Cordial Labeling of One Point Union of Some Graphs

G. V. Ghodasara [1], A. H. Rokad [2]
[1] H. & H. B. Kotak Institute of Science,
Rajkot - 360001, Gujarat - INDIA
gaurang_enjoy@yahoo.co.in
[2] Department of Mathematics, School of Engineering, R. K. University,
Rajkot - 360020, Gujarat - INDIA
rokadamit@rocketmail.com

Abstract

A graph G in which a vertex is distinguished from other vertices is called a rooted graph and the vertex is called the root of G. Let G be a rooted graph. The graph $G^{(n)}$ obtained by identifying the roots of n copies of G is called the one-point union of n copies of the graph G. A function from vertex set of a graph to the set $\{0, 1\}$, which assigns the label $|f(u) - f(v)|$ for each edge uv, is called a cordial labeling of the graph if the number of vertices labeled 0 and number of vertices labeled 1 differ by at most 1, and similar condition is satisfied by the edges of the graph. In this paper we discuss cordial labeling of one point union of grid graph, cycle with one chord and cycle with twin chords.
Key words: Cordial graph, One Point Union
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1 Introduction

Let f be a function from vertex set V of a finite, undirected graph G to the set $\{0, 1\}$ and for each edge $e = uv$, assign the label $|f(u) - f(v)|$. Then f is called a cordial labeling of graph G if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1, and similarly the number of edges labeled 0 and the number of edges labeled 1 differ by at most 1. In this paper C_n denotes cycle with n vertices and $P_n \times P_n$ denotes grid graph with n^2 vertices. For graph theoretical terminology and notations we follow Gross and Yellen[5].

2 Literature survey and Previous work

The concept of cordial graphs was introduced by Cahit[1]. Shee and Ho[6] prove that the one-point union of n copies of flag Fl_m (with the common point being the root) is cordial. Selvaraju[7] proved that the one-point union of any number of copies of a complete bipartite graph is cordial. Benson and Lee[8] investigated the regular windmill graphs $K_m^{(n)}$ and determined precisely which ones are cordial for $m < 14$. A dynamic survey of graph labeling is published and updated every year by Gallian[3]. In this paper we prove that the one point union of grid graph, cycle with one chord and cycle with twin chords are cordial graphs.

3 Main Results

Theorem 3.1 The one point union of grid graph $P_n \times P_n$ is cordial.

Proof: Let G be the one point union of k copies $G_1, G_2, \ldots, G_k$ of grid graph $P_n \times P_n$, where $|G_i| = n^2$, $i = 1, 2, \ldots, k$. Let us denote the successive vertices (in clockwise spiral direction) of graph G_i by $\{u_{i1}, u_{i2}, \ldots, u_{in^2}\}$, where u_{i1} is considered as the root vertex of G. Here we define labeling function $f : V(G) \to \{0, 1\}$ as follows.

Case 1: $n \equiv 0, 2(mod 4)$

$f(u_{i1}) = 1$,

$f(u_{1j}) = 0$; if $j \equiv 2, 3(mod4)$

$= 1$; if $j \equiv 0, 1(mod4)$, $2 \le j \le n^2$

Subcase I: i is odd

$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$

$= 1$; if $j \equiv 2, 3(mod4)$, $2 \le j \le n^2$, $2 \le i \le k$

Subcase II: i is even

$f(u_{ij}) = 0$; if $j \equiv 0, 3(mod4)$
$= 1$; if $j \equiv 1, 2(mod4), 2 \leq j \leq n^2, 1 \leq i \leq k$

Case 2: $n \equiv 1, 3(mod4)$
$f(u_{i1}) = 1$

Subcase I: i is odd
$f(u_{ij}) = 0$; if $j \equiv 2, 3(mod4)$
$= 1$; if $j \equiv 0, 1(mod4), 2 \leq j \leq n^2, 1 \leq i \leq k$

Subcase II: i is even
$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4), 2 \leq j \leq n^2, 1 \leq i \leq k$

The labeling pattern defined in above cases satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ in each case which is shown in *Table 1*. Hence the graph under consideration is cordial graph.

Let $n = 4a + b, k = 4c + d$, where $n, k \in N$.

Table 1: Table for Theorem 3.1

b	d	vertex conditions	edge conditions
0,2	0,2	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
	1,3	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1)$
1,3	0,1,2,3	$v_f(0) + 1 = v_f(1)$	$e_f(0) = e_f(1)$

Illustration 3.1 Cordial labeling of one point union of three copies of grid graph $P_4 \times P_4$ is shown in *Fig. 1* as an illustration for the *Theorem 3.1*. It is the case related to $n \equiv 0(mod4)$.

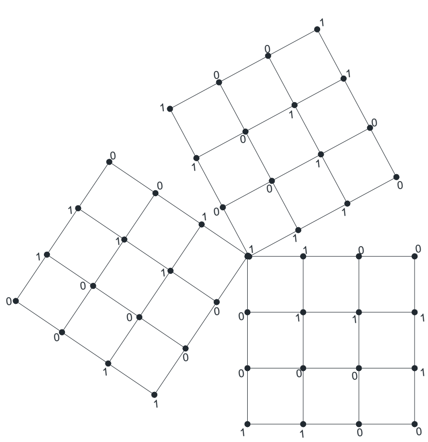

Figure 1: Cordial labeling of one point union of three copies of grid graph $P_4 \times P_4$

Theorem 3.2 The one point union of cycle with one chord is cordial.

Proof: Let G be the one point union of k copies $G_1, G_2, \ldots, G_k$ of cycle C_n with one chord. Let $u_{i1}, u_{i2}, \ldots, u_{in}$ denote the vertices of G_i and let $e_i = u_{i2}u_{in}$ be the chord in G_i, $i = 1, 2, \ldots, k$. Here u_{i1} is considered as the root vertex of G, $i = 1, 2, \ldots, k$.

To define labeling function $f : V(G) \rightarrow \{0, 1\}$ we consider following cases.

Case 1: $n \equiv 0(mod4)$
$f(u_{i1}) = 1$

Subcase I: i is odd, $1 \leq i \leq k$
$f(u_{ij}) = 0$; if $j \equiv 2, 3(mod4)$
$= 1$; if $j \equiv 0, 1(mod4), 2 \leq j \leq n$

Subcase II: i is even, $1 \leq i \leq k$
$f(u_{in}) = 1$
$f(u_{ij}) = 0$; if $j \equiv 0, 3(mod4)$
$= 1$; if $j \equiv 1, 2(mod4), 2 \leq j \leq n - 1$

Case 2: $n \equiv 1(mod4)$
$f(u_{i1}) = 1$

Subcase I: i is odd, $1 \leq i \leq k$
$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4), 2 \leq j \leq n$

Subcase II: i is even, $1 \leq i \leq k$
$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4), 2 \leq j \leq n$

Case 3: $n \equiv 2(mod4)$
$f(u_{i1}) = 1$

Subcase I: i is odd, $1 \leq i \leq k$
$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4), 2 \leq j \leq n$

Subcase II: i is even, $1 \leq i \leq k$
$f(u_{in}) = 0$
$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4), 2 \leq j \leq n - 1$

Case 4: $n \equiv 3(mod4)$
$f(u_{i1}) = 1$
$f(u_{1j}) = 0$; if $j \equiv 2, 3(mod4)$
$= 1$; if $j \equiv 0, 1(mod4), 2 \leq j \leq n$

For $2 \leq i \leq k$:
$f(u_{in-1}) = 1$,
$f(u_{ij}) = 0$; if $j \equiv 2, 3(mod4)$
$= 1$; if $j \equiv 0, 1(mod4), 2 \leq j \leq n, j \neq n - 1$

The labeling pattern defined above satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ in each case which is shown in *Table 2*. Hence the graph under consideration is cordial graph.

Let $n = 4a + b, k = 4c + d$, where $n, k \in N$.

Table 2: Table for the graph G in Theorem 3.2

b	d	vertex conditions	edge conditions
0	0,2	$v_f(0) + 1 = v_f(1)$	$e_f(0) = e_f(1)$
	1,3	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$
1,3	0,1,2,3	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
2	0,2	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
	1,3	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$

Illustration 3.2 Cordial labeling of one point union of three copies of cycle C_5 with one chord is shown in *Fig. 2* as an illustration for the *Theorem 3.2*. It is the case related to $n \equiv 1(mod4)$.

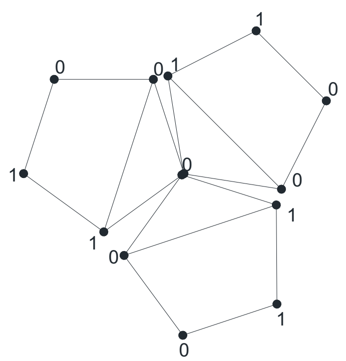

Figure 2: Cordial labeling of one point union of three copies of cycle C_5 with one chord

Theorem 3.3 The one point union of cycle with twin chords is cordial.

Proof: Let G be the one point union of k copies $G_1, G_2, \ldots, G_k$ of cycle C_n with twin chords. Let $u_{i1}, u_{i2}, \ldots, u_{in}$ denotes the vertices of G_i, $i = 1, 2, \ldots, k$. Let $e_i = u_{i2}u_{in}$ and $e'_i = u_{i3}u_{in}$ be the chords in G_i, $i = 1, 2, \ldots, k$. Here u_{i1} is considered as the root vertex of G_i, $i = 1, 2, \ldots, k$. To define labeling function $f : V(G) \to \{0, 1\}$ we consider following cases.

Case 1: $n \equiv 0(mod4)$
$f(u_{i1}) = 1$
$f(u_{1j}) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4)$, $2 \leq j \leq n$
Subcase I: i is odd, $2 \leq i \leq k$
$f(u_{in-1}) = 0$,
$f(u_{ij}) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4)$, $2 \leq j \leq n, i \neq n-1$
Subcase II: i is even, $2 \leq i \leq k$
$f(u_{ij}) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4)$, $2 \leq j \leq n$
Case 2: $n \equiv 1(mod4)$
$f(u_{i1}) = 1$
Subcase I: i is odd, $1 \leq i \leq k$
$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4)$, $2 \leq j \leq n$
Subcase II: i is even, $1 \leq i \leq k$
$f(u_{ij}) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4)$, $2 \leq j \leq n$
Case 3: $n \equiv 2(mod4)$
$f(u_{i1}) = 1$
Subcase I: i is odd, $1 \leq i \leq k$
$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4)$, $2 \leq j \leq n$
Subcase II: i is even, $1 \leq i \leq k$
$f(u_{in}) = 0$
$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4)$, $2 \leq j \leq n-1$
Case 4: $n \equiv 3(mod4)$
$f(u_{i1}) = 1$
Subcase I: i is odd, $1 \leq i \leq k$
$f(u_{ij}) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4)$, $2 \leq j \leq n$
Subcase II: i is even, $1 \leq i \leq k$
$f(u_{in-1}) = 0$
$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4)$, $2 \leq j \leq n, j \neq n-1$

The labeling pattern defined in above cases satisfies the conditions of cordial labeling which is shown in *Table 3*. Hence the graph under consideration is cordial graph.
Let $n = 4a + b, k = 4c + d$, where $n, k \in N$.

Table 3: Table for Theorem 3.3

b	d	vertex conditions	edge conditions
0	0,2	$v_f(0) + 1 = v_f(1)$	$e_f(0) = e_f(1)$
	1,3	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1)$
1	0,1,2,3	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
2	0,2	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
	1,3	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1)$
3	0,2	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
	1,3	$v_f(0) = v_f(1) + 1$	$e_f(0) + 1 = e_f(1)$

Illustration 3.3 Cordial labeling of one point union of three copies of cycle C_5 with twin chords is shown in *Fig. 3* as an illustration for *Theorem 3.3*. It is the case related to $n \equiv 1(mod4)$.

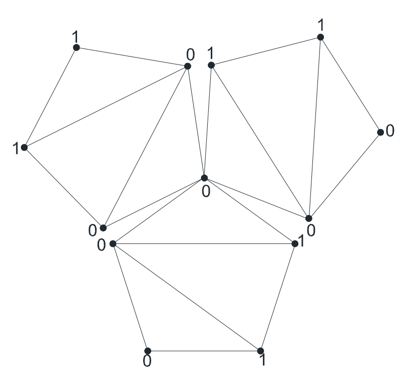

Figure 3: Cordial labeling of one point union of three copies of cycle C_5 with twin chord

4 Conclusion

We have discussed cordiality of grid, cycle with one chord, cycle with twin chords in context of one point union of graphs. We contribute three new graph families in the theory of cordial graphs.

References

[1] I. Cahit, ”Cordial Graphs: A weaker version of graceful and Harmonic Graphs”, Ars Combinatoria, 23(1987) 201-207.

[2] I. Cahit, ”On cordial and 3-equitable labellings of graphs”, Util. Math., 37(1990) 189-198.

[3] J. A. Gallian, ”A dynemic survey of graph labeling”, The Electronics Journal of Combinatorics, 16(2013), $\sharp DS6$ $1 - 308$.

[4] G. V. Ghodasara, A. H. Rokad and I. I. Jadav, ”Cordial labeling of grid related graphs”, International Journal of Combinatorial Graph Theory and Applications, 6(2013) 55-62.

[5] Jonathan L. Gross and Jay Yellen, ”Graph Theory and Its Applications, Second Edition”, CRC Press, 1998.

[6] S. C. Shee and Y. S. Ho, The cordiality of one-point union of n-copies of a graph, Discrete Math., 117 (1993) 225-243.

[7] P. Selvaraju, New classes of graphs with α-valuation, harmonious and cordial labelings, Ph. D. Thesis, Anna University, 2001. Madurai Kamaraj University, 2002.

[8] M. Benson and S. M. Lee, On cordialness of regular windmill graphs, Congress. Numer., 68 (1989) 45-58.