Analysis of Queuing Model for Machine Repairing System with Bernoulli Vacation Schedule

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***Abstract*— In this paper, we consider a queueing model for machine repairing system with Bernoulli vacation schedule. The failure times, repair times and vacation times are all assumed to be exponentially distributed. In congestion, the server may increase the repair rate with pressure coefficient β to reduce the queue length. We assume that the server begins the working vacation when the system is empty. The server may go for a vacation of random length with probability p or may continue to repair the next (if available) failed machine with probability $q = 1 - p$. The whole system is modelled as a finite state Markov Chain and its steady state distribution is obtained by matrix recursive approach.**

***Keywords*— Machine repair, Bernoulli vacation, pressure coefficient, Markov Chain, Matrix-recursive method.**

I. INTRODUCTION

The machining system have pervaded every field of our lives in different activities ensuring our almost total dependence on machines. As the time passes a machine become to prone failure. The failure of machines may result in loss of production, money, goodwill etc. If at any time a machine fails, it is sent to the repair facility for repair. In this system we consider a group of M operating machines and R repairmen (servers) in the repair facility. To avoid any loss of production the plant or company always keeps some standby machines so that a standby machine can immediately act as a substitute when an operating machine fails. To minimize the machine failure duration, we consider a situation where the repair rate is increasing when there are failed machines waiting for repair. The feature is modeled by a "service pressure coefficient β". This coefficient is a positive constant and indicates the degree to which the servers increase the repair rate in order to reduce the number of failed machines immediately.

In modern age, the failure and repairs are coupled events in a typical machining system. The important contribution in the areas of machine repair problem are due to Gross and Harris [4], Gupta, S.M. [5], Jain, M. [7] and Jain, M. [8]. An unreliable retrial queue with two phases of service and Bernoulli admission mechanisms was studied by Choudhary, G. and Daka, K. [2]. A two stage batch arrival queueing system with a modified Bernoulli schedule vacation under N-policy was analysed by Choudhary, G. [3]. Gross and Harris [4] studied fundamentals of queueing theory. Gupta, S.M. [5] developed machine interference problem with warm spaces server vacations and exhaustive service. Jain, M. [9] provided transient analysis of machine repairing system with service interruption mixed standby and priority. Jain, M., Sharma,G.C. and Singh, M.[10] discussed diffusion process for multi-repairman machining system with spares and balking. Jain, M., Sulekha Rani [11] introduced availability analysis of repairable system with warm standby switching failure and reboot delay. Jain, M., Chandra Shekhar and Shalini Shukla [12] discussed queueing analysis of a multi-component machining system having unreliable heterogeneous servers and impatient customers.

Ke, J.C., Hsu, Y.L., Liu, T.H. and Zhang, Z.G. [13] analyzed computational analysis of machine repair problem with unreliable multi-repairmen. Ke, J.C. Lee, S.I. and Liou, C.H. [14] developed machine repair problem in production systems with spares and server vacations. Ke J.C. Wu, C.H. and Pearn, W.L. [15] performed algorithm analysis of the multi-server system with modified Bernoulli schedule. Ke, J.C., Wu,C.H. and Pearn, W.L. [16] gave analysis of an infinite multi-server queue with an optional service.

In machining systems, queueing systems with vacations have many applications working in industrial such as manufacturing and production systems. When there is no failed unit-present in the system, what should a repairman do? For this, instead of remaining idle during this period, the repairmen may go for a vacation and can utilize this time to do some other work such as preventive maintenance, proper arrangement of tools etc. over last four decades, a substantial amount of work has been done to examine queuing systems with vacations. Keilson J., and Servi, L.D. [17] analyzed oscillating random walk models for GI/G/1 vacation systems with Bernoulli schedule. Khorram, E. [18] developed an optimal model by dynamic numbers of repairman infinite population queueing system. On the multi-server machine interference with modified Bernoulli vacations was studied by Liu, Hsin, T. and Ke J.C. [19]. A two server queue with Bernoulli schedule and single vacation policy was developed by Madan, K.C.,W. Abu Dayyah and Taiyyan, F. [20]. Maheshwari Supriya, Ali Sazia [21] studied machine repair problem with mixed spares, balking and reneging. Maheshwari Supriya et al. [22] developed Machine repair problem with K-type warm spares multiple vacations and reneging. Wang, K.H. and Wu, J.D. [23] developed cost analysis of the M/M/R machine repair problem with spares and two modes of failure.

Profit analysis of the M/ M/ /R/ machine repair problem with balking, reneging and standby switching failure was given by Wang, K.H., Ke, J. B. and Ke, J.C., [24]. Wang, K.H. Chen, W.L. and Yang, D.Y. [25] developed optimal management of the machine repair problem with working vacation, Newton's method. Analysis of multi-server queue with a single vacation (e, d) policy was studied by Xu, X and Zhang, Z.G., [26]. Modeling of multi-server repair problem with switching failure and reboot delay under related profit analysis was developed by Ying, Lin, Hsu, Leu J.C.and Tsu-Hsin, Liu and C.H. Wu. [27]. Yue, D. Yue, W., and Qi, H. [28] visualized performance analysis and optimization of a machine repair problem with warm spares and two heterogeneous repairmen.

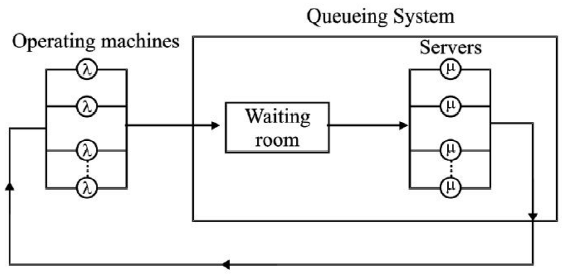

Figure-1. Machine repairmen problem.

II. Model Description

Consider L = M + S homogeneous machines, where M machines are operating S machines are available as standbys. There are R repairmen, The repair times are assumed to follow independent and identically exponential distribution with rate μ. Each of the operating machines fails according to a Poisson process with parameter λ (or α where $0 \le \alpha \le \lambda$) . When an active (or standby) machine fails, it is immediately replaced by any available standby and is repaired in the order of breakdowns by any available server. At each repair completion instant of a server, the server inspects the system state and decides whether to leave for a vacation of random length with probability p or continue to repair the next failed machine if any with probability q = 1 – p. When the system is empty, the server begins a working vacation, and the vacation duration follows an exponential distribution with mean duration 1/η. When a working vacation terminates and the system is empty, the server starts another working vacation. Repair times during a vacation period are according to exponential distribution with mean 1/μ.

Repair rate is defined as follows:

$$\mu_n = \begin{cases} n_\mu; & n = 0,1,2,\dots,R-1 \\ \left[\dfrac{n(R+1)}{R(n+1)}\right]^\beta R\mu; & \text{where, } \beta > 0,\ R \le n \le L. \end{cases}$$

where β is a pressure coefficient representing the degree to which the repair rate is effected by the number of failed machines in the system. We consider a situation in which servers are under work pressure. When numerous failed machines are waiting for repair services and few servers are available, the servers may perform better under reasonable pressure. In this paper we have studied model presented by Liu, Hsin, T. and Ke,J.C. [19].

III. 3. THE GOVERNING EQUATIONS

Consider a multi-server machine interference problem with modified Bernoulli vacation under a single vacation policy. In steady state, we have following probabilities.

$P_r(n)$: Prob. that there are n failed machines in the system when there are r vacationing servers, n = 0, 1, ..,L and r = 0, 1, 2,.., R.

The failure rate λ_n and repair rate μ_n^r are defined as follows:

$$\lambda_n = \begin{cases} M\lambda + (S-n)\alpha; & 0 \le n \le S \\ (L-n)\lambda; & S+1 \le n \le L \end{cases}$$

and

$$\mu_n^r = \begin{cases} n\mu; & n = 1, 2....R - r - 1 \\ \left[\dfrac{n(R-r+1)}{(R-r)(n+1)}\right]^{\beta}(R-r)\mu; & n = R-r, R-r+1, \end{cases}$$

The state transition rate diagram for a multi-server machine repair system with modified Bernoulli vacation schedule is given below:

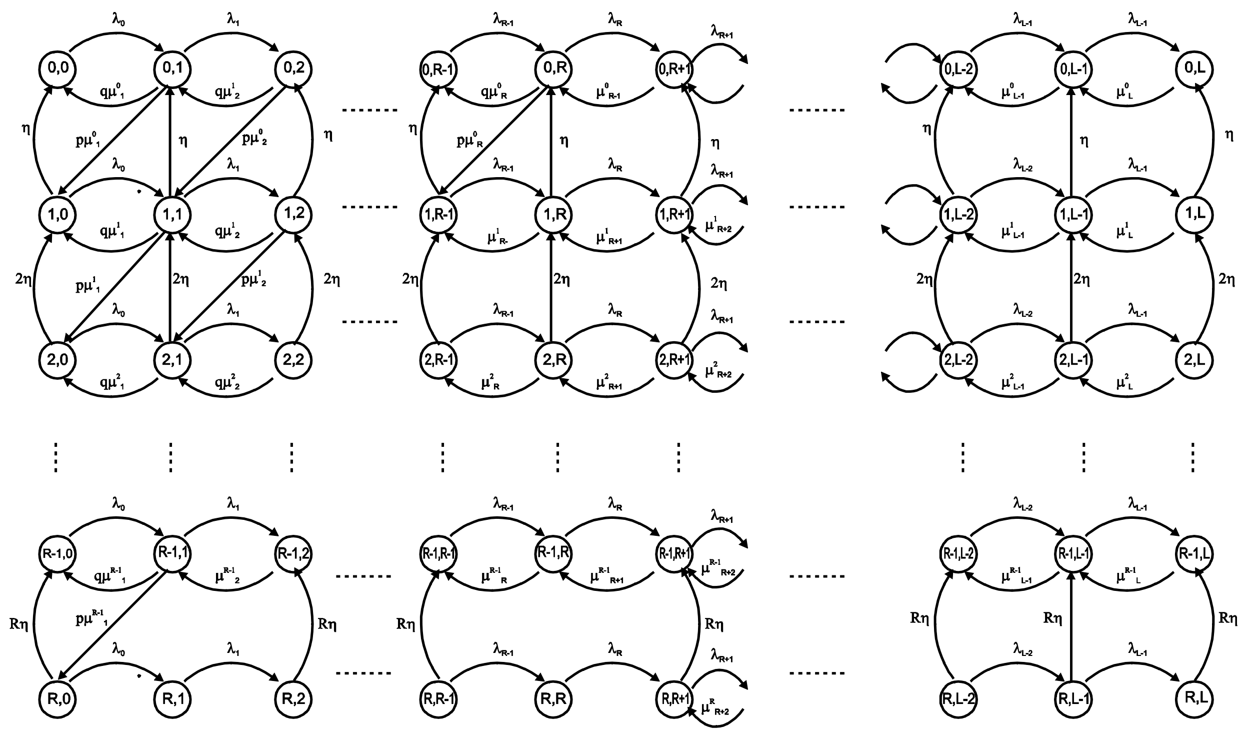

Figure-2. State-transition rate diagram for a multi-server machine repair system with modified Bernoulli vacation schedule.

According to figure-2 state transition rate diagram where (m, n) of circle denotes the state that there are m vacation servers are n failed machines in the system, steady state equations of the machine repair model are given as follows:

(i) When $r = 0$

$$\lambda_0 P_0(0) = q\mu_1^{(0)} P_0(1) + \eta P_1(0) \tag{1}$$

$$(\lambda_n + \mu_n^0) P_0(n) = \lambda_{n-1} P_0(n-1) + \eta P_1(n) + q\mu_{n+1}^{(0)} P_0(n+1); \qquad 1 \le n \le R-1 \tag{2}$$

$$(\lambda_R + \mu_R^0) P_0(R) = \lambda_{R-1} P_0(R-1) + \eta P_1(R) + \mu_{R+1}^{(0)} P_0(R+1); \tag{3}$$

$$(\lambda_n + \mu_n^0) P_0(n) = \lambda_{n-1} P_0(n-1) + \eta P_1(n) + \mu_{n+1}^{(0)} P_0(n+1); \; R+1 \le n \le L-1 \tag{4}$$

$$\mu_L^0 P_0(L) = \lambda_{L-1} P_0(L-1) + \eta P_1(L) \tag{5}$$

(ii) When, $1 \le K \le R - 1$

$$(\lambda_0 + r\eta) P_r(0) = q\mu_1^r P_r(1) + (r+1)\eta P_{r+1}(0) + p\mu_1^{r-1} P_{r-1}(0) \tag{6}$$

$$(\lambda_n + r\eta + \mu_n^r)P_r(n) = \lambda_{n-1}P_r(n-1) + (r+1)\eta P_{r+1}(n) + q\mu_{n+1}^r P_r(n+1)$$
$$+ p\mu_{n+1}^{r-1}P_{r-1}(n+1);\ 1 \le n \le R-r-1 \tag{7}$$

$$(\lambda_{R-r} + r\eta + \mu_{R-r}^r)P_r(R-r) = \lambda_{R-r-1}P_r(R-r-1) + (r+1)\eta P_{r+1}(R-r)$$
$$+ \mu_{R-r+1}^r P_r(R-r+1) + p\mu_{R-r+1}^{r-1}P_{r-1}(R-r+1); \tag{8}$$

$$(\lambda_n + r\eta + \mu_n^r)P_r(n) = \lambda_{n-1}P_r(n-1) + (r+1)\eta P_{r+1}(n) + \mu_{n+1}^r P_r(n+1)$$
$$R-r+1 \le n \le L-1 \tag{9}$$

$$(r\eta + \mu_L^r)P_r(L) = \lambda_{L-1}P_r(L-1) + (r+1)\eta P_{r+1}(L) \tag{10}$$

(iii) When, $r = R$

$$(\lambda_0 + R\eta)P_R(0) = p\mu_1^{R-1}P_{R-1}(1) \tag{11}$$
$$(\lambda_n + R\eta)P_R(n) = \lambda_{n-1}P_R(n-1); \qquad 1 \le n \le L-1 \tag{12}$$
$$R\eta P_R(L) = \lambda_{L-1}P_R(L-1) \tag{13}$$

Since the closed form probability solutions of equations (1)-(13) are too complicate to develop explicit expressions by using a recursive method. Hence we use matrix-geometric methods to analysis this problem we find that the equations (1)-(13) of the present model can be expressed in the matrix form.

To solve the flow balance equations (1)-(13) for the stationary distribution, we construct the transition rate matrix Q which is as follows:

$$Q = \begin{bmatrix} D_0 & U_0 & 0 & 0 & \cdots & \cdots & 0 \\ L_0 & D_1 & U_1 & 0 & \cdots & \cdots & \cdots \\ 0 & L_1 & D_2 & U_2 & \cdots & \cdots & \cdots \\ \vdots & \vdots & L_2 & D_3 & U_3 & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & L_{R-2} & D_{R-1} & U_R \\ 0 & \vdots & \vdots & \vdots & 0 & L_{R-1} & D_R \end{bmatrix}$$

Matrices D_0 and D_1 filter those parts of the Markov process which correspond to no-arrival and arrival transitions respectively.

Sub-matrices of matrix Q are given as follows:

$$U_i = \begin{bmatrix} 0 & 0 & 0 & \cdots & \cdots & 0 \\ p\mu_1^{i-1} & 0 & 0 & \cdots & \cdots & 0 \\ \vdots & 0 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 0 & p\mu_{R-i}^{i-1} & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$D_i = \begin{bmatrix} v_{0,i} & \lambda_0 & 0 & 0 & 0 & 0 \\ u_{1,i} & v_{1,i} & \lambda_1 & 0 & 0 & 0 \\ 0 & u_{2,i} & v_{2,i} & 0 & 0 & 0 \\ & & & & & \\ 0 & 0 & 0 & u_{L-1,i} & v_{L-i,i} & \lambda_L \\ 0 & 0 & 0 & 0 & u_{L,i} & v_{L,i} \end{bmatrix}$$

$$v_{j,i}=\begin{cases}-(\lambda_0+i\eta) & ; \quad 1\le i\le R-1 \ \ and \ \ j=0\\ -(\lambda_0+i\eta+u_j^i); & 1\le i\le R-1 \ \ and \ \ 2\le j\le L-1\\ -(i\eta+\mu_L^i) & ; \quad 1\le i\le R-1 \ \ and \ \ j=L\\ -(\lambda_j+R\eta) & ; \quad i=R \ and \ 0\le j\le L-1\\ R\eta & ; \quad i=R \ and \ \ j=L\end{cases}$$

and

$$v_{j,i}=\begin{cases}q\mu_j^i & ; \quad 1\le i\le R-1 \ and \ 1\le j\le R-i-1\\ \mu_j^i & ; \quad 1\le i\le R-1 \ \ and \ \ R-i\le j\le L\\ 0 & ; \quad otherwise\end{cases}$$

Let π be partitioned conformally with Q, i.e.

$$\pi=[\pi_0,\ \pi_1,....\pi_R] \tag{14}$$

Where, π_i is a $1\times(L+1)$ row vector which is given below:

$$\pi_i=[P_i(0),\ P_i(1),....P_i(L)];\ i=0,1,...,R$$

Now $\quad \pi Q=0$ (15)

$$\pi_0 D_0+\pi_1 L_0=0 \quad (16)$$

$$\pi_{i-1}U_i+\pi_i D_i+\pi_{i+1}L_i=0;\quad 1\le i\le R-1 \tag{17}$$

and

$$\pi_{R-1}U_R+\pi_R D_R=0 \tag{18}$$

After some routine substitutions, we have

$$\pi_R=-\pi_{R-1}U_R D_R^{-1} \tag{19}$$

$$\pi_i=\pi_{i-1}X_i;\qquad 1\le i\le R-1 \tag{20}$$

$$\pi_0(D_0+X_1L_1)=0 \tag{21}$$

Where, $\quad X_i=-U_i(D_i-X_{i-1}L_i)^{-1};\quad 1\le i\le R-2$

And, $\quad X_{R-1}=-U_{R-1}(D_{R-1}-U_R D_k^{-1}L_{R-1})^{-1}$

Equation (21) determines π_0 and equations (19) and (20) determined $\pi_R \ \ \pi_{R-1},...,\pi_1$ upto a constant. This constant can be determined by the normalizing equation

$$\sum_{i=0}^{R}\ \pi_i e=1 \tag{22}$$

Where, e is a column vector with all elements equal to one i.e. $e'=(1,1,...,1)$.

IV. SYSTEM PERFORMANCE MEASURES AND COST ANALYSIS

Various performance measures of machining system in terms of probabilities can be obtained. We define assumptions and notations which are computed s follows:

The average no. of failed machines in the system

$$= E(f) = \sum_{i=0}^{R} \sum_{n=0}^{L} nP_i(n)$$

The average no. of vacationing servers in the system

$$E(v) = \sum_{i=0}^{R} \sum_{n=0}^{R-i} iP_i(n)$$

The average no. of idle servers in the system

$$E(i) = \sum_{i=0}^{R-1} \sum_{n=0}^{R-i} (R-i)P_i(n)$$

The average no. of busy servers in the system

$$E(b) = R - E(w) - E(i)$$

The average no. of operating machines in the system

$$E(O) = M - \sum_{i=0}^{R} \sum_{n=S+1}^{L} (n-S)P_i(n)$$

The average no. of standby machines in the system

$$E(s) = \sum_{i=0}^{R} \sum_{n=0}^{S} (S-n)P_i(n)$$

Bension and Cox [1] defined the machine availability and the server utilization as follows:

$$\text{Machine availability (M.A.)} = \frac{L - E(f)}{L} \tag{23}$$

Server utilization (the fraction of busy servers)

$$\text{O.U.} = \frac{E(b)}{R}. \tag{24}$$

Now we determine the optimal amount of resources to maintain the system availability at a certain level.

Define, the steady state probability that at least M machines are in operation (system availability) = Av;

and the minimum fraction of Av is given by A_0 :

The production system requires a minimum of M machines in operation. The cost per unit time of each machine downtime occurs when there are less than M operating machines in the system.

We select the following cost elements.

c(h) = Cost per unit time for a failed machine.

c(e) = Cost per unit time of a failed machine after all standbys are exhausted.

c(w) = Cost per unit time for a machine functioning as standby.

c(b) = cost per unit time for a busy server.

c(i) = Cost per unit time for an idle server.

c(v) = Cost per unit time for a vacationing server.

c(s) = Cost per unit time for offering service rate μ for a failed machine.

c(O) = Cost of (loss) operative utilization.

With these costs, we consider R, s,μ as decision variables and write the total cost function as follows:

$$\text{Cost function } F(R, S, \mu) = c(h)\, E(f) - c(e)\, [\, M - E(O)] + c(w)\, E(s) + c(b)\, E(b) + c(i)\, E(i) - c(v)\, E(v) + c(s)\mu + c(O)\, (1 - O.U.). \quad (26)$$

$$\text{Subject to } Av \geq A_0 \quad (27)$$

There is a trade-off between the cost and the availability of the system. The cost function (26) is highly nonlinear and complex.

Our aim is to designate optimal values of some controllable parameters for this system under the cost function given in equation (26).

Two discrete variables R, S and one continuous variable μ are considered. Our main objective is to determine the optimal values of R, S and μ so as to minimize this cost function F(R, S, μ) subject to equation (27).

V. CONCLUSIONS

In present paper, we have studied a machine repairing system with Bernoulli vacation schedule. In this article we examine the effect of repair pressure on the system performance, such a system can be frequently used to model as a system with an electronic equipment. We have developed a Markov Chain model and obtained the stationary distribution using matrix recursive approach also we have developed the expected cost function.

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