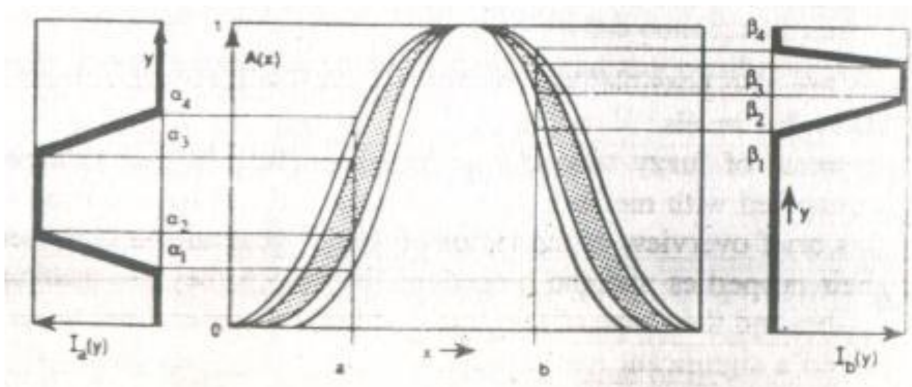


Illustration of the concept of a fuzzy set of type 2.

2.4 Interval Type-2 Fuzzy set: Interval type-2 fuzzy set is defined to be a T2FS where all its secondary grade are of unity for all $f_x(u) = 1$.

2.5Footprint Of Uncertainty: Uncertainty in the primary membership of a type-2 fuzzy set, $\tilde{A}$, consists of a bounded region that we call the footprint of uncertainty (FOU), It is the union of all primary membership.

ie,FOU($\tilde{A}$) = $\int_{x\varepsilon X} J_x$. The FOU can be described in terms of its upper and lower membership function. FOU ($\tilde{A}$) = $\int_{x\varepsilon X} [\mu_{\bar{a}}(x)^l, \mu_{\bar{A}}(x)^u]$.

2.6 Principle Membership Function: The principal membership function defined as the union of all the primary membership having secondary grades equal to 1.

ie. $P_r(\tilde{A}) = \int_{x\varepsilon X} u/x/f_x(u) = 1$.

2.7 Type-2 Fuzzy Number: Let $\tilde{A}$ be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied.

1. $\tilde{A}$ is normal,
2. $\tilde{A}$ is a convex set,
3. The support of $\tilde{A}$ is closed and bounded, then $\tilde{A}$ is called a type-2 fuzzy number.

2.8 Normal Type-2 Fuzzy Number: A T2FS , $\tilde{A}$ is said to be normal if its FOU is normal IT2FS and it has a primary membership function.

2.9 Type-2 Trapezoidal Fuzzy Number : Let a = (a_1 , a_2, a_3, a_4) be a trapezoidal fuzzy number. A normal type-2 trapezoidal fuzzy number $\tilde{A}$ ={ (x, $\mu_A^L(x)$, $\mu_A^M(x)$, $\mu_A^N(x)$, $\mu_A^U(x)$)}, xε R and $\mu_A^L(x) \leq \mu_A^M(x) \leq \mu_A^N(x) \leq \mu_A^U(x)$, for all x ε R.Denote $\tilde{A}$ =(A^L ,A^M, A^N, A^U), where $\tilde{A}$ = ((a_1^L, a_2^L, a_3^L, a_4^L),(a_1^M, a_2^M, a_3^M, a_4^M),(a_1^N, a_2^N, a_3^N, a_4^N) (a_1^U, a_2^U, a_3^U, a_4^U)).

3. Arithmetic Operations On Type-2 Trapezoidal Fuzzy Number:

3.1 Arithmetic operations:

Let $\tilde{A} = \bigcup_{foarall} \overline{\alpha} FOU(\overline{A_\alpha})$

$= (A^L , A^M, A^N, A^U)$

$= ((a_1^L, a_2^L, a_3^L, a_4^L),(a_1^M, a_2^M, a_3^M, a_4^M),(a_1^N, a_2^N, a_3^N, a_4^N), (a_1^U, a_2^U, a_3^U, a_4^U)).$

and Let $\tilde{B} = \bigcup_{foralll\,\alpha} \overline{\alpha} FOU(\overline{B_\alpha})$

$= (B^L , B^M, B^N, B^U)$

$=((b_1^L, b_2^L, b_3^L, b_4^L),(b_1^M, b_2^M, b_3^M, b_4^M),(b_1^N, b_2^N, b_3^N, b_4^N), (b_1^U, b_2^U, b_3^U, b_4^U)).$

be two type-2 trapezoidal fuzzy numbers, then we define

$\tilde{A} + \tilde{B} = [\bigcup_{forall\,\overline{\alpha}} \overline{\alpha} FOU(\overline{A_\alpha})] * [\bigcup_{forall\,\overline{\alpha}} \overline{\alpha} FOU(\overline{B_\alpha})]$ Where * = { +, -, *, / }.

$= \bigcup_{forall\,\overline{\overline{\alpha}}} \overline{\alpha}$ [FOU($\overline{A_\alpha}$) * FOU ($\overline{B_\alpha}$)]

$\widetilde{A} + \widetilde{B} = (A^L + B^L , A^M + B^M, A^N + B^N , A^U + B^U)$

Therefore,

(i)**Addition:**

$\tilde{A} + \tilde{B} = ((a_1^L + b_1^L , a_2^L + b_2^L , a_3^L + b_3^L , a_4^L + b_4^L), (a_1^M + b_1^M , a_2^M + b_2^M , a_3^M + b_3^M, a_4^M + b_4^M),$
$(a_1^N + b_1^N , a_2^N + b_2^N , a_3^N + b_3^N, a_4^N + b_4^N), (a_1^U + b_1^U , a_2^U + b_2^U , a_3^U + b_3^U, a_4^U + b_4^U)).$

(ii) Subtraction :

$\tilde{A} - \tilde{B} = ((a_1^L - b_4^L , a_2^L - b_3^L , a_3^L - b_2^L , a_4^L - b_1^L), (a_1^M - b_4^M , a_2^M - b_3^M , a_3^M - b_2^M, a_4^M - b_1^M),$
$(a_1^N - b_4^N , a_2^N - b_3^N , a_3^N - b_2^N, a_4^N - b_1^N), (a_1^U - b_4^U , a_2^U - b_3^U , a_3^U - b_2^U, a_4^U - b_1^U)).$

(iii) **Multiplication:**

$\tilde{A} * \tilde{B} = ((a_1^L * b_1^L , a_2^L * b_2^L , a_3^L * b_3^L , a_4^L * b_4^L), (a_1^M * b_1^M , a_2^M * b_2^M , a_3^M * b_3^M, a_4^M * b_4^M),$
$(a_1^N * b_1^N , a_2^N * b_2^N , a_3^N * b_3^N, a_4^N * b_4^N), (a_1^U * b_1^U , a_2^U * b_2^U , a_3^U * b_3^U, a_4^U * b_4^U)).$

(iv) Division:

$\tilde{A} / \tilde{B} = ((a_1^L / b_4^L , a_2^L / b_3^L , a_3^L / b_2^L , a_4^L / b_1^L), (a_1^M / b_4^M , a_2^M / b_3^M , a_3^M / b_2^M, a_4^M / b_1^M),$
$(a_1^N / b_4^N , a_2^N / b_3^N , a_3^N / b_2^N, a_4^N / b_1^N), (a_1^U / b_4^U , a_2^U / b_3^U , a_3^U / b_2^U, a_4^U / b_1^U)).$

3.2 Ranking On Type-2 Trapezoidal Fuzzy Number:

Let F (R) be the set of all type-2 normal trapezoidal fuzzy numbers. On Convenient approach for solving numerical value problem is based on the concept of comparison of fuzzy numbers by use of ranking function. An effective approach for ordering the elements of F (R) is to define a linear ranking function $\bar{R}$: F (R) $\rightarrow$ R which maps each fuzzy number into R.

Suppose if $\tilde{A} = (A^L , A^M, A^N , A^U)$

$= ((a_1^L , a_2^L, a_3^L, a_4^L), (a_1^M, a_2^M, a_3^M, a_4^M), (a_1^N, a_2^N, a_3^N, a_4^N),(a_1^U, a_2^U, a_3^U, a_4^U))$

$R(\tilde{A}) = (a_1^L + a_2^L + a_3^L + a_4^L + a_1^M + a_2^M + a_3^M + a_4^M + a_1^N + a_2^N + a_3^N + a_4^N + a_1^U + a_2^U + a_3^U + a_4^U) / 16.$

Also we define orders on F(R) by

$R(\tilde{A}) \geq R(\tilde{\boldsymbol{B}})$ if and only if $\tilde{A} \geq \tilde{B}$

$R(\tilde{A}) \leq R(\tilde{\boldsymbol{B}})$ if and only if $\tilde{A} \leq \tilde{B}$

$R(\tilde{A}) = R(\tilde{\boldsymbol{B}})$ if and only if $\tilde{A} = \tilde{B}$

3.4 Algorithm : A new method is proposed to find the fuzzy optimal solution to the Time-Cost Trade-Off problems using by Type-2 Trapezoidal fuzzy numbers.

Step:1 Construct a network.

Step:2 Determine the critical path, by any normal time duration.

Step:3 Compute the normal duration of the project completion and normal total cost.

Step:4 Find the project cost by the formula

Project cost = Direct cost + (Indirect cost * Project duration).

Step: 5 Calculate the cost slope for the different activities by the formula

$$\text{Cost Slope} = \frac{Crash\ cost - Normal\ cost}{Normal\ time - Crash\ time}$$

Step:6 Rank the activities in the ascending order of cost slope.

Step:7 Determine the Crash time and Crash cost for each activity to compute the cost slope.

Step:8 Identify the activity with the minimum cost slope and crash that activity. Identify the new critical path and find the cost of the project by formula

Project Cost = (Project Direct cost + Crashing cost of crashed activity + Indirect cost * Project duration).

Step: 9 Crash all activities in the project simultaneously.

Step: 10 Draw the project network after crashing all activities.

Step: 11 Determine the critical path and noncritical paths. Also, identify the critical activities.

Step: 12 In the new critical path select the activity with the next minimum cost slope, and repeat this step until all the activities along the critical path are crashed upto desired time.

Step: 13 At this point all the activities are crashed and further crashing is not possible. The crashing of non-critical activities does not alter the project duration time and is of no use.

4. NUMERICAL ILLUSTRATION:

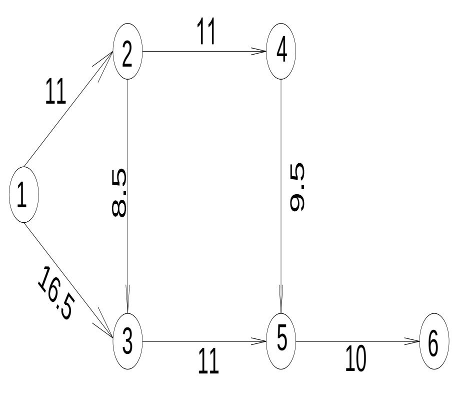
The normal time and crash time of an activity are Type-2 Trapezoidal fuzzy numbers and normal cost and crash cost also are type-2 trapezoidal fuzzy numbers are as shown in Table 1 and Table 2.

Table-1 : Details of the Project Duration

Activity Number	Activity	Normal Time (N_T)	Crash Time (C_T)
A	$1\rightarrow2$	**(8,10,12,14),(6,10,12,16), (4,8,14,18),(2,8,14,20)**	**(4,5,6,7),(3,5,6,8), (2,4,7,9),(1,4,7,10)**
B	$1\rightarrow3$	**(12,15,18,21),(9,15,18,24), (6,12,21,27)(3,12,21,30)**	**(8,10,12,14).(6,10,12,16), (4,8,14,18),(2,8,14,20)**
C	$2\rightarrow3$	**(7,8,9,10),(6,8,9,11), (5,7,10,12),(4,7,10,13)**	**(4,5,6,7),(3,5,6,8), (2,4,7,9),(1,4,7,10)**
D	$2\rightarrow4$	**(8,10,12,14),(6,10,12,16), (4,8,14,18),(2,8,14,20)**	**(4,5,6,7),(3,5,6,8), (2,4,7,9),(1,4,7,10)**
E	$3\rightarrow5$	**(8,10,12,14),(6,10,12,16), (4,8,14,18),(2,8,14,20)**	**(4,5,6,7),(3,5,6,8), (2,4,7,9),(1,4,7,10)**
F	$4\rightarrow5$	**(8,9,10,11),(7,9,10,12), (6,8,11,13),(5,8,11,14)**	**(4,5,6,7),(3,5,6,8), (2,4,7,9),(1,4,7,10)**
G	$5\rightarrow6$	**(7,9,11,13),(5,9,11,15), (3,7,13,17),(1,7,13,19)**	**(5,6,7,8),(4,6,7,9), (3,5,8,10),(2,5,8,11)**

Table-2: Details of the project cost

Activity Number	Activity	Normal Cost (N_C)	Crash Cost (C_C)
A	$1\rightarrow 2$	**(1300,1400,1500,1600), (1200,1400,1500,1700), (1100,1300,1600,1800), (1000,1300,1600,1900)**	**(2700,2800,2900,3000), (2600,2800,2900,3100), (2500,2700,3000,3200), (2400,2700,3000,3300)**
B	$1\rightarrow 3$	**(1600,1700,1800,1900), (1500,1700,1800,2000), (1400,1600,1900,2100), (1300,1600,1900,2200)**	**(3600,3800,4000,4200), (3400,3800,4000,4400), (3200,3600,4200,4600), (3000,3600,4200,4800)**
C	$2\rightarrow 3$	**(1500,1600,1700,1800), (1400,1600,1700,1900), (1300,1500,1800,2000), (1200,1500,1800,2100)**	**(2600,2800,3000,3200), (2400,2800,3000,3400), (2200,2600,3200,3600), (2000,2600,3200,3800)**
D	$2\rightarrow 4$	**(1600,1800,2000,2200), (1400,1800,2000,2400), (1200,1600,2200,2600), (1000,1600,2200,2800)**	**(4000,5000,6000,7000), (3000,5000,6000,8000), (2000,4000,7000,9000), (1000,4000,7000,10000)**
E	$3\rightarrow 5$	**(1300,1400,1500,1600), (1200,1400,1500,1700), (1100,1300,1600,1800), (1000,1300,1600,1900)**	**(2500,3000,3500.4000), (2000,3000,3500,4500), (1500,2500,4000,5000), (1000,2500,4000,5500)**
F	$4\rightarrow 5$	**(1400,1500,1600,1700), (1300,1500,1600,1800), (1200,1400,1700,1900), (1100,1400,1700,2000)**	**(3200,3500,3800,4100), (2900,3500,3800,4400), (2600,3200,4100,4700), (2300,3200,4100,5000)**
G	$5\rightarrow 6$	**(1600,1800,2000,2200), (1400,1800,2000,2400), (1200,1600,2200,2600), (1000,1600,2200,2800)**	**(2500,3000,3500,4000), (2000,3000,3500,4500), (1500,2500,4000,5000), (1000,2500,4000,5500)**



Critical path = $1\rightarrow 2\rightarrow 4\rightarrow 5\rightarrow 6$ Project duration = 11 +11 + 9.5 + 10 = 41.5

Table-3: Details of the slope cost

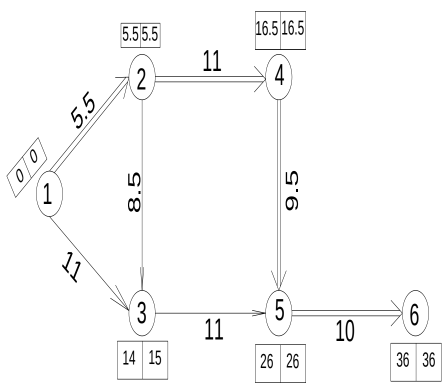
Activity Number	Activity	N_T	C_T	N_C	C_C	$\Delta_T = N_T - C_T$	$\Delta_C = C_C - N_C$	$\frac{\Delta_c}{\Delta_T}$
A	$1 \rightarrow 2$	**11**	**5.5**	**1450**	**2850**	**5.5**	**1400**	**255**
B	$1 \rightarrow 3$	**16.5**	**11**	**1750**	**3900**	**5.5**	**2150**	**391**
C	$2 \rightarrow 3$	**8.5**	**5.5**	**1650**	**2900**	**3**	**1250**	**417**
D	$2 \rightarrow 4$	**11**	**5.5**	**1900**	**5500**	**5.5**	**3600**	**655**
E	$3 \rightarrow 5$	**11**	**5.5**	**1450**	**3250**	**5.5**	**1800**	**327**
F	$4 \rightarrow 5$	**9.5**	**5.5**	**1550**	**3650**	**4**	**2100**	**525**
G	$5 \rightarrow 6$	**10**	**6.5**	**1900**	**3250**	**3.5**	**1350**	**386**
				11650				

Direct Cost = Rs. 11650

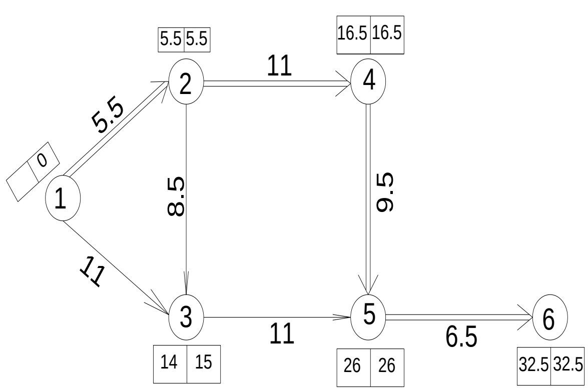
Indirect cost = Rs.0

Total Cost = Rs. 11650.

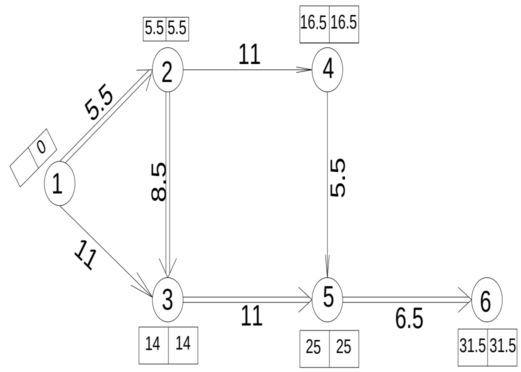
Step-1 :



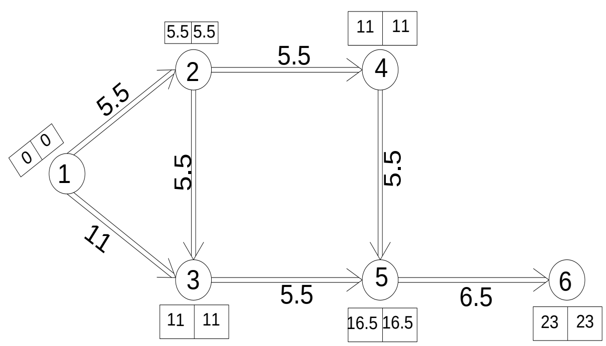
Step-2 :



Step-3:



Step-4:



Step Size	Critical Path	Project Duration	Total Cost
Step-1	$1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 6$	**36**	**12296**
Step-2	$1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 6$	**32.5**	**12682**
Step-3	$1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 6$	**31.5**	**13207**
Step-4	$1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 6$ $1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 6$	**23**	**14606**

5.CONCLUSION:

This paper investigated the Fuzzy Time-cost Trade- Off problems using Type-2 Trapezoidal Fuzzy Numbers. We can consider this approach faster and easier to complete the project in shortest possible duration, which is very much useful in many industrial problems.

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