Wave Propagation at Liquid/Fluid Saturated Incompressible Porous Solid Interface

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Abstract

The present paper is concerned with the reflection and transmission of elastic waves from a plane surface separating liquid half space and fluid saturated incompressible porous half space when longitudinal wave (P-wave) or transverse wave (SV-wave) impinge obliquely at the interface. Amplitude ratios of various reflected and transmitted waves are obtained. These amplitude ratios have been computed numerically for a specific model and results obtained are depicted graphically with angle of incidence of incident wave. It is found that these amplitude ratios depend on angle of incidence of the incident wave and material properties. A particular case of reflection at free surface of fluid saturated porous half space has been deduced and discussed. A special case in which fluid saturated porous half space reduced to empty porous solid is obtained and discussed from the present investigation.

Keywords: Porous solid, reflection, refraction, longitudinal wave, transverse wave, amplitude ratios.

1. Introduction

Based on the work of Fillunger model (1913), Bowen (1980) and de Boer and Ehlers (1990a, 1990b) developed an interesting theory for porous medium having all constituents to be incompressible. There are sufficient reasons for considering the fluid saturated porous constituents as incompressible. For example, consider the composition of soil in which the solid constituents as well as liquid constituents which are generally water or oils are incompressible. Therefore, the assumption of incompressible constituents meet the properties appearing the in many branches of engineering.

Based on this theory, many researchers like de Boer and Liu (1994, 1995), de Boer and Liu (1996), Liu (1999), Yan et.al. (1999), de Boer and Didwania (2004), Tajuddin and Hussaini (2006), Kumar and Hundal (2007), Kumar et.al. (2011) etc. studied some problems of wave propagation in fluid saturated porous media.

In the present paper, using the theory of de Boer and Ehlers (1990b) for fluid saturated porous medium, the reflection and transmission phenomenon of plane waves at an interface between liquid half space and fluid saturated porous half space is studied. Amplitude ratios of various reflected and transmitted waves have been obtained using suitable boundary conditions at the interface and computed numerically for a specific model. The results obtained are depicted graphically with the angle of incidence and discussed. Reflection at free surface of fluid saturated porous half space is also derived as a particular case of the problem. A special case in which fluid saturated porous half space reduced to empty porous solid is also obtained and discussed with the help of graphs from the present investigation.

2. Basic equations and their solutions

2.1 For medium M_1 (Fluid saturated incompressible porous medium)

Following de Boer and Ehlers (1990b), the governing equations in a fluid-saturated incompressible porous medium are

$$\mathrm{div}(\eta^S \dot{\mathbf{x}}_S + \eta^F \dot{\mathbf{x}}_F) = 0. \quad (1)$$

$$\mathrm{div}\mathbf{T}_E^S - \eta^S \,\mathrm{grad}\, p + \rho^S(\mathbf{b} - \ddot{\mathbf{x}}_s) - \mathbf{P}_E^F = 0, \quad (2)$$

$$\mathrm{div}\mathbf{T}_E^F - \eta^F \,\mathrm{grad}\, p + \rho^F(\mathbf{b} - \ddot{\mathbf{x}}_F) + \mathbf{P}_E^F = 0, \quad (3)$$

where $\dot{\mathbf{x}}_i$ and $\ddot{\mathbf{x}}_i$ $(i = S, F)$ denote the velocities and accelerations, respectively of solid (S) and fluid (F) phases of the porous aggregate and p is the effective pore pressure of the incompressible pore fluid. ρ^S and ρ^F are the densities of the solid and fluid phases respectively and $\mathbf{b}$ is the body force per unit volume. $\mathbf{T}_E^S$ and $\mathbf{T}_E^F$ are the effective stress in the solid and fluid phases respectively, $\mathbf{P}_E^F$ is the effective quantity of momentum supply and η^S and η^F are the volume fractions satisfying

$$\eta^S + \eta^F = 1. \quad (4)$$

If $\mathbf{u}_S$ and $\mathbf{u}_F$ are the displacement vectors for solid and fluid phases, then

$$\dot{x}_S = \dot{\mathbf{u}}_S, \quad \ddot{\mathbf{x}}_s = \ddot{\mathbf{u}}_s, \quad \dot{\mathbf{x}}_F = \dot{\mathbf{u}}_F, \quad \ddot{\mathbf{x}}_F = \ddot{\mathbf{u}}_F. \quad (5)$$

The constitutive equations for linear isotropic, incompressible porous medium are given by de Boer, Ehlers and Liu (1993) as

$$\mathbf{T}_E^S = 2\mu^S \mathbf{E}_S + \lambda^S (E_S.\mathbf{I})\mathbf{I}, \quad (6)$$

$$\mathbf{T}_E^F = 0, \quad (7)$$

$$\mathbf{P}_E^F = -\mathbf{S}_v(\dot{\mathbf{u}}_F - \dot{\mathbf{u}}_S), \quad (8)$$

where λ^S and μ^S are the macroscopic Lame's parameters of the porous solid and $\mathbf{E}_S$ is the linearized Langrangian strain tensor defined as

$$\mathbf{E}_S = \frac{1}{2}(\mathrm{grad}\, \mathbf{u}_S + \mathrm{grad}^T \mathbf{u}_S), \quad (9)$$

In the case of isotropic permeability, the tensor $\mathbf{S}_v$ describing the coupled interaction between the solid and fluid is given by de Boer and Ehlers (1990b) as

$$\mathbf{S}_v = \frac{(\eta^F)^2 \gamma^{FR}}{K^F} \mathbf{I}, \quad (10)$$

where γ^{FR} is the specific weight of the fluid and K^F is the Darcy's permeability coefficient of the porous medium.

Making the use of (5) in equations (1)-(3), and with the help of (6)-(9), we obtain

$$\mathrm{div}(\eta^S \dot{\mathbf{u}}_S + \eta^F \dot{\mathbf{u}}_F) = 0, \tag{11}$$

$$(\lambda^S + \mu^S)\,\mathrm{grad\,div}\,\mathbf{u}_S + \mu^S \mathrm{div\,grad}\,\mathbf{u}_S - \eta^S \mathrm{grad}\,p + \rho^S(\mathbf{b} - \ddot{\mathbf{u}}_s) + S_v(\dot{\mathbf{u}}_F - \dot{\mathbf{u}}_S) = 0, \tag{12}$$

$$-\eta^F \mathrm{grad}\,p + \rho^F(\mathbf{b} - \ddot{\mathbf{u}}_F) - S_v(\dot{\mathbf{u}}_F - \dot{\mathbf{u}}_S) = 0. \tag{13}$$

For the two dimensional problem, we assume the displacement vector $\mathbf{u}_i$ $(i = F, S)$ as

$$\mathbf{u}_i = (u^i, 0, w^i) \quad \text{where} \quad i = F, S. \tag{14}$$

Equations (11) - (13) with the help of eq. (14) in the absence of body forces take the form

$$\eta^S\left[\frac{\partial^2 u^S}{\partial x\,\partial t} + \frac{\partial^2 w^S}{\partial z\,\partial t}\right] + \eta^F\left[\frac{\partial^2 u^F}{\partial x\,\partial t} + \frac{\partial^2 w^F}{\partial z\,\partial t}\right] = 0, \tag{15}$$

$$\eta^F\frac{\partial p}{\partial x} + \rho^F\frac{\partial^2 u^F}{\partial t^2} + S_v\left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{16}$$

$$\eta^F\frac{\partial p}{\partial z} + \rho^F\frac{\partial^2 w^F}{\partial t^2} + S_v\left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \tag{17}$$

$$(\lambda^S + \mu^S)\frac{\partial \theta^S}{\partial x} + \mu^S\nabla^2 u^S - \eta^S\frac{\partial p}{\partial x} - \rho^S\frac{\partial^2 u^S}{\partial t^2} + S_v\left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{18}$$

$$(\lambda^S + \mu^S)\frac{\partial \theta^S}{\partial z} + \mu^S\nabla^2 w^S - \eta^S\frac{\partial p}{\partial z} - \rho^S\frac{\partial^2 w^S}{\partial t^2} + S_v\left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \tag{19}$$

where

$$\theta^S = \frac{\partial(u^S)}{\partial x} + \frac{\partial(w^S)}{\partial z}, \tag{20}$$

and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}. \tag{21}$$

Also, ${t_{zz}}^S$ and ${t_{zx}}^S$ the normal and tangential stresses in the solid phase are as under

$${t_{zz}}^S = \lambda^S\left(\frac{\partial u^S}{\partial x} + \frac{\partial w^S}{\partial z}\right) + 2\mu^S\frac{\partial w^S}{\partial z}, \tag{22}$$

$${t_{zx}}^S = \mu^S\left(\frac{\partial u^S}{\partial z} + \frac{\partial w^S}{\partial x}\right). \tag{23}$$

The displacement components u^j and w^j are related to the dimensional potential ϕ^j and ψ^j as

$$u^j = \frac{\partial \phi^j}{\partial x} + \frac{\partial \psi^j}{\partial z}, \quad w^j = \frac{\partial \phi^j}{\partial z} - \frac{\partial \psi^j}{\partial x}, \qquad j = S, F. \tag{24}$$

Using eq. (24) in equations (15)-(19), we obtain the following equations determining ϕ^S, ϕ^F, ψ^S, ψ^F and p as:

$$\nabla^2 \phi^S - \frac{1}{C_1^{\,2}} \frac{\partial^2 \phi^S}{\partial t^2} - \frac{S_v}{(\lambda^S + 2\mu^S)(\eta^F)^2} \frac{\partial \phi^S}{\partial t} = 0, \tag{25}$$

$$\phi^F = -\frac{\eta^S}{\eta^F} \phi^S, \tag{26}$$

$$\mu^S \nabla^2 \psi^S - \rho^S \frac{\partial^2 \psi^S}{\partial t^2} + S_v \left[\frac{\partial \psi^F}{\partial t} - \frac{\partial \psi^S}{\partial t} \right] = 0, \tag{27}$$

$$\rho^F \frac{\partial^2 \psi^F}{\partial t^2} + S_v \left[\frac{\partial \psi^F}{\partial t} - \frac{\partial \psi^S}{\partial t} \right] = 0, \tag{28}$$

$$(\eta^F)^2 p - \eta^S \rho^F \frac{\partial^2 \phi^S}{\partial t^2} - S_v \frac{\partial \phi^S}{\partial t} = 0, \tag{29}$$

where

$$C_1 = \sqrt{\frac{(\eta^F)^2(\lambda^S + 2\mu^S)}{(\eta^F)^2 \rho^S + (\eta^S)^2 \rho^F}}. \tag{30}$$

Assuming the solution of the system of equations (25) - (29) in the form

$$(\phi^S, \phi^F, \psi^S, \psi^F, p) = (\phi_1^{\,S}, \phi_1^{\,F}, \psi_1^{\,S}, \psi_1^{\,F}, p_1) \exp(i\omega t), \tag{31}$$

where ω is the complex circular frequency.

Making the use of (31) in equations (25)-(29), we obtain

$$\left[\nabla^2 + \frac{\omega^2}{C_1^{\,2}} - \frac{i\omega S_v}{(\lambda^S + 2\mu^S)(\eta^F)^2} \right] \phi_1^{\,S} = 0, \tag{32}$$

$$[\mu^S \nabla^2 + \rho^S \omega^2 - i\omega S_v] \psi_1^{\,S} = -i\omega S_v \psi_1^{\,F}, \tag{33}$$

$$[-\omega^2 \rho^F + i\omega S_v] \psi_1^{\,F} - i\omega S_v \psi_1^{\,S} = 0, \tag{34}$$

$$(\eta^F)^2 p_1 + \eta^S \rho^F \omega^2 \phi_1^{\,S} - i\omega S_v \phi_1^{\,S} = 0, \tag{35}$$

$$\phi_1^{\,F} = -\frac{\eta^S}{\eta^F} \phi_1^{\,S}. \tag{36}$$

Equation (32) corresponds to longitudinal wave propagating with velocity $\overline{V}_1$, given by

$$\overline{V}_1^{\,2} = \frac{1}{G_1}, \tag{37}$$

where

$$G_1 = \left[\frac{1}{C_1^{\,2}} - \frac{iS_v}{\omega(\lambda^S+2\mu^S)(\eta^F)^2}\right]. \tag{38}$$

From equation (33) and (34), we obtain

$$\left[\nabla^2 + \frac{\omega^2}{\overline{V}_2^{\,2}}\right]\psi_1^{\,S} = 0, \tag{39}$$

Equation (39) corresponds to transverse wave propagating with velocity$\overline{V}_2$, given by $\overline{V}_2^{\,2} = 1/G_2$

where

$$G_2 = \left\{\frac{\rho^S}{\mu^S} - \frac{iS_v}{\mu^S\omega} - \frac{S_v^{\,2}}{\mu^S(-\rho^S\omega^2+i\omega S_v)}\right\}, \tag{40}$$

For medium M_2 (Liquid half space)

The equation of motion in terms of displacement potential ϕ^1 for liquid half space is given by

$$\frac{\partial^2\phi^1}{\partial x^2} + \frac{\partial^2\phi^1}{\partial z^2} = \frac{1}{\alpha^{1\,2}}\frac{\partial^2\phi^1}{\partial t^2}, \tag{41}$$

where $\alpha^1 = \sqrt{\frac{\lambda^1}{\rho^1}}$ is the velocity of the liquid.

The displacement components $u_1^{\,1}, u_3^{\,1}$and pressure p^1 are given by

$$u_1^{\,1} = \frac{\partial\phi^1}{\partial x},\quad u_3^{\,1} = \frac{\partial\phi^1}{\partial z},\quad p^1 = -\rho^1\frac{\partial^2\phi^1}{\partial t^2}, \tag{42}$$

3. Formulation of the problem

Consider a fluid saturated incompressible porous half space as medium M_1 and homogeneous inviscid liquid half space medium M_2 in welded contact along a plane interface. Rectangular cartesian coordinate system (x,y,z) is taken in such a way that the plane interface z=0 separates both the medium and z-axis is pointing into the medium M_1. The medium M_1 through which incident takes place occupies the region z>0 and the region z<0 is occupied by the medium M_2. The problem is two dimensional in the xz plane. The geometry of the problem is as shown in figure 1.

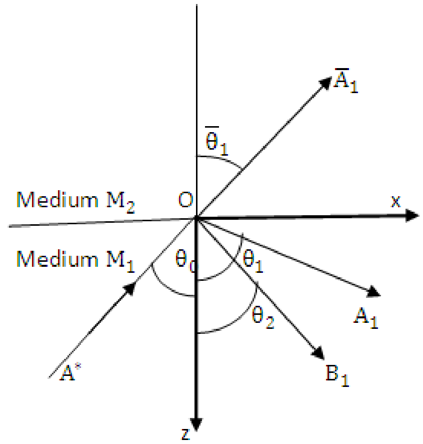

Fig.1 Geometry of the problem.

4. Reflection and transmission of the waves

Consider a longitudinal wave (P-wave) or transverse wave (SV-wave) is propagating through the fluid saturated porous medium M_1 and incident at the plane z=0 and making an angle θ_0 with normal to the surface. Corresponding to incident longitudinal or transverse wave, we get two reflected waves P-wave or SV-wave in medium M_1 and one refracted P-wave in the medium M_2.

The potential function satisfying the equations (25)-(29) can be taken as

$$\{\phi^S, \phi^F, p\} = \{1, m_1, m_2\}[A_{01}\exp\{ik_1(x\sin\theta_0 - z\cos\theta_0) + i\omega_1 t\} + A_1\exp\{ik_1(x\sin\theta_1 + z\cos\theta_1) + i\omega_1 t\}], \tag{43}$$

$$\{\psi^S, \psi^F\} = \{1, m_3\}[B_{01}\exp\{ik_2(x\sin\theta_0 - z\cos\theta_0) + i\omega_2 t\} + B_1\exp\{ik_2(x\sin\theta_2 + z\cos\theta_2) + i\omega_2 t\}], \tag{44}$$

where

$$m_1 = -\frac{\eta^S}{\eta^F}, \quad m_2 = -\left[\frac{\eta^S\omega_1{}^2\rho^F - i\omega_1 S_v}{(\eta^F)^2}\right], \quad m_3 = \frac{i\omega_2 S_v}{i\omega_2 S_v - \omega_2{}^2\rho^F}, \tag{45}$$

We assume the solution of the eq. (41) in the form

$$\phi^1 = \bar{A}_1\exp\{i\bar{k}_1(x\sin\bar{\theta}_1 - z\cos\bar{\theta}_1) + i\bar{\omega}_1 t\}, \tag{46}$$

where A_{01} and B_{01} are amplitudes of the incident P-wave and SV-wave, respectively and A_1, B_1 are amplitudes of the reflected P-wave and SV-wave respectively and $\bar{A}_1$ is the amplitude of transmitted P-wave.

5. Boundary conditions

The appropriate boundary conditions for the two dimensional motion, at the interface z=0 are the continuity of normal force stress, normal displacement and vanishing of the tangential force stress. Mathematically, these boundary conditions can be expressed as:

$$t_{zz}{}^{S} - p = -p^{1}, \quad t_{zx}{}^{S} = 0, \quad u_3 = u_3{}^{1}, \tag{47}$$

In order to satisfy the boundary conditions, the extension of the Snell's law will be

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\bar{\theta}_1}{\alpha^1}, \tag{48}$$

For P-wave ,

$$V_0 = V_1, \quad \theta_0 = \theta_1, \tag{49}$$

For SV-wave,

$$V_0 = V_2, \quad \theta_0 = \theta_2, \tag{50}$$

Also, frequencies of all the waves must be equal at the interface z=0 for all positions and time.

i.e

$$k_1V_1 = k_2V_2 = \bar{k}_1\alpha^1 = \omega, \text{ at } z = 0. \tag{51}$$

Making the use of potentials given by equations(43)-(44) and (46) in equations(22)-(24)and (42) and then using the boundary conditions given by (47) as well as the equations (48)-(51), we get a system of three non homogeneous which can be written as

$$\sum_{j=1}^{3} a_{ij} Z_j = Y_i, \qquad (i = 1,2,3) \tag{52}$$

where

$$Z_1 = \frac{A_1}{A^*}, \quad Z_2 = \frac{B_1}{A^*}, \quad Z_3 = \frac{\bar{A}_1}{A^*}, \tag{53}$$

where Z_1 to Z_3 are the amplitude ratios of reflected P- wave, reflected SV-wave and refracted P-wave, respectively.

Also

$$a_{11} = k_1{}^2(\lambda^s + 2\mu^s\cos^2\theta_1) + m_2, \quad a_{12} = -2k_2{}^2\mu^s\sin\theta_2\cos\theta_2, \qquad a_{13} = -\rho^1\bar{\omega}_1{}^2,$$

$$a_{21} = 2k_1{}^2\mu^s\sin\theta_1\cos\theta_1, \qquad a_{22} = k_2{}^2(\cos^2\theta_2 - \sin^2\theta_2), \quad a_{23} = 0,$$

$$a_{31} = k_1\cos\theta_1, \qquad a_{32} = -k_2\sin\theta_2, \qquad a_{33} = \bar{k}_1\cos\bar{\theta}_1. \quad (54)$$

For incident longitudinal wave:

$$A^* = A_{01},\ B_{01} = 0,\ Y_1 = -a_{11},\ Y_2 = a_{21},\ Y_3 = a_{31}, \quad (55)$$

For incident transverse wave:

$$A^* = B_{01},\ A_{01} = 0,\ Y_1 = a_{12},\ Y_2 = -a_{22},\ Y_3 = -a_{32}, \quad (56)$$

6. Particular cases

CASE-1

If pore is absent or gas is filled in the pores then ρ^F is very small as compared to ρ^S and can be neglected, so the relation (30) reduces to

$$C_0 = \sqrt{\frac{\lambda^S + 2\mu^S}{\rho^S}}. \quad (57)$$

Then fluid saturated incompressible porous medium reduces to empty porous solid.

CASE-2

When upper half space is not present in the given formulation.

Considering a fluid saturated incompressible porous half space with free surface boundary. A plane wave (P-wave or SV-wave) propagating through the fluid saturated incompressible porous half space making an angle θ_0 with z-axis. Corresponding to each incident wave we get two reflected waves. Boundary conditions for this case reduces to

$$t_{zz}{}^S - p = 0, \quad t_{zx}{}^S = 0, \quad (58)$$

And hence we obtain a system of two non-homogeneous equations which can be written as

$$\sum_{j=1}^{2} a_{ij} Z_j = Y_i, \qquad (i = 1,2) \quad (59)$$

where a_{11}, a_{12}, a_{21}, a_{22} are given by equation (54)

7. Numerical results and discussion

The theoretical results obtained above indicate that the amplitude ratios Z_i $(i = 1,2,3)$ depend on the angle of incidence of incident wave and material properties of half spaces. In order to study in more detail the behaviour of various amplitude ratios, we have computed them numerically for a particular model for which the values of relevant elastic parameters are as follow

In medium M_1,

Following Ewing, Jardetzky and Press (1957), the parameters for inviscid liquid half space are taken as

$$\rho^1 = 1.0\,\text{gm/cm}^3, \quad \lambda^1 = 2.14\,\text{dyne/cm}^2$$

In medium M_2, the physical constants for fluid saturated incompressible porous medium are taken from de Boer, Ehlers and Liu (1993) as

$$\eta^S = 0.67, \quad \eta^F = 0.33, \quad \rho^S = 1.34\,\text{Mg/m}^3, \quad \rho^F = 0.33\,\text{Mg/m}^3,$$

$$\lambda^S = 5.5833\text{MN/m}^2, \qquad K^F = 0.01\text{m/s},$$

$$\gamma^{FR} = 10.00\text{KN/m}^3, \qquad \mu^S = 8.3750\text{N/m}^2, \qquad (60)$$

With these values of constants, we have solved the system of equations given by (52) for different values of angle of incidence from 0 to 90 degrees.

Figures (2)-(4) shows the variation of amplitude ratios of reflected P-wave reflected SV-wave and refracted P-wave respectively when longitudinal wave (P-wave) is made incident. In these figures solid lines show the variations of amplitude ratios when medium-I is incompressible fluid saturated porous medium (FS) and medium-II is liquid half space whereas dotted lines show the variations of amplitude ratios when medium-I becomes empty porous solid (EPS). Figures (5)-(7) depicts the case of incident transverse wave (SV-wave) under similar situations.

Figures (8) and (9) describe the variation of amplitude ratios of reflected P-wave, reflected SV-wave from free surface boundary when longitudinal wave (P-wave) is made incident. In these figures solid lines show the variations of amplitude ratios when medium is incompressible fluid saturated porous medium (FS) whereas dotted lines show the variations of amplitude ratios when the medium becomes empty porous solid (EPS). Figures (10)-(11) depicts the case of incident transverse wave (SV-wave) under the same conditions of free surface.

The effect of fluid filled in the pores of fluid saturated porous medium can be observed from figures (2)-(11). The figures (2)-(8) show that in case of incidence SV-wave, the magnitude values of amplitude ratios is maximum in comparison to incidence P-wave in both the situations either the medium-I is incompressible fluid saturated porous medium (FS) or the medium-I is empty porous solid (EPS).Also in case of incidence P-wave or SV-wave, the amplitude ratios for reflected P-wave $|Z_1|$ and refracted P-wave $|Z_3|$ first increases with the increase in angle of incidence very sharply and goes to a maximum value and there after they start decreasing uniformly to approach to zero .Figure (6) shows that the variation for reflected SV-wave is oscillatory in case of incidence SV-wave.

Figures (8) and (9) show the effect of boundary. In case of incident P-wave, the magnitude values of amplitude rati os for reflected waves are more in case of free surface.

Figures (10) and (11) show that the behaviour of reflected P-wave and reflected SV-wave is oscillatory when SV-wave is incident.

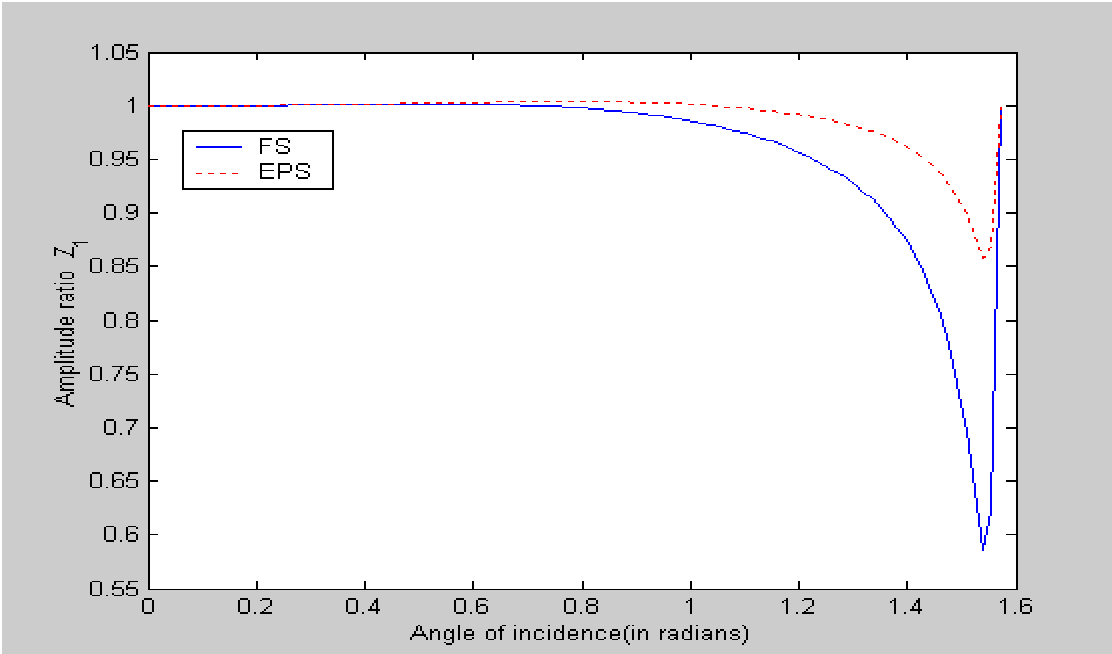

Fig.2 Variation of the amplitude ratio $|Z_1|$ with angle of incidence of the incident P-wave

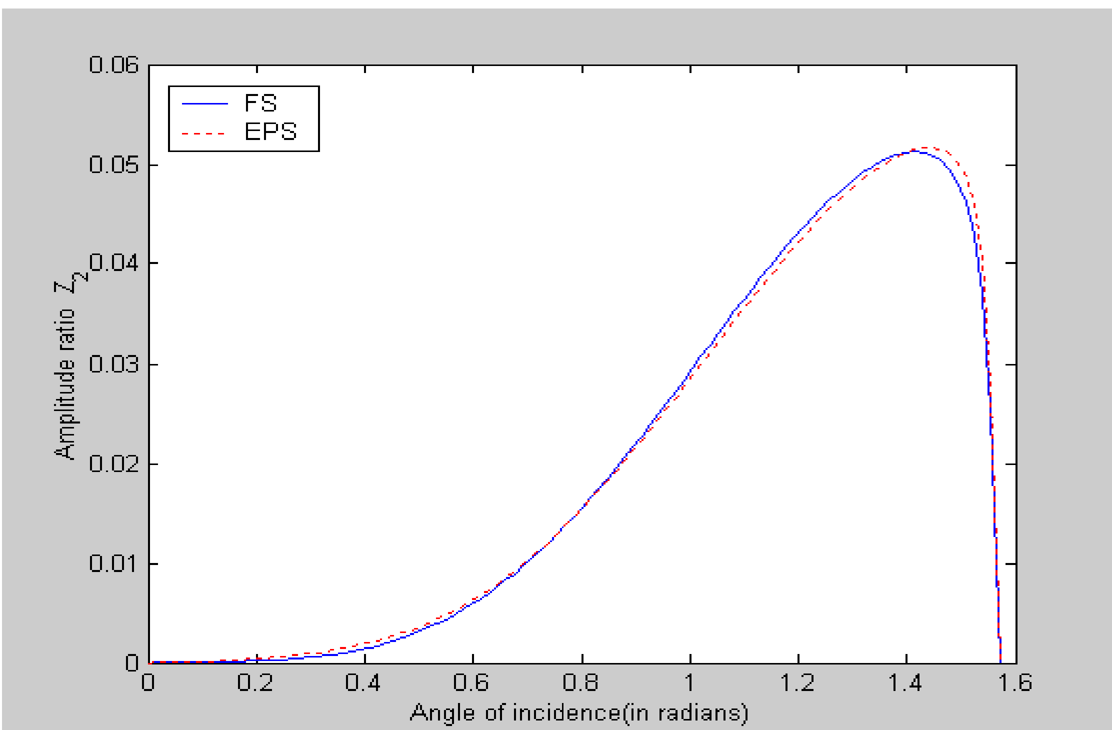

Fig 3 Variation of the amplitude ratio $|Z_2|$ with angle of incidence of the incident P-wave

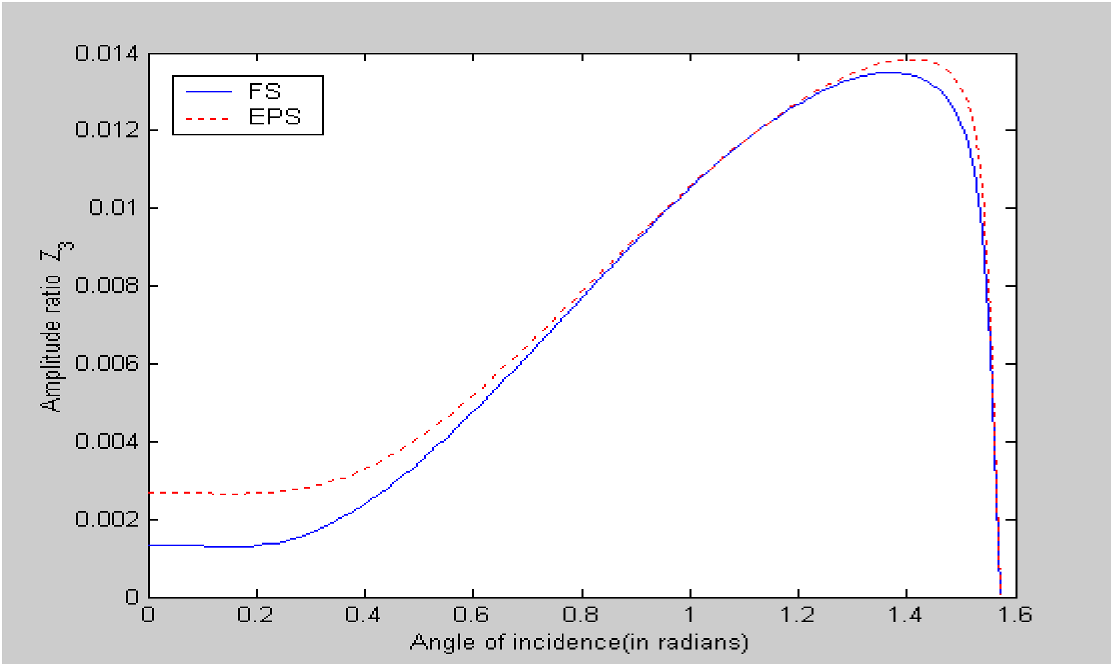

Fig.4 Variation of the amplitude ratio $|Z_3|$ with angle of incidence of the incident P-wave

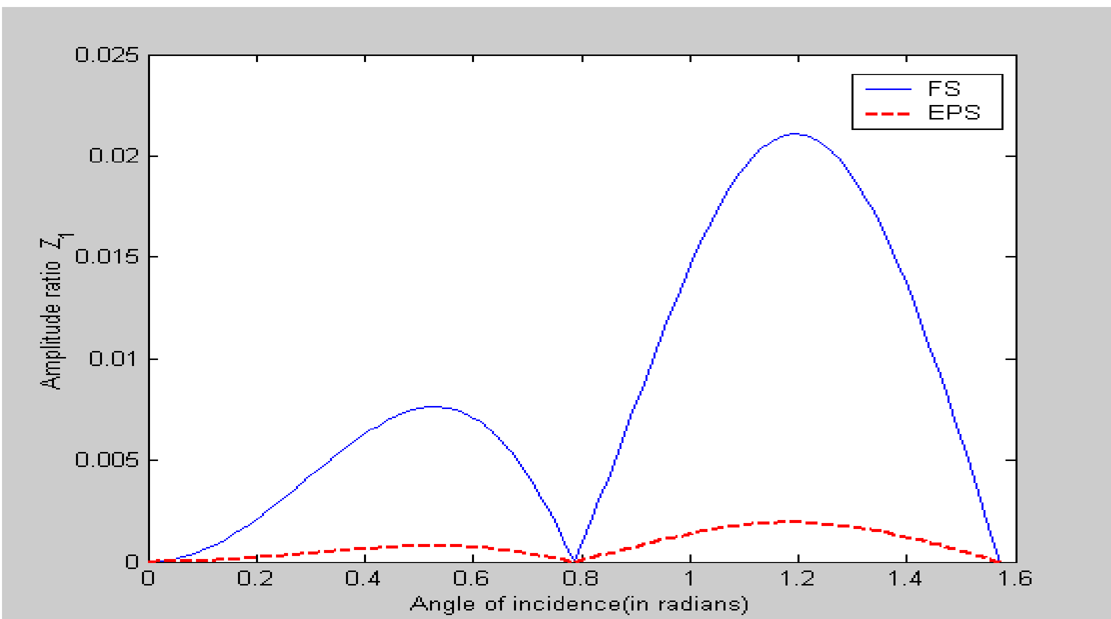

Fig.5 Variation of the amplitude ratio $|Z_1|$ with angle of incidence of the incident SV-wave

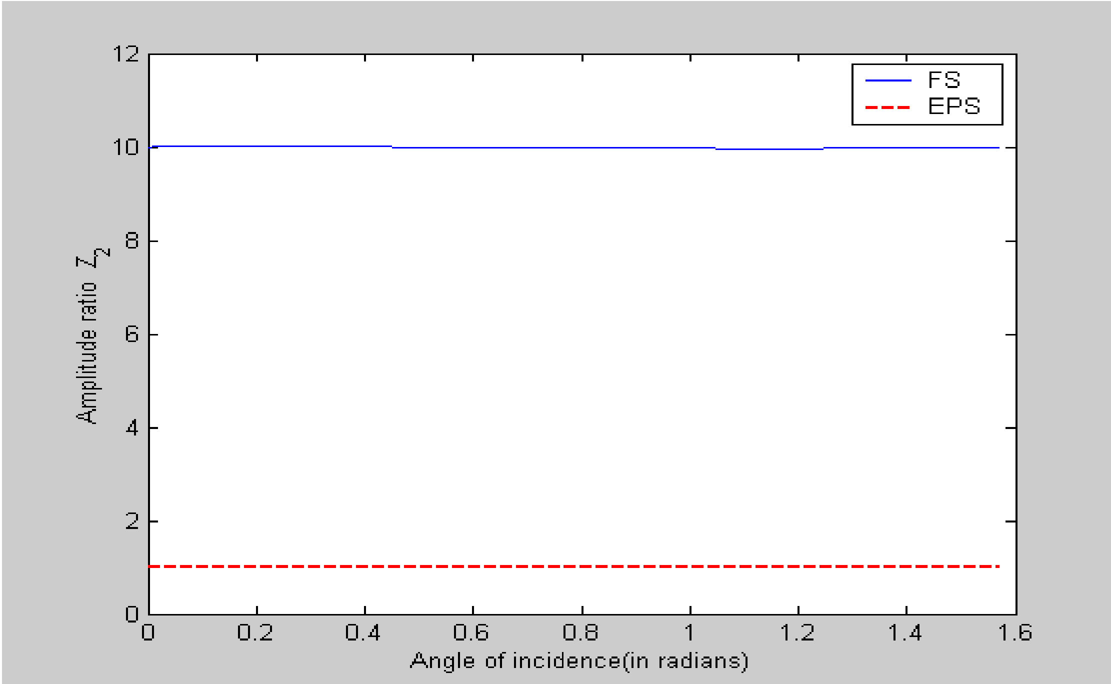

Fig.6 Variation of the amplitude ratio $|Z_2|$ with angle of incidence of the incident SV-wave

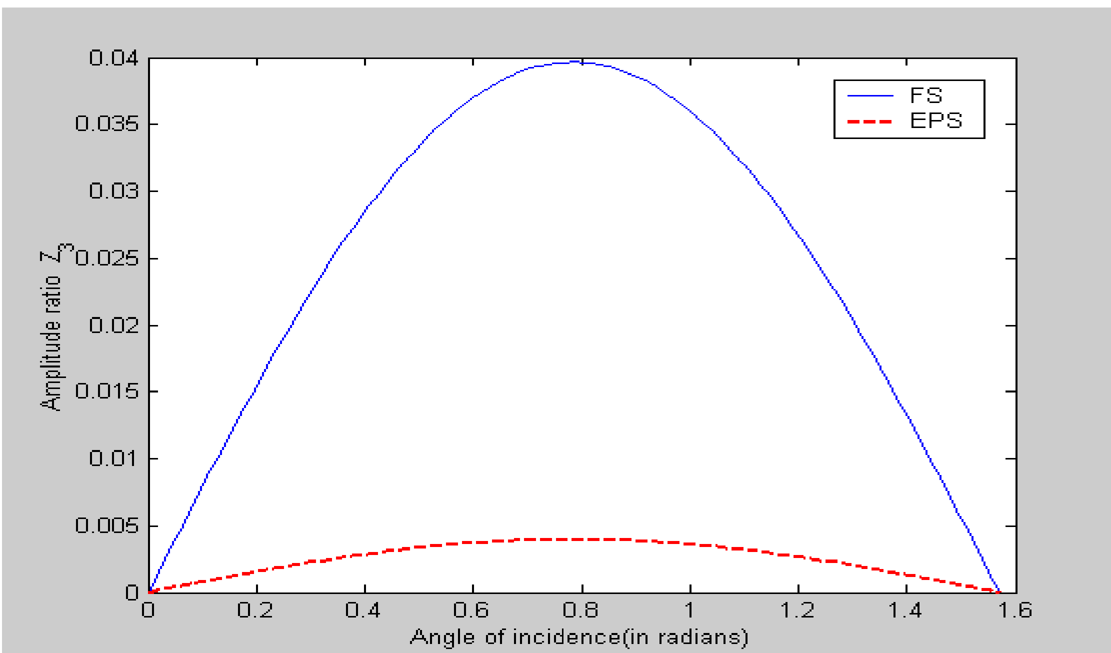

Fig.7 Variation of the amplitude ratio $|Z_3|$ with angle of incidence of the incident SV-wave

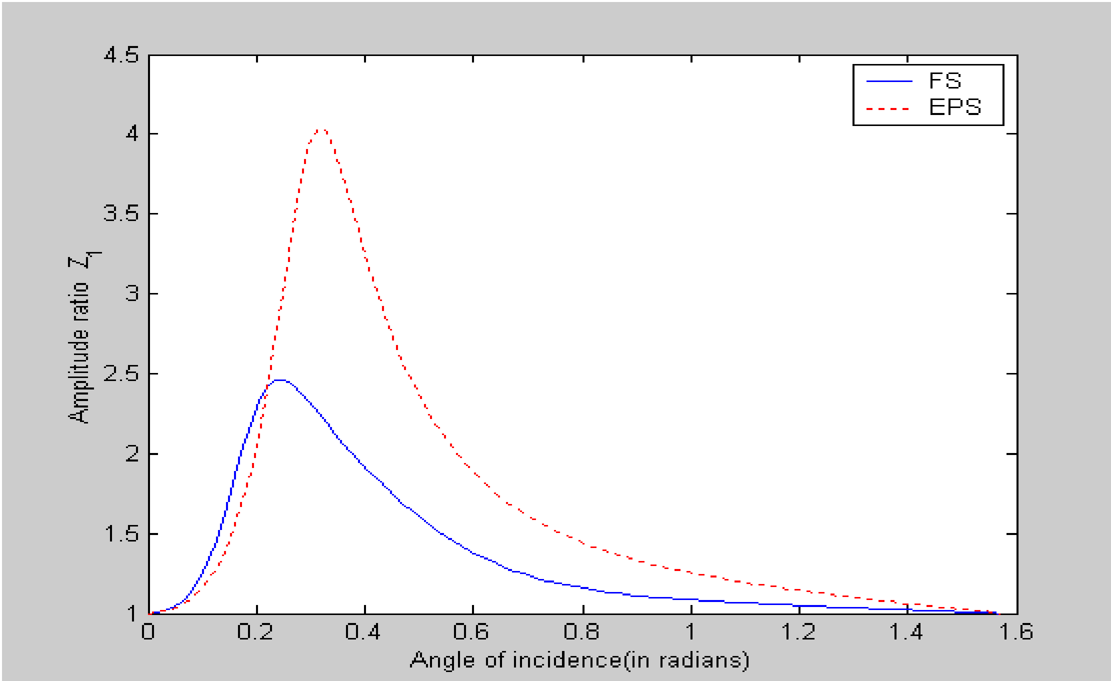

Fig.8 Variation of the amplitude ratio $|Z_1|$ with angle of incidence of the incident P-wave (free surface)

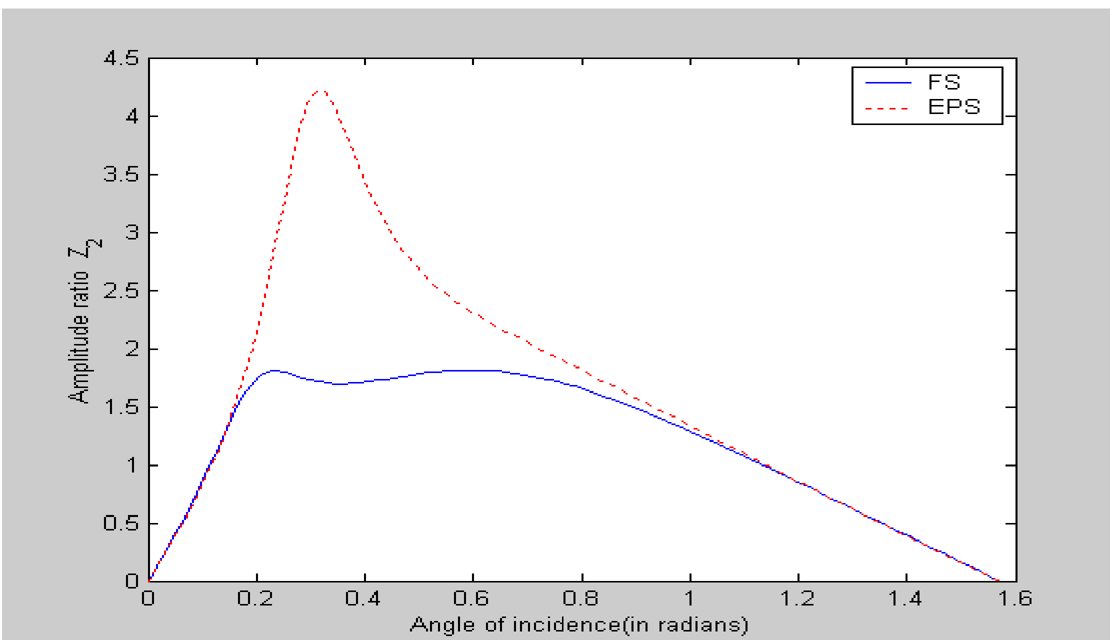

Fig.9 Variation of the amplitude ratio $|Z_1|$ with angle of incidence of the incident P-wave (free surface)

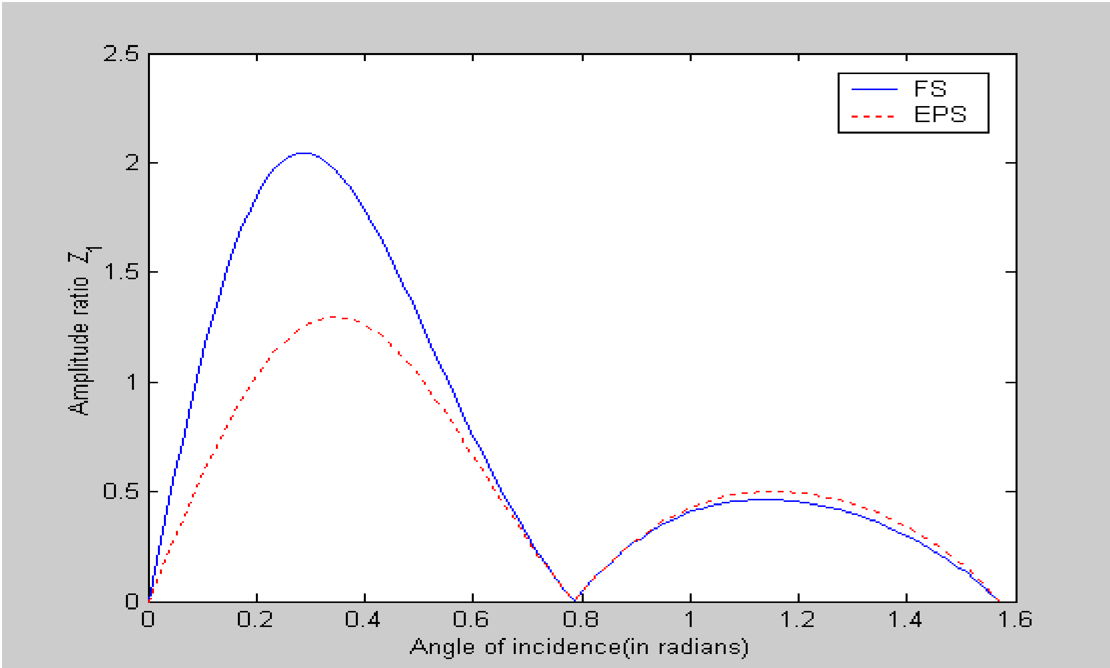

Fig.10 Variation of the amplitude ratio $|Z_1|$ with angle of incidence of the incident SV-wave (free surface)

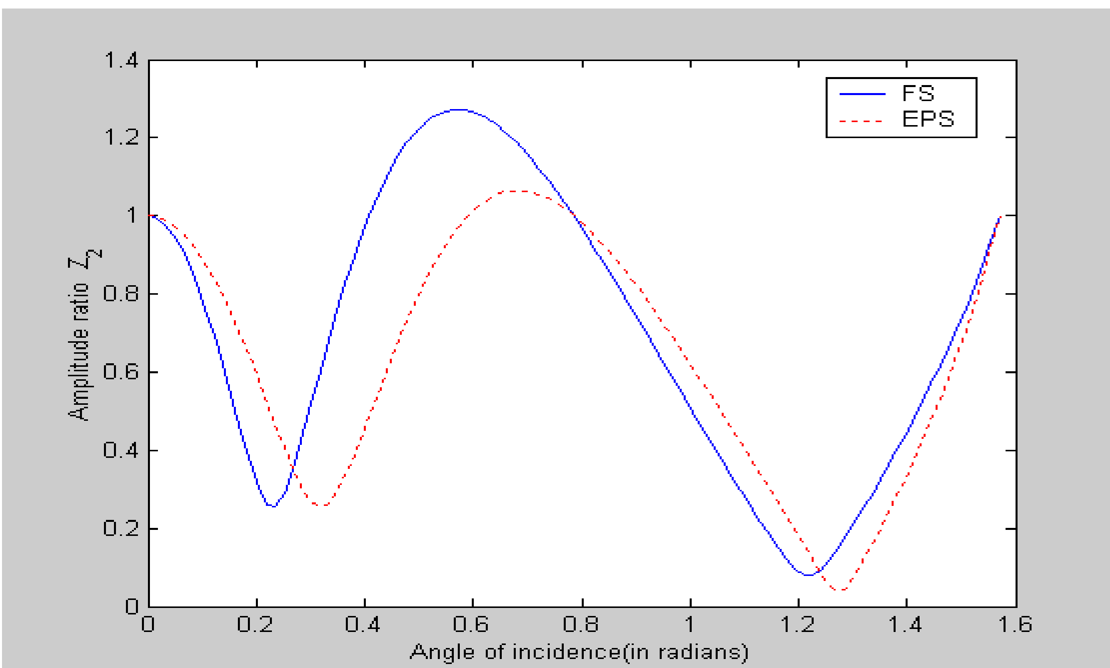

Fig.11 Variation of the amplitude ratio $|Z_1|$ with angle of incidence of the incident SV-wave (free surface)

8. Conclusions

In conclusion, a mathematical study of reflection and refraction coefficients at an interface separating liquid half space and fluid saturated incompressible porous half space is made when longitudinal wave (P-wave) or transverse wave (SV –wave) is incident. It is observed that

(i) The amplitudes ratios of various reflected and refracted waves depend on the angle of incidence of the incident wave and material properties of half spaces.

(ii) The effect of fluid filled in the pores of incompressible fluid saturated porous medium is significant on the amplitudes ratios.

(iii) Significant difference in magnitude values of amplitudes ratios is observed in case of incident P-wave and SV-wave.

(iv) A significant difference in the values of amplitudes ratios corresponding to reflected waves is observed in both the cases (i) when upper half space is present (ii) when upper half space is not present.

The model presented in this paper is one of the more realistic forms of the earth models. Such a model may be found in the earth's crust, and the results of this problem can be applicable to the earth's crust, to mantle-crust interface. It may also be of some use in engineering, seismology and geophysics etc.

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