Decision Making Approach for Solving Fuzzy Soft Matrix

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Abstract— **In this work, we investigate fuzzy soft matrix and their operations. We then investigate fuzzy soft algorithm that allows constructing those efficient decision making method. Finally, we give an example which shows that the method can be successfully applied to many problems that contains uncertainties.**

Keywords— **Soft set, Fuzzy soft set, Fuzzy soft matrix, min, Average.**

I. INTRODUCTION

Molodtsov [1] introduced the concept of soft sets that can be seen as a new mathematical theory for dealing with uncertainty. Majiet.al[2] worked on theoretical study of soft sets in detail, and [3] presented an application of soft set in the decision making problem using the reduction of soft sets[3].

Majiet.al[2] presented the concept of fuzzy soft sets by embedding the ideas of fuzzy sets. By using this definition of fuzzy soft sets many interesting applications of soft set theory have been expanded by some Researchers. Roy and Maji[3] gave some applications of fuzzy soft sets. Aktas and cagman[6] compared soft sets with the related concepts of Fuzzy sets and Rough sets. Cagman and Enginoglu [5] defined soft matrices which were a matrix representation of the soft sets and constructed a soft max-min decision making method. Cagman and Enginoglu [6] defined fuzzy soft matrices and constructed a decision making problem. Borahet al.[7] extended fuzzy soft matrix theory and its application. Maji and Roy [8] presented a novel method of object from an imprecise multi-observer data to deal with decision making based on fuzzy soft sets. Majumdar and Samanta[9] generalized the concept of fuzzy soft sets.

II. PRELIMINARIES

This section, briefly reviews the basic characteristics of fuzzy set and fuzzy soft sets.

Definition-2.1:

Let U be an initial universe, P (U) be the power set of U, E be the set of all parameters and A⊆ E. A soft set (f_A, E) on the universe U is defined by the set of order pairs (f_A, E) = {(e, f_A (e)): e∈ E, f_A∈ P (U)} where f_A : E → P (U) such that f_A (e) = ϕ if e ∉ A.

Here f_A is called an approximate function of the soft set.

Example:

Let U = {u_1,u_2,u_3,u_4} be a set of four shirts and E = {white(e_1),red(e_2),blue (e_3)} be a set of parameters. If A = {e_1 ,e_2} ⊆ E. Let $f_A(e_1)$ = {u_1, u_2 ,u_3, u_4} and $f_A(e_2)$= {u_1,u_2,u_3} then we write the soft set (f_A , E)= {(e_1, {u_1, u_2 ,u_3, u_4}), (e_2,{u_1,u_2,u_3})} over U which describe the “colour of the shirts” which Mr. X is going to buy. We may represent the soft set in the following form:

U	e_1	e_2	e_3
u_1	1	1	0
u_2	1	1	0
u_3	1	1	0
u_4	1	0	0

Definition -2.2:

Let U be an initial universe, E be the set of all parameters and A⊆ E. A pair (F, A) is called a fuzzy soft set over U where F: A → P (U) is a mapping from A into P (U) , where P (U) denotes the collection of all subsets of U.

Example:

Consider the above example, here we cannot express with only two real numbers 0 and 1, we can characterized it by a membership function instead of crisp number 0 and 1, which associate with each element a real number in the interval [0,1].Then

$(f_A, E) = \{f_A(e_1) = \{(u_1, 0.7), (u_2, 0.5), (u_3, 0.4), (u_4, 0.2)\}, f_A(e_2) = \{(u_1, 0.5), (u_2, 0.1), (u_3, 0.5)\}$

is the fuzzy soft set representing the "colour of the shirts" which Mr. X is going to buy. We may represent the fuzzy soft set in the following

U	e_1	e_2	e_3
u_1	0.7	0.5	0
u_2	0.5	0.1	0
u_3	0.4	0.5	0
u_4	0.2	0	0

Definition- 2.3:

Let (f_A, E) be fuzzy soft set over U. Then a subset of U × E is uniquely defined by $R_A = \{(u, e): e \in A, u \in f_A(e)\}$, which is called relation form of (f_A, E) .The characteristic function of R_A is written by $\mu_{RA} : U \times E \to [0, 1]$, where $\mu_{RA}(u, e) \in [0,1]$ is the membership value of $u \in U$ for each $e \in E$. If $\mu_{ij} = \mu_{R_A}(u_i, e_j)$, we can write the matrix by,

$$[\mu_{ij}]_{m\times n} = \begin{pmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \mu_{m1} & \mu_{m2} & \cdots & \mu_{mn} \end{pmatrix}$$

which is called an m x n soft matrix of the soft set (f_A, E) over U. Therefore we can say that a fuzzy soft set (f_A, E) is uniquely characterized by the matrix $[\mu_{ij}]_{m\times n}$ and both concepts are interchangeable. The set of all $m\times n$ fuzzy soft matrices over U will be denoted by FSM$m\times n$.

Example:

Assume that U= $\{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and E= $\{e_1, e_2, e_3, e_4\}$ is a set all parameters. If $A \subseteq E = \{e_2, e_3, e_4\}$ and

$$f_A(e_2) = \left\{\frac{u_1}{0.4}, \frac{u_2}{0.5}, \frac{u_3}{1.0}, \frac{u_4}{0.3}, \frac{u_5}{0.6}\right\}, f_A(e_3) = \left\{\frac{u_1}{0.3}, \frac{u_2}{0.4}, \frac{u_3}{0.6}, \frac{u_4}{0.5}, \frac{u_5}{1.0}\right\}, f_A(e_4) = \left\{\frac{u_1}{0.5}, \frac{u_2}{0.5}, \frac{u_3}{0.4}, \frac{u_4}{0.3}, \frac{u_5}{0.9}\right\}.$$

Then the fuzzy soft set *(f_A, E)* is a parameterized family *$\{f_A(e_2), f_A(e_3), f_A(e_4)\}$*of all fuzzy sets over U. Then the relation form of *(f_A, E)* is written by

R_A	e_1	e_2	e_3	e_4
u_1	0	0.4	0.3	0.5
u_2	0	0.5	0.4	0.5
u_3	0	1.0	0.6	0.4
u_4	0	0.3	0.5	0.3
u_5	0	0.6	1.0	0.9

Hence the fuzzy soft matrix $[\mu_{ij}]$ is written as

$$[\mu_{ij}] = \begin{bmatrix} 0 & 0.4 & 0.3 & 0.5 \\ 0 & 0.5 & 0.4 & 0.5 \\ 0 & 1.0 & 0.6 & 0.4 \\ 0 & 0.3 & 0.5 & 0.3 \\ 0 & 0.6 & 1.0 & 0.9 \end{bmatrix}$$

Definition- 2.4: (ROW- FUZZY SOFT MATRIX)

A fuzzy soft matrix of order $1\times n$ i.e., with a single row is called a row-fuzzy soft matrix.

Definition- 2.5: (COLUMN -FUZZY SOFT MATRIX)

A fuzzy soft matrix of order $m\times 1$ i.e.,with a single column is called a column-fuzzy soft matrix.

Definition- 2.6: (COMPLEMENT FUZZY SOFT MATRIX)

Let (a_{ij}) be an $m \times n$ fuzzy soft matrix.Then the complement of (a_{ij}) is denoted by $(a_{ij})^{o}$ and defined by, $(a_{ij})^{o} = (c_{ij})$ is also an fuzzy soft matrix of order $m \times n$ and c_{ij} = 1-a_{ij} for all i, j.

Example :

Let $(a_{ij})=\begin{bmatrix} 0 & 0.4 & 0.3 & 0.5 \\ 0 & 0.5 & 0.4 & 0.5 \\ 0 & 1.0 & 0.6 & 0.4 \\ 0 & 0.3 & 0.5 & 0.3 \\ 0 & 0.6 & 1.0 & 0.9 \end{bmatrix}$. The complement of (a_{ij}) is $(a_{ij})^{o}=\begin{bmatrix} 1 & 0.5 & 0.7 & 0.5 \\ 1 & 0.4 & 0.6 & 0.5 \\ 1 & 0.0 & 0.4 & 0.6 \\ 1 & 0.7 & 0.5 & 0.7 \\ 1 & 0.4 & 0.0 & 0.1 \end{bmatrix}$.

Definition- 2.8: (SUM OF THE FUZZY SOFT MATRICES)

Two fuzzy soft matrices A and B are said to be conformable for addition, if they be of the same order. The addition of two fuzzy soft matrices (a_{ij}) and (b_{ij}) of order $m\times n$ is defined by, $(a_{ij}) \oplus (b_{ij}) = (c_{ij})$ is also an $m\times n$ fuzzy soft matrix and c_{ij} = max$\{a_{ij}, b_{ij}\}$for all i, j.

Example :

Let U be the set of four cities, given by, U = $\{u_1, u_2, u_3, u_4, u_5\}$. Let E be the set of parameters given by, E = {highly, *immensely, moderately, average, less}* = $\{e_1, e_2, e_3, e_4, e_5\}$ (say). Let A$\subset E$, given by, $A = \{e_1, e_2, e_3, e_5\}$ and

$$F_A(e_1)=\left\{\frac{u_1}{0.2}, \frac{u_2}{0.8}, \frac{u_3}{0.4}, \frac{u_4}{0.6}, \frac{u_5}{0.7}\right\},\ F_A(e_2)=\left\{\frac{u_1}{0}, \frac{u_2}{0.9}, \frac{u_3}{0.3}, \frac{u_4}{0.4}, \frac{u_5}{0.6}\right\},$$

$$F_A(e_3)=\left\{\frac{u_1}{0.3}, \frac{u_2}{0.4}, \frac{u_3}{0.8}, \frac{u_4}{0.8}, \frac{u_5}{0.3}\right\},\ F_A(e_5)=\left\{\frac{u_1}{0.9}, \frac{u_2}{0.1}, \frac{u_3}{0.5}, \frac{u_4}{0.3}, \frac{u_5}{0.1}\right\}.$$

Hence the fuzzy soft matrix (a_{ij}) is written by,

$$(a_{ij})=\begin{pmatrix} 0.2 & 0.0 & 0.3 & 0 & 0.9 \\ 0.8 & 0.9 & 0.4 & 0 & 0.1 \\ 0.4 & 0.3 & 0.8 & 0 & 0.5 \\ 0.6 & 0.4 & 0.8 & 0 & 0.3 \\ 0.7 & 0.6 & 0.3 & 0 & 0.1 \end{pmatrix}.$$

Now consider another intuitionistic fuzzy soft matrix (b_{ij}) associated with the intuitionistic fuzzy soft set over the same universe U. Let $B = \{e_1, e_4, e_5\} \subset E$ and

$$F_B(e_1)=\left\{\frac{u_1}{0.3}, \frac{u_2}{0.9}, \frac{u_3}{0.4}, \frac{u_4}{0.7}, \frac{u_5}{0.6}\right\},\ F_B(e_4)=\left\{\frac{u_1}{0.2}, \frac{u_2}{0.3}, \frac{u_3}{0.7}, \frac{u_4}{0.2}, \frac{u_5}{0.3}\right\},\ F_B(e_5)=\left\{\frac{u_1}{0.8}, \frac{u_2}{0.2}, \frac{u_3}{0.6}, \frac{u_4}{0.3}, \frac{u_5}{0.2}\right\}.$$

Hence the intuitionistic fuzzy soft matrix (b_{ij}) is written by

$$(b_{ij}) = \begin{pmatrix} 0.3 & 0 & 0 & 0.2 & 0.8 \\ 0.9 & 0 & 0 & 0.3 & 0.2 \\ 0.4 & 0 & 0 & 0.7 & 0.6 \\ 0.7 & 0 & 0 & 0.2 & 0.3 \\ 0.6 & 0 & 0 & 0.3 & 0.2 \end{pmatrix}.$$

Therefore the sum of the intuitionistic fuzzy soft matrices (a_{ij}) and (b_{ij}) is,

$$(a_{ij}) \oplus (b_{ij}) \begin{pmatrix} 0.3 & 0.0 & 0.3 & 0.2 & 0.9 \\ 0.9 & 0.9 & 0.4 & 0.3 & 0.2 \\ 0.4 & 0.3 & 0.8 & 0.7 & 0.6 \\ 0.7 & 0.4 & 0.8 & 0.2 & 0.3 \\ 0.7 & 0.6 & 0.3 & 0.3 & 0.2 \end{pmatrix}.$$

Definition- 2.9: (SUBTRACTION OF THE FUZZY SOFT MATRICES)

Two fuzzy soft matrices A and B are said to be conformable for subtraction, if they be of the same order. For any two intuitionistic fuzzy soft matrices (a_{ij}) and (b_{ij}) of order $m\times n$, the subtraction of (b_{ij}) from (a_{ij}) is defined as $(a_{ij}) \ominus (b_{ij}) = (c_{ij})$ is also an $m\times n$ fuzzy soft matrix and c_{ij} = min$\{a_{ij}, b_{ij}^0\}$ for all i, j, where (b_{ij}^0) is the complement of (b_{ij}).

Example :

Consider the fuzzy soft matrices (a_{ij}) and (b_{ij}) in the previous example,

$$(a_{ij})=\begin{pmatrix} 0.2 & 0.0 & 0.3 & 0 & 0.9 \\ 0.8 & 0.9 & 0.4 & 0 & 0.1 \\ 0.4 & 0.3 & 0.8 & 0 & 0.5 \\ 0.6 & 0.4 & 0.8 & 0 & 0.3 \\ 0.7 & 0.6 & 0.3 & 0 & 0.1 \end{pmatrix}, (b_{ij}) = \begin{pmatrix} 0.3 & 0 & 0 & 0.2 & 0.8 \\ 0.9 & 0 & 0 & 0.3 & 0.2 \\ 0.4 & 0 & 0 & 0.7 & 0.6 \\ 0.7 & 0 & 0 & 0.2 & 0.3 \\ 0.6 & 0 & 0 & 0.3 & 0.2 \end{pmatrix}.$$

.

$$\text{Now } (b_{ij})^{o} = \begin{pmatrix} 0.7 & 1 & 1 & 0.8 & 0.2 \\ 0.1 & 1 & 1 & 0.7 & 0.8 \\ 0.6 & 1 & 1 & 0.3 & 0.4 \\ 0.3 & 1 & 1 & 0.8 & 0.7 \\ 0.4 & 1 & 1 & 0.7 & 0.8 \end{pmatrix}$$

∴ the subtraction of the fuzzy soft matrix (b_{ij}) from (a_{ij}) is

$$(a_{ij})\Theta(b_{ij})=\begin{pmatrix} 0.2 & 0.0 & 0.3 & 0 & 0.2 \\ 0.1 & 0.9 & 0.4 & 0 & 0.1 \\ 0.4 & 0.3 & 0.8 & 0 & 0.4 \\ 0.3 & 0.4 & 0.8 & 0 & 0.3 \\ 0.4 & 0.6 & 0.3 & 0 & 0.1 \end{pmatrix}$$

Definition- 2.10: (PRODUCT OF AN FUZZY SOFT MATRIX WITH A CHOICE MATRIX)

Let U be the set of universe and E be the set of parameters. Suppose that A be any fuzzy soft matrix and B be any choice matrix of a decision maker concerned with the same universe U and E. Now if the number of columns of the fuzzy soft matrix A be equal to the number of rows of the choice matrix B, then A and B are said to be conformable for the product (A⊗B) and the product (A⊗B) becomes an fuzzy soft matrix.

If A = $(a_{ij})_{m\times n}$ and $B = (b_{jk})_{n\times p}$, then $(A\otimes\beta)=(c_{ik})$ where $c_{ik}= (max_{j=1}^{n}\ \{\mu_{a_{ij}}, \mu_{b_{jk}}\})$.

Example :

Let U be the set of four dresses, given by, U = $\{d_1, d_2, d_3, d_4\}$. Let E be the set of parameters, given by, E = {*cheap, beautiful, comfortable, gorgeous*} = $\{e_1, e_2, e_3, e_4\}$ (say). Suppose that the set of choice parameters of Mr.X be, $A= \{e_1, e_3\}$. Now let according to the choice parameters of Mr.X, we have the intuitionistic fuzzy soft set (*F*, *A*) which describes the attractiveness of the dresses and the intuitionistic fuzzy soft matrix of the intuitionistic fuzzy soft set (*F*, *A*) be,

$$(a_{ij}) = \begin{pmatrix} 0.8 & 0.2 & 0.7 & 0.3 \\ 0.3 & 0.7 & 0.4 & 0.8 \\ 0.7 & 0.4 & 0.5 & 0.6 \\ 0.5 & 0.1 & 0.9 & 0.2 \end{pmatrix}$$

Again the choice matrix of Mr.X is,

$$(\beta_{ij})_A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Since the number of columns of the fuzzy soft matrix (a_{ij}) is equal to the number of rows of the choice matrix $(\beta_{ij})_A$, they are conformable for the product.

$$\text{Therefore } (a_{ij})\otimes(\beta_{ij})_A = \begin{pmatrix} 1.0 & 0.8 & 1.0 & 0.8 \\ 1.0 & 0.8 & 1.0 & 0.8 \\ 1.0 & 0.7 & 1.0 & 0.7 \\ 1.0 & 0.9 & 1.0 & 0.9 \end{pmatrix}$$

III.DECISION MAKING MODEL OF FUZZY SOFT MATRICES

In this section, we put forward fuzzy soft matrices in decision making by using different operators.
Input : Fuzzy soft set of r objects, each of which has s parameters.
Output: An optimum result.

ALGORITHM

Step- 1: Choose the set of parameters.
Step -2: Construct the fuzzy soft matrix for the set of parameters.
Step -3: Compute min matrix of the three fuzzy soft matrices as M_m
Step- 4: Compute the arithmetic mean of membership value of fuzzy soft matrix as $A(M_m)$
Step-5: Find the decision with highest membership value.

PROBLEM:

Let U= $\{M_1, M_2, M_3, M_4\}$ be a set of four candidates and C = {Highest qualification, Knowledge, Previous experience, Hard work} be the set of parameters, given by P = $\{P_1, P_2, P_3, P_4\}$. A set of three experts E = $\{ e_1, e_2, e_3\}$ want to evaluate the best candidates as per knowledge base. The fuzzy decision matrices of experts e_1, e_2 *and* e_3 are given in the following table.
The fuzzy soft decision matrices of four candidates is given by X, Y and Z,

$$X = \begin{pmatrix} 0.8 & 0.7 & 0.5 & 0.9 \\ 0.4 & 0.3 & 0.8 & 0.4 \\ 0.6 & 0.4 & 0.2 & 0.6 \\ 0.7 & 0.6 & 0.6 & 0.7 \end{pmatrix},\ Y = \begin{pmatrix} 0.7 & 0.6 & 0.6 & 0.8 \\ 0.3 & 0.3 & 0.4 & 0.4 \\ 0.5 & 0.5 & 0.7 & 0.6 \\ 0.6 & 0.6 & 0.9 & 0.6 \end{pmatrix},\ Z = \begin{pmatrix} 0.1 & 0.8 & 0.2 & 0.9 \\ 0.8 & 0.3 & 0.9 & 0.4 \\ 0.3 & 0.5 & 0.6 & 0.7 \\ 0.5 & 0.6 & 0.4 & 0.6 \end{pmatrix}.$$

SOLUTION:

$$\text{Given } X = \begin{pmatrix} 0.8 & 0.7 & 0.5 & 0.9 \\ 0.4 & 0.3 & 0.8 & 0.4 \\ 0.6 & 0.4 & 0.2 & 0.6 \\ 0.7 & 0.6 & 0.6 & 0.7 \end{pmatrix},\ Y = \begin{pmatrix} 0.7 & 0.6 & 0.6 & 0.8 \\ 0.3 & 0.3 & 0.4 & 0.4 \\ 0.5 & 0.5 & 0.7 & 0.6 \\ 0.6 & 0.6 & 0.9 & 0.6 \end{pmatrix},\ Z = \begin{pmatrix} 0.1 & 0.8 & 0.2 & 0.9 \\ 0.8 & 0.3 & 0.9 & 0.4 \\ 0.3 & 0.5 & 0.6 & 0.7 \\ 0.5 & 0.6 & 0.4 & 0.6 \end{pmatrix}.$$

$$\text{Therefore, } M_m = \begin{pmatrix} 0.1 & 0.6 & 0.2 & 0.8 \\ 0.3 & 0.3 & 0.4 & 0.4 \\ 0.3 & 0.4 & 0.2 & 0.6 \\ 0.5 & 0.6 & 0.4 & 0.6 \end{pmatrix} \text{ and } A(M_m) = \begin{pmatrix} 0.043 \\ 0.035 \\ 0.038 \\ \mathbf{0.053} \end{pmatrix}$$

Now the candidate C_4 associated with the fourth row of the resultant fuzzy soft matrix has the highest value.
Therefore the **candidate C_4 has the best candidate as per the knowledge base.**

IV. CONCLUSION

To develop the theory, in this work, first we explain FSM and their operations. We then presented the decision making method for FSM. Finally we provided an example demonstrating the successfully application of this method.

FUTURE WORK: It may be applied to many fields with problems that contain uncertainty, and it would be beneficial to extend the proposed method to subsequent studies.

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