On Normal Fuzzy Soft Group

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Abstract

In this paper, we introduce the concept of normal fuzzy soft group. We also define the level subsets of a normal fuzzy soft subgroup and discussed some of its properties.

Keywords

Fuzzy group, Fuzzy Soft Group, Fuzzy Normal, Normal Fuzzy Soft Group, Fuzzy Normalizer.

1. Introduction

The concept of fuzzy sets was initiated by Zadeh**[11].** Then it has become a vigorous area of research in engineering, medical science, social science, graph theory etc. Rosenfeld **[7]** gave the idea of fuzzy subgroups. In this paper we define a new algebraic structure of normal fuzzy soft subgroups and study some of their properties.

2. Preliminaries

In this section, the fundamental definitions that will be used in the sequel.

Definition 2.1

Let X be a group and μ be a soft set over X. then μ be a soft set over X. Then μ is said to be a soft group over X iff $F(a) < X$, $\forall a \in A$

Definition 2.2 [11]

Let X be any non empty set. A fuzzy subset μ of X is a function $\mu : X \rightarrow [0,1]$.

Definition 2.3 [7]

Let G be any sets. A mapping $\mu : G \rightarrow [0, 1]$ is called a fuzzy set in G.

Definition 2.4

Let G be a group. A fuzzy subgroup μ of G is said to be normal if for all $x,y \in G$, $\mu(xyx^{-1}) = \mu(y)$ or $\mu(xy) \geq \mu(yx)$.

3. Normal Fuzzy soft Subgroups

In this section, we define normal fuzzy soft groups and study some of their basic properties.

Definition 3.1

A fuzzy set μ is called a fuzzy soft subgroup of a group G, if for x, y $\in$ G,

$$\text{(i) } \mu(xy) \geq T\{\mu(x), \mu(y)\}$$

$$\text{(ii) } \mu(x^{-1}) \geq \mu(x)$$

Definition 3.2

Let G be a group. A fuzzy soft subgroup μ of G is said to be normal fuzzy soft subgroup ,if for all x,y $\in$ G and

$$\mu(xyx^{-1}) = \mu(y) \text{ or } \mu(xy) \geq \mu(yx)$$

Theorem 3.1:

Let μ and λ be two fuzzy soft subgroups of G. Then $\mu\cap\lambda$ is a fuzzy soft subgroup of G.

Proof :

Let μ and λ be two fuzzy soft subgroups of G

$$\begin{aligned}\text{(i) } (\mu\cap\lambda)(xy^{-1}) &= T\{\mu(xy^{-1}), \lambda(xy^{-1})\} \\ &\geq T\{T\{\mu(x), \mu(y^{-1})\}, T\{\lambda(x), \lambda(y^{-1})\}\} \\ &\geq T\{T\{\mu(x), \lambda(x)\}, T\{\mu(y^{-1}), \lambda(y^{-1})\}\} \\ &= T\{(\mu\cap\lambda)(x), (\mu\cap\lambda)(y^{-1})\}\end{aligned}$$

Thus $(\mu\cap\lambda)(xy^{-1}) \geq T\{(\mu\cap\lambda)(x),(\mu\cap\lambda)(y^{-1})\}$

$$\begin{aligned}\text{(ii) } (\mu\cap\lambda)(x) &= \{\mu(x), \lambda(x)\} \\ &= \{\mu(x^{-1}), \lambda(x^{-1})\} \\ &= \{(\mu\cap\lambda)(x^{-1})\}.\end{aligned}$$

Hence $\mu\cap\lambda$ is a fuzzy soft subgroup of G.

Theorem 3.2

The intersection of any two normal fuzzy soft subgroups of G is also a normal fuzzy soft subgroup of G.

Proof :

Let μ and λ be two normal fuzzy soft subgroups of G.

According to theorem 3.1, $\mu\cap\lambda$ is a fuzzy soft subgroup of G.

Now for all x , y in G , we have

$$(\mu\cap\lambda)(yxy^{-1}) = T(\mu(yxy^{-1}), \lambda(yxy^{-1}))$$
$$= T(\mu(x), \lambda(x))$$
$$=(\mu\cap\lambda)(x)$$

Hence $\mu\cap\lambda$ is a normal fuzzy soft subgroup of G.

Remark :

If μ_i, $i\in\Delta$ are normal fuzzy soft subgroup of G, then $\cap_{i\in\Delta} \mu_i$ is a normal fuzzy soft subgroup of G.

Theorem 3.3

Let μ is a normal fuzzy soft subgroup of G, then for any $y \in G$, we have

$$\mu(y^{-1} x y) = \mu(y x y^{-1})$$

Proof:

Let μ is a normal fuzzy soft subgroup of G, then for any $y \in G$

Now

$$\mu(y^{-1} x y) = \mu(x y^{-1} y)$$
$$= \mu(x)$$
$$= \mu(y y^{-1} x)$$
$$= \mu(y x y^{-1})$$

Hence the theorem.

Theorem 3.4

If μ is a normal fuzzy soft subgroup of G, then $g\mu g^{-1}$ is also a normal fuzzy soft subgroup of G , for all $g\in G$.

Proof :

Let μ be a normal fuzzy soft subgroup of G. then $g\mu g^{-1}$ is a subgroup of G.

Now

$$g\mu g^{-1}(yxy^{-1}) = \mu(g^{-1}(yx y^{-1})g)$$
$$= \mu(yx y^{-1})$$
$$= \mu(x)$$
$$= \mu(g x g^{-1})$$
$$= g \mu g^{-1}(x).$$

Hence the theorem

Definition 3.3

Let μ be a fuzzy soft subgroup of a group G. For any $t \in [0,1]$, we define the level subset of μ is the set,

$$\mu^t = \{ x \in G / \mu(x) \geq t \}.$$

Theorem 3.5

Let G be a group and μ be a fuzzy subset of G. Then μ is a normal fuzzy soft subgroup of G iff the level subsets μ^t, $t \in [0,1]$, are subgroup of G.

Proof

Let μ be a normal fuzzy soft subgroup of G and the level subset

$$\mu^t = \{x \in G \,/\, \mu(x) \geq t, t \in [0, 1]\}.$$

Let $x, y \in \mu^t$. Then $\mu(x) \geq t$ & $\mu(y) \geq t$.

$$\text{Now } \mu(xy^{-1}) \geq T\{\mu(x), \mu(y^{-1})\}$$

$$= T\{\mu(x), \mu(y)\}$$

$$\geq T\{t, t\}$$

Therefore, $\mu(xy^{-1}) \geq t$

This implies $xy^{-1} \in \mu^t$.

Thus μ^t is a subgroup of G.

Conversely, let us assume that μ^t be a subgroup of G.

Let $x, y \in \mu^t$. Then $\mu(x) \geq t$ and $\mu(y) \geq t$.

$$\text{Also, } \mu(xy^{-1}) \geq t, \text{ since } xy^{-1} \in \mu^t$$

$$= T\{t, t\}$$

$$= T\{\mu(x), \mu(y)\}$$

That is, $\mu(xy^{-1}) \geq T\{\mu(x), \mu(y)\}$

Hence μ is a normal fuzzy soft subgroup of G.

Definition 3.4

Let G be a group and μ be a normal fuzzy soft subgroup of G.

Let $N(\mu) = \{ a \in G \,/\, \mu(axa^{-1}) = \mu(x), \text{ for all } x \in G \}$. Then $N(\mu)$ is called the fuzzy Normalizer of μ.

Theorem 3.6

Let G be a group and μ be a fuzzy subset of G. Then μ is a normal fuzzy soft subgroup of G iff the level subsets μ^t, $t \in [0,1]$, are normal subgroup of G.

Proof:

Let μ be a normal fuzzy soft subgroup of G and the level subsets μ^t, $t \in [0,1]$, is a subgroup of G.

Let $x \in G$ and $a \in \mu^t$, then $\mu(a) \geq t$.

Now, $\mu(xax^{-1}) = \mu(a) \geq t$,

Since μ is a normal fuzzy soft subgroup of G.

That is , $\mu\,(xax^{-1}) \geq t$.

Therefore, $xax^{-1} \in \mu^t$.

Hence μ^t is a normal subgroup of G.

Theorem 3.7

If 'μ' is a normal fuzzy soft subgroup of G, iff μ^C is an anti normal fuzzy soft subgroup of G.

Proof

Suppose μ is a normal fuzzy soft subgroup of G. Then for all x, y G .

$$\mu\,(xy) \geq T\,\{\mu\,(x), \mu\,(y)\}$$

$$\Leftrightarrow 1 - \mu^C\,(xy) \geq T\,\{ (1- \mu^C\,(x)),(1- \mu^C\,(y))\}$$

$$\Leftrightarrow \mu^C\,(xy) \leq 1- T\,\{ (1- \mu^C\,(x)),(1- \mu^C\,(y))\}$$

$$\Leftrightarrow \mu^C\,(xy) \leq \max\,\{ \mu^C\,(x), \mu^C\,(y)\}.$$

By defn 3.1 , $\mu\,(x) = \mu\,(x^{-1})$ for all x in G .

$$1- \mu^C\,(x) = 1- \mu^C\,(x^{-1}) .$$

Therefore, $\mu^C\,(x) = \mu^C\,(x^{-1})$.

Hence μ^C is an anti normal fuzzy soft subgroup of G.

μ is a normal fuzzy soft subgroup of G

iff $\mu\,(xyx^{-1}) = \mu\,(y)$.

iff $1- \mu\,(xyx^{-1}) = 1- \mu\,(y)$.

iff $\mu^c\,(xyx^{-1}) =\mu^c(y)$.

iff μ^c is an anti normal fuzzy soft subgroup of G.

Hence, μ is a normal fuzzy soft subgroup of G, iff μ^C is an anti normal fuzzy soft subgroup of G.

4. Conclusion

In this article we have discussed Normal fuzzy soft subgroups, fuzzy Normalizer . Interestingly, It has been observed that fuzzy concept adds an another dimension to the defined fuzzy normal subgroups. This concept can further be extended for new results.

References

[1] Akta.H, Çagman.N, *Soft sets and soft group, Information Science 177 (2007) 2726_2735.*

[2] Klir .G.J, Yuan.B, *Fuzzy sets and Fuzzy logic in Theory and Applications, Prentice-Hall Inc, New Jersey, 1995.*

[3] Kumbhojkar . H.V, *On normal fuzzy subgroups, preprint*.

[4] Maji P.K., Biswas .R, A.R. Roy, *Soft set theory, Computers & Mathematics with Applications 45 (2003) 555_562*.

[5] Molodsov. D, *Soft set theory_First results, Computers & Mathematics with Applications 37 (4/5) (1999) 19_31.*

[6] Mukherjee .N. P and Bhattacharya.P, *Fuzzy normal subgroups and fuzzy cosets, Inform. Sci.,* ***34****(1984), 225–239*

[7] Rosenfeld.A., *fuzzy groups, J. math. Anal. Appl. 35 (1971),512-517 ",Antarctica journal of mathematics.vol 5,2008.*

[8] Sarala .N and Suganya .B, Some *Properties of Fuzzy Soft Groups,International Organization of scientific research , vol 10(apr 2014), pp 36-40.*

[9] Vasntha Kandasamy.W.B., *Smarandache Fuzzy Algebra, American Research Press,2003*.

[10] Wangjin Liu, *Fuzzy invariant subgroups and fuzzy ideals, Fuzzy Sets and Systems,* ***8****(1982), 133–139.*

[11] Zadeh. L.A.*Fuzzy sets, Information and Control,8(1965),338-353*.