Special $D(k^2+1)$ Dio-quadruple Involving Jacobsthal Lucas and Thabit-ibn-kurrah Numbers

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ABSTRACT

We search for three distinct integers a,b,c such that product of any two from the set added with k-times their sum and increased by k^2+1 is a perfect square.Also, we show that the triple can be extended to the quadruple with property $D(k^2+1)$

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KEYWORDS: Diophantine m-tuples, Thabit-ibn-kurrah number,Kynea number ,Jacobsthal lucas number,Pellian equation

NOTATIONS

TK_n : Thabit-ibn-kurrah number of rank n

j_n : Jacobsthal Lucas number of rank n

Ky_n : Kynea number of rank n

1. INTRODUCTION

Let n be an integer. A set of m positive integers $(a_1,a_2,a_3,....a_m)$ is said to have the property $D(n)$,if a_ia_j+n is a perfect square for all $1\le i<j\le m$ .Such a set is called a diophantine m-tuple or P_n set of size m.The problem of construction of such set was studied by Diophantus

Many mathematicians considered the construction of different formulations of Diophantine triples,quadruple and quintuples with property $D(n)$ for any arbitrary integer n and also,for any linear polynomials in n. In this context,one may refer $[1-21]$ for an extensive review of various problems on Diophantine triples,quadruple and quintuples

This paper aims using at constructing special dio-quadruple where the product of any two members of the quadruple with the addition of k-times their sum and increased by k^2+1 satisfies the required property.

II. METHOD OF ANALYSIS

Let $a=3j_{2n}-4$, $b=TK_{2n}+2$ be any two integers such that $ab+ka+kb+k^2+1$ is a perfect square.

Let $C_s(n)$ be any non-zero integer such that

$$(3j_{2n}-4+k)C_s(n)+(3j_{2n}-4)k+k^2+1=\alpha_s^2 \qquad (1)$$

$$(TK_{2n}+k+2)C_s(n)+(TK_{2n}+2)+k^2+1=\beta_s^2 \qquad (2)$$

Eliminating $C_s(n)$ between (1) and (2), we get

$$(TK_{2n}+k+2)\alpha_s^2-(3j_{2n}+k-4)\beta_s^2=2 \qquad (3)$$

Introducing the linear transformations

$$\left.\begin{aligned}\alpha_s&=X_s+(3j_{2n}+k-4)T_s\\ \beta_s&=X_s+(TK_{2n}+k+2)T_s\end{aligned}\right\} \qquad (4)$$

in (3),we get

$$X_s^2=(3j_{2n}+k-4)(TK_{2n}+k+2)T_s^2+1 \qquad (5)$$

This is a well known pellian equation,whose general solution is given by

$$\left.\begin{aligned}X_s&=\frac{1}{2}\left\{\left[3(j_{2n}-1)+k+\sqrt{D}\right]^{s+1}+\left[3(j_{2n}-1)+k-\sqrt{D}\right]^{s+1}\right\}\\ T_s&=\frac{1}{2\sqrt{D}}\left\{\left[3(j_{2n}-1)+k+\sqrt{D}\right]^{s+1}-\left[3(j_{2n}-1)+k-\sqrt{D}\right]^{s+1}\right\}\end{aligned}\right\} \qquad (6)$$

where $D=(3j_{2n}+k-4)(TK_{2n}+k+2)$, $s=-1,0,1,2.......$

Taking $s=0$ in (6) and using (1),we get

$$C_0(n)=12j_{2n}+3k-12$$

Note that $(a,b,C_0(n))$ is the special Diophantine triple with property $D(k^2+1)$

Now, substituting $s=1$ in (6) we have

$$\left.\begin{aligned}X_1&=[3(j_{2n}-1)+k]^2+D\\ T_1&=2[3(j_{2n}-1)+k]\end{aligned}\right\} \qquad (7)$$

and using (1),we see that

$$C_1(n)=16(TK_{2n}+k+1)^3-12j_{2n}-5k+12$$

Thus,we obtain

$(3j_{2n}-4, TK_{2n}+2, 12j_{2n}+3k-12, 16(TK_{2n}+k+1)^3-12j_{2n}-5k+12)$ as a diophantine quadruple with the property $D(k^2+1)$

II.1 :Properties:

1. $4[C_1(n)+C_0(n)+2k]$ is a cubic integer

2.Each of the following is a nasty number

(a) $3[a(n)+b(n)+C_0(n)-3k]$

(b) $6[4b(n)-C_0(n)+3k]$

3. $C_0(n)-4b(n)-3k\equiv 0(\bmod 4)$

4. $a(n)+b(n)+C_1(n)+6j_{2n}+5k\equiv 0(\bmod 2)$

5. $a(n)b(n)+C_0(n)-9Ky_{2n}+2Tk_{2n}-3k\equiv 0(\bmod 2)$

Again,taking $s = 2,3$ in(6) and using (1),we get

$C_2(n) = 64[(3(j_{2n}-1)+k]^5 - 48[Tk_{2n}+k+1]^3 + 8[(3(j_{2n}-1)+k] - k$

$C_3(n) = 256[(3(j_{2n}-1)+k]^7 - 320[Tk_{2n}+k+1]^5 + 112[(3(j_{2n}-1)+k]^3 - 8[Tk_{2n}+k+1] - k$ It is seen that $[a,b,C_1(n),C_2(n)]$ represents special $D(k^2+1)$ dio-quadruple

Repeating the above process,it is observed that $[a,b,C_{m-1}(n),C_m(n)], m = 1,2,3......$ represents special dio-quadruple with the property $D(k^2+1)$ .A few illustrations are given below:

(n,k)	$(a,b,C_0(n),C_1(n))$	$(a,b,C_1(n),C_2(n))$	$(a,b,C_2(n),C_3(n))$
(1,1)	(11,13,51,35099)	(11,13,35099,23657399)	(11,13,23657399,15945052551)
(1,2)	(11,13,54,43846)	(11,13,43846,34289106)	(11,13,34289106,26814060558)
(1,3)	(11,13,57,53937)	(11,13,53937,48438117)	(11,13,48438117,43497377877)
(2,2)	(47,49,198,1999798)	(47,49,1999798,19994000398)	(47,49,19994000398,199900013999598)
(2,3)	(47,49,201,2122209)	(47,49,2122209,22075249221)	(47,49,22075249221,229626740306037)

3. CONCLUSION

In the construction of the special dio-quadruple we have assumed the product ab added with k-times their sum and increased by k^2+1 is a perfect square.One may search for special dio-quadruples consisting of other special numbers with suitable property.

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REFERENCES

1. Brown E,sets in which $xy + k$ is always a square,Math,Comp.45,613-620,1985

2. Bashmakova I.G,(Ed),Diophantus of Alexandria,Arithmetics and the Book of polygonal numbers,Nauka,Moscow,1974

3. Bugeaud Y Dujella A and Mignotte,On the family of Diophantine triples , $(k-1, k+1, 16k^3 - 4k)$ Glasgow Math.J.49 333- 344,2007.

4. Dujella A,On Diophantine Quintuple,Acta Arith.81 69-79,1997.

5. Filipin A,An irregular D(4) quadruple cannot be extended to a quintuple,Acta Arith.136, 167-176,2009

6. Filipin A Fujita Y,The number of Diophantine Quintuples II.Pupl.Math.Debrecen 82 293-308.2013.

7. Fujita Y,The number of Diophantine Quintuples,Glas.Mat.Ser.III 45 15-29,2010.

8.Gopalan M A Srividhya .G,Some non-extentable P_{-5} sets ,Diophantus J.Math Vol 1 issue1 19-22,2012.

9. Gopalan M A Srividhya G,Two special Diophatine Triples ,Diophantus J.Math Vol 1 issue1 23-27,2012.

10. Gopalan M A and Pandichelvi V ,On the extendibility of the Diophantine triple involving Jacobsthal numbers $(J_{2n-1}, J_{2n+1} - 3, 2J_{2n} + J_{2n-1} + J_{2n+1} - 3)$, Indian Journal of Mathematical and Computing Applications vol 5 issue 2 83-85,2013.

11.Gopalan M A Sumathi G and Vidhyalakshmi S,Special Dio-quadruple involving jacobsthal and Jacobsthal lucas number with the Property $D(k^2+1)$,International J.of .Math.Sci and Engg.Appls Vol 8 NoIII pp 221-225,2014.

12. Gopalan M AVidhyalakshmi.S Premalatha E and Presenna R,On the extendibility of 2 tuple to 4 tuple with property $D(4)$,Bulletin of Mathematical Sciences and Applications Vol 3, No 2 pp 100-104,2014.

13.Gopalan M A Mallika S and Vidhyalakshmi S ,On the extendibility of the Gaussian Diophantine 2-tuple to Gaussian Diophantine Quadruple with property $D(1)$,International journal of Innovation in science and Mathematics, Vol 2, issue 3 pp 310-311,2014.

14. Jukic Matic Lj.,Non-existence of certain Diophantine Quadruples in rings of integers of pure cubic fields,Proc.Japan,Acad.Ser.A.Math.Sci.88 163-167, 2012.

15. Meena K Vidhyalakshmi S Gopalan M A Akila G and Presenna R ,Formation of Special Diophantine Quadruples with property $D(6kpq)^2$,The International Journal of Science and Technoledge Vol 2,Issue 2 11-14, 2014.

16.Rich W,Regular Diophantine m-tuple and their extension,Phd Thesis,Central Michigan University, 2012.

17.Vidhyalakshmi S Gopalan M A and Lakshmi K,Gaussian Diophantine Quadruples with property $D(1)$,International organization of Scientific Research,Vol 10, issue 3 Ver II pp 12-14, (May-june2014).

18. web.math.pmf.unizg.hr/~duje/dtuples.html

19. Dujella A,On Diophantine quintuples,Acta Arith.81,69-79, 1997

20. Gopalan M A Sumathi G and Vidhyalakshmi S,Diophantine Quadruple involving Jacobsthal lucas number and Thabit-ibn-kurrah number with the Property $D(1)$,International Journal of InnovativeResearch and Review,Vol 2(2),pp 47-50, (2014).

21. M A Gopalan Vidhyalakshmi S and A.Kavitha ,Special Dio-Quadruples with the Property $D(2)$,International Journal of Science and Technology Vol.2,Issue 5,pp 51-52, (May2014).