A Study of an Inflationary Inventory Model with Stock-Dependent Demand and Shortages

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***Abstract* - An inventory replenishment problem for a deteriorating item is considered over a finite time-horizon with shortages in all cycles and stock-dependent demand rate. The effects of inflation and time-value of money are also taken into account. The advantage of allowing shortages in all cycles is illustrated with an example. A sensitivity analysis of the optimal solution with respect to the parameters of the system is carried out. The results for a non-deteriorating item are also derived as a limiting case of the general model.**

***Keywords* - Inventory, Shortages in all cycles, Deterioration, Stock-dependent demand, Inflation, Time-discounting**

I. INTRODUCTION

During the last forty years, the economic situation of most of the countries has changed to such an extent due to large-scale inflation and consequent sharp decline in the purchasing power of money, that it is not possible to ignore the effects of inflation and time-value of money. Buzacott [7] was the first to develop an EOQ model taking inflation into consideration. Several other researchers like Misra ([17], [18]), Bierman and Thomas [5], Aggarwal [1], Chandra and Bahner [8], Sarker and Pan [25], etc., have considered various interesting situations like the time-value of money, different inflation rates for the internal and external costs, finite replenishment rate, shortage etc. in their models. The market demand rate was however assumed to be constant in their models. Dutta and Pal [10] investigated a finite time-horizon inventory model with a linearly time-dependent demand rate, allowing shortages and considering the effects of inflation and time-value of money. An EOQ model for deteriorating items incorporating the effects of inflation, time value of money, a linearly time-dependent demand rate and shortages was also developed by Bose et. al. [6].

Generally for a consumer – goods type of inventory especially in supermarkets or shopping malls, the demand rate may go up and down if the on-hand inventory level increases or decreases respectively. Large piles of goods displayed in the supermarkets and malls usually tempt the customer to buy more. Silver and Peterson [24], Gupta and Vrat [11], Baker and Urban [3], Mandal and Phaujder [13], Datta and Pal [9], Pal et. al. [21], etc. have developed economic order quantity models with inventory-level-dependent demand rate. However, these authors did not consider the inflationary effects in their models. Ray and Chaudhuri [22] were the first to develop an inventory model with stock-dependent demand rate incorporating the concepts of inflation and time - value of money. They developed this model without taking deterioration of items into account and allowing shortages in all cycles except the last one. Valliathal and Uthayakumar [30] have however discussed the effects of inflation and time value of money on an EOQ model for deteriorating items under stock-dependent demand and time- dependent partial backlogging. This inventory model is studied under the replenishment policy starting with no shortages.

Researchers are continuously modifying the inventory models so as to make them more practicable. Basu and Sinha [4] have taken into account the impact of inflation for an inventory model with time-dependent demand and deterioration, considering permissible delay in payments and partial backlogging. Kumar and Rajput [12] also developed a similar type of general inventory model with constant demand and studied the effects of inflation therein. Tripathi [29] dealt with economic ordering policies for perishable items with inflation dependent demand rate under permissible delay in payments. Shortages are not allowed in this model. Tripathi, Misra and Shukla [28] also developed a cash-flow oriented EOQ model under permissible delay in payments without shortages, incorporating inflationary effects. Mehta and Shah [15] however have studied the effect of inflation and time-discounting for an inventory model for deteriorating items with exponentially increasing demand and shortages, without considering the permissible delay in payments. Mishra et. al. [16] have investigated the influences of inflation and time-value of money on an inventory system with power demand of deteriorating items without allowing shortages. Misra et. al. [20] derived an optimal inventory replenishment policy with constant demand and no shortages for two- parameter Weibull deteriorating items with a permissible delay in payment under inflation. Tripathi [27] in his paper establishes an inventory model for non-deteriorating items with time-dependent demand rate under inflation when the supplier offers a permissible delay to the purchaser. In this paper also, shortages are not allowed. However, Misra et. al. [19] investigated the inventory system for perishable items with quadratic demand pattern and shortages, under the influence of inflation and time-value of money. In the recent interesting works of Mandal and De [14], Amutha and Chandrasekaran [2], Singh, Tripathi and Mishra [26], Ray [23], the inflationary effects can be taken into consideration for further realistic development of their models.

In the present study, the model of Ray and Chaudhuri [22] is reconsidered. This model has been modified by allowing shortages in all replenishment cycles. The effect of deterioration is also taken into account. On comparing the optimal solutions for the two models by considering the same numerical example as considered by Ray and Chaudhuri [22], we see that the system cost decreases considerably and the duration of no-shortage period increases, as a result of allowing shortages in all cycles. We also consider the limiting case of this model when there is no deterioration and compare the optimal result with that of the model developed by Ray and Chaudhuri [22].

II. THE MODEL

The inventory model is developed for a deteriorating item. The costs considered in this model are (i) replenishment cost per cycle, (ii) purchase cost, (iii) holding cost, (iv) shortage cost and (v) deterioration cost. These costs are influenced by the rate of inflation. We consider here two distinct inflation rates – the *internal* (company) inflation rate and the *external* (general economy) inflation rate. Generally, the replenishment cost increases at the *internal inflation rate* and the unit purchase cost at the *external inflation rate*. The holding cost which comprises of costs in the form of taxes, insurance and costs of storage, etc. increases with the *external inflation rate.* The storage cost may be however affected by both the inflation rates depending on whether the company has its own warehouse or has a rented warehouse. The shortage cost is also affected by both the rates. Lastly, the deterioration cost, which depends on the cost of purchasing, increases at the *external inflation rate*. The classification of costs in this manner may vary, but in the present model, we assume a clear-cut categorization of the costs and the costs are determined accordingly. Shortages are allowed in all the replenishment cycles. In this model, our purpose is to find out the optimal reorder and shortage points that minimize the total cost over the time-horizon $[0, H]$.

III. ASSUMPTIONS AND NOTATIONS

The model is developed with the following assumptions and notations:

(i) H is taken to be the fixed time-horizon.

(ii) The demand rate is assumed to be stock-dependent. If $R(i)$ be the rate of demand for the item when the on-hand inventory level is i, then

$$R(i) = \begin{cases} D, (j-1)T \le t \le (K+j-1)T; j = 1,2, \dots n-1 \\ \alpha i^{\beta}, \quad (K+j-1)T \le t \le jT;\ j = 1,2, \dots n \end{cases}$$

where $\alpha > 0$ and $0 < \beta < 1$ are scale and shape parameters respectively and $D(> 0)$ is a constant. β is (Pal et. al. [21]) the elasticity of the demand rate with respect to the inventory level. In other words, it is called the stock-elasticity of the demand and is thus equal to the ratio of the percentage change in the quantity demanded to the percentage change in the stock-level.

(iii) A is the internal replenishment cost per cycle.

(iv) The rate of replenishment is infinite, i.e., replenishment is instantaneous.

(v) Lead time is taken to be zero for the sake of simplicity.

(vi) The internal and external inflation rates are denoted by i_1 and i_2 respectively.

(vii) r is the discount rate representing the time-value of money.

(viii) C_{11} and C_{12} are respectively the internal and external holding costs per unit item per unit time at time $t = 0$; C_{21} and C_{22} are respectively the internal and external shortage costs per unit item per unit time .

(ix) p is the unit purchase cost of the item.

(x) θ $(0 < \theta < 1)$ is a constant fraction of the on-hand inventory that deteriorates per unit of time.

The time-horizon H is divided into n equal parts, each of length T, so that $T = \frac{H}{n}$.

The reorder times over the time-horizon H are $(K + j - 1)T$ $(j = 1,2, \dots, n)$. The initial and final inventories are both zero in each cycle within the planning horizon. We assume that the period for which there is no shortage in each interval $[jT, (j+1)T]$ is a fraction of the scheduling period and is equal to $(1 - K)T$ $(0 < K < 1)$. Shortages occur at times jT $(j = 0,1,2, \dots, n-1)$. In this inventory policy, shortages occur in every cycle for a period KT, $0 < K < 1$. A pictorial description of the inventory policy is given in Figure 1.

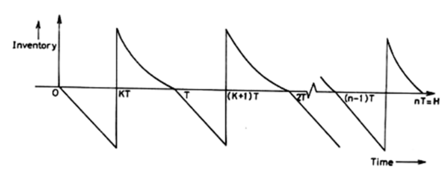

Fig. 1: Inventory Cycle

IV. THE PROBLEM AND THE SOLUTION PROCEDURE

Problem:

The instantaneous state of the on-hand inventory $i(t)$ in the present model is described by the following differential equations:

$$\frac{di}{dt} = -D, (j-1)T \le t \le (K+j-1)T,\ j = 1,2, \dots \dots, n \tag{1}$$

with the initial conditions $i\{(j-1)T\} = 0,\ j = 1,2, \dots, n$ (2)

and $\frac{di}{dt}+\theta i=-\alpha i^{\beta}$, $(K+j-1)T\le t\le jT$,
$j=1,2,\ldots\ldots,n.$ (3)

with the terminal conditions $i\{jT\}=0$, $j=1,2,\ldots,n.$ (4)

The solution of (1) using (2) is obtained as

$$i(t)=D\{(j-1)T-t\},\ (j-1)T\le t\le (K+j-1)T,\quad j=1,2,\ldots,n-1 \tag{5}$$

Again, the solution of (3) using (4) is

$$i(t)=\left[\frac{\alpha}{\theta}\{e^{\theta(1-\beta)(jT-t)}-1\}\right]^{\frac{1}{1-\beta}},\ (K+j-1)T\le t\le jT,\quad j=1,2,\ldots\ldots,n \tag{6}$$

The costs involved in the system are:

(i) Holding cost: The present worth of the total holding cost during the entire time horizon H is given by (see *Appendix I*)

$$C_H=\sum_{j=1}^{n}H_j=\sum_{j=1}^{n}\left(\sum_{m=1}^{2}I_{jm}\right) \tag{7}$$

(ii) Shortage cost: The total shortage cost during the entire time horizon is given by (see *Appendix II*)

$$C_S=\sum_{j=1}^{n}G_j=\sum_{j=1}^{n}\left(\sum_{m=1}^{2}J_{jm}\right) \tag{8}$$

(iii) Purchase cost: The present worth of the total purchase cost is (see *Appendix III*)

$$C_P=\sum_{j=1}^{n}P_n \tag{9}$$

(iv) Replenishment cost: The total replenishment cost is (see *Appendix IV*)

$$C_R=\frac{Ae^{-R_1KT}(1-e^{-R_1H})}{(1-e^{-R_1T})} \tag{10}$$

(v) Deterioration cost: The total cost of deterioration in the entire time horizon H is (see *Appendix V*)

$$C_D=p\sum_{j=1}^{n}D_j \tag{11}$$

Thus, the total cost of the inventory system over the entire time horizon H is given by

We will now determine the optimum values of n and K that minimize the total cost C of the inventory system.

Solution Procedure:

$$C(n,K)=C_H+C_S+C_P+C_R+C_D \tag{12}$$

We solve the non-linear equation (13) (using Newton-Raphson method) for $n=1,2,\ldots$ to get the corresponding values of K, when the other parameter values are prescribed. Again the corresponding values of C would be obtained in this process from (12). The minimum of these values of C would be the optimum value of C and the values of n and K obtained for this minimum C are then the optimum values of n and K

The cost function $C(n,K)$ is a function of two variables n (discrete) and K (continuous). This function $C(n,K)$ being complicated, it's not possible to prove its convexity analytically. However, using the parameter values of the numerical example, we see that $C(n,K)$ is a strict convex function of K (see Figure 2) for any given value of n. Therefore, the optimum value of K for C to be minimum is obtained by setting the derivative of C at zero i.e., $\frac{dC}{dK}=0$. By applying the rule of differentiation under the sign of integration, $\frac{dC}{dK}=0$ leads to the equation

$$\sum_{j=1}^{n}\sum_{m=1}^{2}TC_{1m}\left(\frac{\alpha}{\theta\beta}\right)^{\frac{1}{1-\beta}}\int_{l}^{jT}\{e^{\theta(1-\beta)(jT-t)}-1\}^{\frac{\beta}{1-\beta}}e^{-R_m t}dt-\sum_{j=1}^{n}\sum_{m=1}^{2}\frac{DC_{2m}T}{R_m}(1-e^{-R_mKT})$$
$$-\sum_{j=1}^{n}Tp\,e^{-R_2(K+j-1)T}\left[D-\left(\frac{\alpha}{\theta\beta}\right)^{\frac{1}{1-\beta}}\{e^{\theta(1-\beta)(jT-l)}-1\}^{\frac{\beta}{1-\beta}}\right]$$
$$+\sum_{j=1}^{n}p\,R_2Te^{-R_2(K+j-1)T}\left[DKT+\left(\frac{\alpha}{\theta\beta}\right)^{\frac{1}{1-\beta}}\int_{l}^{jT}\{e^{\theta(1-\beta)(jT-t)}-1\}^{\frac{\beta}{1-\beta}}dt\right]+$$
$$\frac{AR_1Te^{-R_1KT}(1-e^{-R_1H})}{(1-e^{-R_1T})}-Tp\left[\frac{\alpha}{\theta\beta}\{e^{\theta(1-\beta)(1-K)T}-1\}\right]^{\frac{1}{1-\beta}}=0, \tag{13}$$

where $l=(K+j-1)T$.

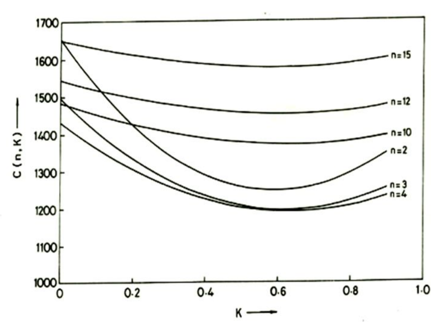

Fig. 2.

respectively. Simpson's one-third rule is applied while solving (13) in order to obtain the value of the integrals that appear in the equation.

A particular case:

The present inventory policy will reduce to a policy without any deterioration if the limiting value of the deteriorating fraction, θ becomes zero. Thus, by taking this limit $(\theta\to 0)$, the system cost in (12) reduces to (see *Appendix VI*)

$$C'(n,K) = \sum_{j=1}^{n}\sum_{m=1}^{2} C_{1m}\{\alpha(1-\beta)^{\beta}\}^{\frac{1}{1-\beta}} \int_{l}^{jT} (t-l)\,(jT - t)^{\frac{\beta}{1-\beta}} e^{-R_m t} dt + \sum_{j=1}^{n}\sum_{m=1}^{2} \frac{DC_{2m}}{R_m^2} e^{-R_m(j-1)T}\{e^{-R_m KT} + R_m KT - 1\} + \sum_{j=1}^{n} pe^{-R_2(K+j-1)T}\left[DKT + \{\alpha(1-\beta)(jT-l)\}^{\frac{1}{1-\beta}}\right] + \frac{Ae^{-R_1 KT}(1-e^{-R_1 H})}{(1-e^{-R_1 T})} \quad (14)$$

Thus, for a fixed value of n, the necessary condition for the cost C' to be minimum is obtained by taking $\theta \to 0$ in equation (13) (see *Appendix VI*)

$$\sum_{j=1}^{n}\sum_{m=1}^{2}\left[TC_{1m}\{\alpha(1-\beta)^{\beta}\}^{\frac{1}{1-\beta}}\int_{l}^{jT}(jT-t)^{\frac{\beta}{1-\beta}}e^{-R_m t}dt - \frac{DC_{2m}T}{R_m}(1-e^{-R_m KT})\right] - \sum_{j=1}^{n} Tp\, e^{-R_2(K+j-1)T}\left[D - \{\alpha(1-\beta)^{\beta}(jT-l)^{\beta}\}^{\frac{1}{1-\beta}}\right] + \sum_{j=1}^{n} p\, R_2 T e^{-R_2(K+j-1)T}\left[DKT + \{\alpha(1-\beta)(jT-l)\}^{\frac{1}{1-\beta}}\right] + \frac{AR_1 Te^{-R_1 KT}(1-e^{-R_1 H})}{(1-e^{-R_1 T})} = 0, \quad (15)$$

where $l = (K + j - 1)T$ and $R_m = r - i_m$, $m = 1,2$.

We solve this non-linear equation as before for $n = 1,2, \ldots \ldots$ to get the corresponding values of K for which the cost C' is minimum. A list of values of C' is obtained for different values of n from (14) and minimum cost in this list is the optimal cost when there is no deterioration.

V. NUMERICAL EXAMPLE

Case – I: Shortages in all cycles

Here we consider the same numerical example of Ray and Chaudhuri [22] so that a clear-cut comparison can be made. With the help of this numerical example, we will illustrate the advantage of allowing *shortages in all cycles*. The parameter values as considered by Ray and Chaudhuri [22] are as follows:

$A = 80$, $i_1 = 0.08$, $i_2 = 0.14$, $r = 0.2$, $C_{11} = 0.2$, $C_{12} = 0.4$, $C_{21} = 0.8$, $C_{22} = 0.6$, $\alpha = 20$, $\beta = 0.1$, $p = 5$, $H = 10$ *and* $D = 23.92$ *in appropriate units*.

In the present model, we have incorporated the factor of deterioration of goods over time; let us take $\theta = 0.01$.

Equation (13) is solved for K $(0 < K < 1)$ for different values of n and the corresponding values of the cost C are obtained from (12) by substituting these values of n and K. The results for the case of *shortages in all cycles* with the deterioration factor are shown in Table-I.

We see from this table that for $n = 4$, the system cost C is minimum. Thus, the optimum values of n and K are respectively $n^* = 4$ and $K^* = .5530820$ and the minimum value of cost C becomes $C^*(n^*, K^*) = 1197.8146$. Again $T^* = \frac{H}{n^*} = 2.50$.

Case – II: No Shortages

If no shortage in inventory is allowed in any cycle, the length of the stock-out interval $[(j-1)T, (K+j-1)T]$ should be zero. Thus, $KT = 0$. this implies that $K = 0$ for all cycles. Therefore, by putting $K = 0$ in (12), the model reduces to the case of no-shortage. Using the same parameter values as in Case-I and putting $K = 0$, we obtain the total cost from (12) for different values of n and then find the minimum cost. This is shown in Table II. We see that in the case of no-shortage $(K = 0)$, the optimal results are $n^* = 6$, $T^* = 1.667$ and $C^* = 1405.4302$.

TABLE I
OPTIMAL SOLUTION FOR A DETERIORATING ITEM WITH SHORTAGES IN ALL CYCLES

n	K	T	$C(n,K)$
2	.5257308	5.000	1254.3388
3	.5413294	3.333	1202.7837
4*	**.5530820***	**2.500***	**1197.8146***
5	.5615760	2.000	1211.9988
6	.5672284	1.667	1236.0625
7	.5705195	1.429	1265.9126
8	.5718800	1.250	1299.4748
9	.5716591	1.111	1335.5577
10	.5701382	1.000	1373.4378
11	.5675393	0.909	1412.6351
12	.5640440	0.833	1452.8412
13	.5597932	0.769	1493.8200
14	.5549052	0.714	1535.4181
15	.5494726	0.667	1577.5075

TABLE II
OPTIMAL SOLUTION FOR DETERIORATING ITEMS WITHOUT ANY SHORTAGES ($K = 0$)

n	T	$C(n, K)$
2	5.000	1652.0600
3	3.333	1492.7625
4	2.500	1430.2870
5	2.000	1407.6149
6*	**1.667***	**1405.4302***
7	1.429	1415.2606
8	1.250	1432.8569
9	1.111	1455.7800
10	1.000	1482.5527
11	0.909	1512.1889
12	0.833	1544.0589
13	0.769	1577.6684
14	0.714	1612.7012
15	0.667	1648.8846

The table shows that the total cost C becomes minimum for $n = 4$, $K = 0.5637721$.
Hence, the optimal values of n, K and C become respectivel $n^* = 4$, $K^* = 0.5637721$, $C^* = 1204.9451$.
Again, $T^* = \frac{H}{n^*} = 2.500$.
The corresponding optimal values obtained by Ray and Chaudhuri [22] are $n^* = 5$, $K^* = 0.365518$, $T^* = 2.000$, $C^* = 1278.523$, in case of *shortages in all cycles except the last one and for a non-deteriorating item.*
Now comparing the two results, we find that the system cost as obtained by Ray and Chaudhuri [22] has reduced by 5.76 %.

On comparing the results of the *shortage* and *no-shortage* case, we see that the system cost and the reorder number n increase whereas the scheduling period T decreases in the case of no shortage.

Particular case:
Now we will consider the particular case of our model when there is no physical deterioration of goods with time i.e. θ converges to zero $(\theta \to 0)$. The optimal solution in this case is obtained from (14) and (15) and is prescribed in Table III.
Thus, a considerable cost-savings is achieved by allowing shortages in all cycles.

TABLE III
OPTIMAL SOLUTION WITH SHORTAGES IN ALL CYCLES AND NO DETERIORATION [$\theta \to 0$]

n	K	T	$C(n, K)$
2	0.5404212	5.000	1262.0606
3	0.5533100	3.333	1210.1967
4*	**0.5637721***	**2.500***	**1204.9451***
5	0.5712970	2.000	1218.9032
6	0.5762013	1.667	1242.7828
7	0.5788907	1.429	1272.4958
8	0.5797691	1.250	1305.9526
9	0.5791687	1.111	1341.9620
10	0.5773283	1.000	1379.7863
11	0.5744778	0.909	1418.9549
12	0.5704700	0.833	1459.1419
13	0.5663487	0.769	1500.1223
14	0.5613226	0.714	1541.7317
15	0.5557745	0.667	1583.8471

Again on comparing the results of Table I and Table III, we see that the optimal system cost decreases by 0.59% on incorporating the deterioration factor.
Thus, deterioration has little effect on the system cost.

VI. SENSITIVITY ANALYSIS

We will now study the sensitivity of the optimal solution of our model (with shortages in all cycles and deterioration) to changes in the values of the different parameters associated with the system. The results using the same numerical example are listed in Table-IV.

TABLE IV
SENSITIVITY ANALYSIS

Para-meters	% change	n	K	T	$C(n,K)$	Para-meters	% change	n	K	T	$C(n,K)$
A	+50	3	.5575085	3.333	1268.8011	C_{22}	+50	4	.4889393	2.500	1223.7616
	+20	3	.5478240	3.333	1229.2471		+20	4	.5255921	2.500	1208.9179
	-20	4	.5447758	2.500	1162.0807		-20	4	.5834027	2.500	1185.5850
	-50	5	.5365731	2.000	1099.3004		-50	3	.6255476	3.333	1160.2549
i_1	+50	3	.5337797	3.333	1240.2283	θ	+50	4	.5475834	2.500	1198.1102
	+20	4	.5499034	2.500	1214.1149		+20	4	.5509014	2.500	1197.9078
	-20	4	.5559704	2.500	1182.9275		-20	4	.5552409	2.500	1197.7678
	-50	4	.5598488	2.500	1162.9449		-50	4	.5584599	2.500	1197.8435
i_2	+50	4	.4991489	2.500	1592.9476	α	+50	5	.5914344	2.000	1727.0230
	+20	4	.5365097	2.500	1335.7841		+20	4	.5645987	2.500	1409.8926
	-20	3	.5544415	3.333	1081.6519		-20	3	.5303698	3.333	981.4049
	-50	3	.5657104	3.333	934.3526		-50	3	.5156198	3.333	653.6931
r	+50	3	.5869061	3.333	781.5716	β	+50	4	.6210305	2.500	1298.8876
	+20	4	.5754718	2.500	1002.2714		+20	4	.5810555	2.500	1237.1151
	-20	4	.5198663	2.500	1445.2853		-20	4	.5244330	2.500	1160.1231
	-50	4	.4422919	2.500	1943.2798		-50	3	.4672716	2.500	1100.0447
C_{11}	+50	4	.5639450	2.500	1199.5840	p	+50	4	.6443865	2.500	1633.3269
	+20	4	.5575042	2.500	1198.5349		+20	4	.5922353	2.500	1373.2175
	-20	4	.5485521	2.500	1197.0766		-20	4	.5102616	2.500	1020.4208
	-50	4	.5415464	2.500	1195.9350		-50	3	.4274699	3.333	747.3073
C_{12}	+50	4	.5810203	2.500	1202.4370	H	+50	6	.5028996	1.667	1556.1704
	+20	4	.5647634	2.500	1199.7499		+20	5	.5340573	2.000	1355.8585
	-20	4	.5406451	2.500	1195.7494		-20	3	.5718835	3.333	1018.1159
	-50	4	.5302951	2.500	1192.3753		-50	2	.6028562	5.000	703.0042
C_{21}	+50	4	.4730717	2.500	1226.9779	D	+50	5	.1070023	2.000	1399.5156
	+20	4	.5182701	2.500	1210.3473		-20	4	.3769398	2.500	1311.2037
	-20	4	.5923334	2.500	1183.9476		-50	4	.7528545	2.500	1037.0362
	-50	3	.6507168	3.333	1154.1417			3	.9668432	3.333	712.4741

A careful study of Table IV reveals the following:

(i) It is seen that the reorder number n and the scheduling period T are both almost insensitive to changes in any of the parameters.

(ii) K is highly sensitive to changes in the values of p and D. K increases (decreases) with increase (decrease) of p. On the other hand, K decreases (increases) with increase (decrease) of D. K is moderately sensitive to changes in $r, i_2, C_{21}, C_{22}, \alpha, \beta$ and H; and is almost insensitive to changes in the parameters $\theta, i_1, C_{11}, C_{12}$ and A.

(iii) The system cost C^* is almost insensitive to changes in C_{11}, C_{12} and θ; it is moderately sensitive to changes in i_1, C_{21}, C_{22}, β and A whereas it is highly sensitive to changes in i_2, r, α, p, H and D.

VII. CONCLUSION

In the present paper, we have considered an inventory model for a deteriorating item with stock-dependent demand rate and shortages in all cycles. The effects of inflation and time-discounting of money have been incorporated into this model. The limiting case of this model when there is no deterioration is also studied. We have reconsidered the inventory model developed by Ray and Chaudhuri [22] for a non-deteriorating item with shortages in all cycles except the last one. On comparing the optimal result of this model with that of the limiting case of the present model (i.e. in the case of no deterioration), we see that the system cost reduces considerably. This proves that allowing shortages in all cycles is advantageous. The present model also shows that the system cost decreases due to physical decay or deterioration of goods over time. Apparently, the system cost should increase for a deteriorating item if shortages are not allowed.

APPENDIX - I

The holding cost over the period $[(j-1)T, jT]$, $j = 1,2,\ldots,n$ is given by $H_j = I_{j1} + I_{j2}$

where $I_{jm} = C_{1m}\int_{(K+j-1)T}^{jT}\{t-(K+j-1)T\}\,\alpha i^{\beta} e^{-R_m t}dt$

$$= C_{1m}\left(\frac{\alpha}{\theta\beta}\right)^{\frac{1}{(1-\beta)}}\int_{l}^{jT}\{t-l\}\{e^{\theta(1-\beta)(jT-t)}-1\}^{\frac{\beta}{1-\beta}}e^{-R_m t}dt$$

where $\theta \neq 0$, $l = (K+j-1)T$ and $R_m = r - i_m$ $(m = 1,2)$.

APPENDIX - II

The shortage cost during $[(j-1)T, (K+j-1)T]$, $j = 1,2,\ldots,n$ is given by $G_j = J_{j1} + J_{j2}$

where $J_{jm} = C_{2m}\int_{(j-1)T}^{(K+j-1)T}\{(K+j-1)T - t\}\,De^{-R_m t}dt$

$$= \frac{DC_{2m}}{R_m^2}e^{-R_m(j-1)T}[e^{-R_m KT} + R_m KT - 1],\ m = 1,2.$$

During shortage period, there are no items in the stock and therefore, the question of deterioration does not arise.

APPENDIX - III

The total cost of purchasing at time $t = (K+j-1)T$ for the periods $[(j-1)T, (K+j-1)T]$ and $[(K+j-1)T, jT]$, $j = 1,2,\ldots,n$ is given by

$$P_n = pe^{-R_2(K+j-1)T}\left[\int_{(j-1)T}^{(K+j-1)T} D\,dt + \int_{(K+j-1)T}^{jT}\alpha i^{\beta}\,dt\right]$$

$$= pe^{-R_2(K+j-1)T}\left[DKT + \left(\frac{\alpha}{\theta\beta}\right)^{\frac{1}{(1-\beta)}} + \int_{(K+j-1)T}^{jT}\{e^{\theta(1-\beta)(jT-t)} - 1\}^{\frac{\beta}{1-\beta}}dt\right]$$ where $\theta \neq 0$.

APPENDIX - IV

There are n replenishments in the entire time horizon H and hence the total replenishment cost (with /without deterioration) is given by

$$C_R = A\sum_{j=1}^{n}e^{(i_1-r)(K+j-1)T}$$

$$= Ae^{-R_1(K-1)T}\sum_{j=1}^{n}e^{-R_1 jT} \quad where\ R_1 = r - i_1.$$

On summation and simplification, we get

$$C_R = \frac{Ae^{-R_1KT}(1-e^{-R_1H})}{(1-e^{-R_1T})}.$$

APPENDIX - V

The number of items deteriorated during the j^{th} replenishment cycle is

$$D_j = \int_{(K+j-1)T}^{jT}\theta\, i(t)\,dt, \qquad j = 1,2,\ldots,n$$

$$= \int_{(K+j-1)T}^{jT}\left[\left(\frac{\alpha}{\theta\beta}\right)\{e^{\theta(1-\beta)(jT-t)} - 1\}\right]^{\frac{1}{1-\beta}}dt,$$

$$where\ \theta \neq 0.$$

APPENDIX - VI

$$\lim_{\theta\to 0} C(n,K) = \sum_{j=1}^{n}\sum_{m=1}^{2}C_{1m}\alpha^{\frac{1}{1-\beta}}\int_{l}^{jT}(t-l)\ e^{-R_m t}\left\{\lim_{\theta\to 0}\frac{e^{\theta(1-\beta)(jT-t)}-1}{\theta}\right\}^{\frac{\beta}{1-\beta}}dt$$
$$+\sum_{j=1}^{n}\sum_{m=1}^{2}\frac{DC_{2m}}{R_m^2}e^{-R_m(j-1)T}\{e^{-R_mKT} + R_mKT - 1\}$$
$$+\sum_{j=1}^{n}p\,e^{-R_2(K+j-1)T}\left[DKT + \alpha^{\frac{1}{1-\beta}}\int_{l}^{jT}\left\{\lim_{\theta\to 0}\frac{e^{\theta(1-\beta)(jT-t)}-1}{\theta}\right\}^{\frac{\beta}{1-\beta}}dt\right]$$
$$+\frac{Ae^{-R_1KT}(1-e^{-R_1H})}{(1-e^{-R_1T})}$$
$$=\sum_{j=1}^{n}\sum_{m=1}^{2}C_{1m}\{\alpha(1-\beta)^{\beta}\}^{\frac{1}{1-\beta}}\int_{l}^{jT}(t-l)e^{-R_mt}(jT-t)^{\frac{\beta}{1-\beta}}dt$$
$$+\sum_{j=1}^{n}\sum_{m=1}^{2}\frac{DC_{2m}}{R_m^2}e^{-R_m(j-1)T}\{e^{-R_mKT} + R_mKT - 1\}$$
$$+\sum_{j=1}^{n}p\,e^{-R_2(K+j-1)T}\left[DKT + \{\alpha(1-\beta)^{\beta}\}^{\frac{1}{1-\beta}}\int_{l}^{jT}(jT-t)^{\frac{\beta}{1-\beta}}dt\right]$$
$$+\frac{Ae^{-R_1KT}(1-e^{-R_1H})}{(1-e^{-R_1T})}$$

Here the system cost decreases even after incorporating the deterioration factor because shortages are allowed in all cycles. Lastly, the sensitivity of the optimal solution to changes in different parameter values has been discussed.

where, $\lim_{\theta\to 0}\frac{e^{\theta(1-\beta)(jT-t)}-1}{\theta}$

$$= \lim_{\theta\to 0}\frac{(1-\beta)(jT-t)e^{\theta(1-\beta)(jT-t)}}{1}$$
$$= (1-\beta)(jT-t)$$

[by applying $L'Hospital's\;\; Rule$]

Again taking the limit, $\theta \to 0$ on both sides of equation (13), we obtain the necessary condition for the system cost C to be minimum when there is no deterioration, as follows:

$$\sum_{j=1}^{n}\sum_{m=1}^{2} T\,C_{1m}\alpha^{\frac{1}{1-\beta}}\int_{l}^{jT}\left\{\lim_{\theta\to 0}\frac{e^{\theta(1-\beta)(jT-t)}-1}{\theta}\right\}^{\frac{\beta}{1-\beta}} e^{-R_m t}dt$$
$$-\lim_{\theta\to 0}\sum_{j=1}^{n}\sum_{m=1}^{2}\frac{DC_{2m}T}{R_m}(1-e^{-R_m KT})$$
$$-\sum_{j=1}^{n} Tpe^{-R_2(K+j-1)T}\left[\lim_{\theta\to 0} D - \alpha^{\frac{1}{1-\beta}}\left\{\lim_{\theta\to 0}\frac{e^{\theta(1-\beta)(jT-l)}-1}{\theta}\right\}^{\frac{\beta}{1-\beta}}\right]$$
$$+\sum_{j=1}^{n} pR_2\,Te^{-R_2(K+j-1)T}\left[\lim_{\theta\to 0} DKT + \alpha^{\frac{1}{1-\beta}}\int_{l}^{jT}\left\{\lim_{\theta\to 0}\frac{e^{\theta(1-\beta)(jT-t)}-1}{\theta}\right\}^{\frac{\beta}{1-\beta}}dt\right]$$
$$+\lim_{\theta\to 0}\frac{AR_1Te^{-R_1KT}(1-e^{-R_1H})}{(1-e^{-R_1T})}$$
$$-Tp\alpha^{\frac{1}{1-\beta}}\left[\lim_{\theta\to 0}\frac{\{e^{\theta(1-\beta)(1-K)T}-1\}\theta^{1-\beta}}{\theta}\right]^{\frac{1}{1-\beta}} = 0.$$

i.e., $\sum_{j=1}^{n}\sum_{m=1}^{2}\left[TC_{1m}\{\alpha(1-\beta)^{\beta}\}^{\frac{1}{1-\beta}}\int_{l}^{jT}(jT-t)^{\frac{\beta}{1-\beta}}e^{-R_m t}dt - \frac{DC_{2m}T}{R_m}(1-e^{-R_m KT})\right]$

$$-\sum_{j=1}^{n} Tpe^{-R_2(K+j-1)T}\left[D-\{\alpha(1-\beta)^{\beta}(jT-t)^{\beta}\}^{\frac{1}{1-\beta}}\right]$$
$$+\sum_{j=1}^{n} pR_2\,Te^{-R_2(K+j-1)T}\left[DKT + \{\alpha(1-\beta)(jT-l)\}^{\frac{1}{1-\beta}}\right]$$
$$+\frac{AR_1Te^{-R_1KT}(1-e^{-R_1H})}{(1-e^{-R_1T})} = 0$$

since, $\lim_{\theta\to 0}\frac{e^{\theta(1-\beta)(jT-t)}-1}{\theta} = (1-\beta)(jT-t)$ and the last term vanishes as $0 < \beta < 1$.

ACKNOWLEDGMENT

The author is very grateful to the editor(s) and would also like to thank the referees .

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