Square Difference Labeling of Some Union Graphs

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Abstract: Let $G(V,E)$ be a graph with p vertices and q edges. A (p,q) graph $G(V,E)$ is said to be a square difference graph if there exists a bijection $f: V(G) \to \{0,1,2,..., p-1\}$ such that the induced function $f^*: E(G) \to N$, N is a natural number, given by $f^*(uv) = |[f(u)]^2 - [f(v)]^2|$ for every edges uv in G and are all distinct and the function f is a called Square difference labeling of the graph G. In this paper, we prove $P_m \cup P_n$, $P_m \cup C_n$, $P_m \cup S_n$, $P_m \cup (C_n \Theta K_1)$, $(P_m \Theta K_1) \cup P_n$, $(P_m \Theta K_1) \cup C_n$, $(P_m \Theta K_1) \cup S_n$, $(P_m \Theta K_1) \cup L_n$ $(P_m \Theta K_1) \cup (P_n \Theta K_1)$, $(P_m \Theta K_1) \cup (C_n \Theta K_1)$ and $(P_m \Theta K_1) \cup (L_n \Theta K_1)$ are the square difference graphs.

Key Words: Square difference labeling, Square difference graph.

Mathematical subject classification (2010):05C78

1. INTRODUCTION

All graphs in this paper are finite, simple and undirected graphs. Let (p,q) be a graph with $p = |V(G)|$ vertices and $q = |E(G)|$ edges. Graph labeling ,where the vertices and edges are assigned real values or subsets of a set are subject to certain conditions. A detailed survey of graph labeling can be found in [2]. Terms not defined here are used in the sense of Harary in [3].There are different kinds of labelings in the graph labeling such as Graceful, Harmonious, Cordial, Fibonacci, Square sum, etc. The concept of square difference labeling was first introduced in [1] and some results on square difference labeling of graphs are discussed in [1,4]. In this paper we investigate some more graphs for square difference labeling. We use the following definitions in the subsequent sections.

Definition 1.1[1]: A graph G(p, q) is said to be a square difference graph if there exists a bijection f : V(G) →{0,1,2,…, p-1} such that the induced function f*: E(G) → N given by $f^*(uv)=|[f(u)]^2 - [f(v)]^2|$ for every edges uv in G and are all distinct and the function f is a called Square difference labeling of the graph G.

Definition 1.2[2]: The corona $G_1 \Theta G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1(which has p points) and p copies of G_2 and then joining the ith point of G_1 to every point in the ith copy of G_2.

Definition 1.3[2]: A complete bipartite graph $K_{1,n}$ is called a star and it has $n+1$ vertices and n edges and also it is denoted as S_n.

2. Main Results

Theorem 2.1: The graph $P_m \cup P_n$ is a square difference graph.

Proof: Let P_m be the path graph with m vertices and $m-1$ edges. Let P_n be the path graph with n vertices and $n-1$ edges. Let $V(P_m) = \{u_i : 1 \le i \le m\}$.

Let $V(P_n)=\{v_i : 1\le i\le n\}$. Therefore $V(P_m\cup P_n)=\{u_i : 1\le i\le m\ ; v_i : 1\le i\le n\}$.

Let $E(P_m)=\{u_iu_{i+1} : 1\le i\le m-1\}$.

Let $E(P_n)=\{v_iv_{i+1} : 1\le i\le n-1\}$.

Then $E(P_m\cup P_n)=\begin{cases} u_iu_{i+1} : 1\le i\le m-1 \\ v_iv_{i+1} : 1\le i\le n-1 \end{cases}$.

Then we have $|V(P_m\cup P_n)|=m+n$ and $|E(P_m\cup P_n)|=m+n-2$.

Define a bijection from the vertices of $P_m\cup P_n$

to $\{0,1,2,...,m+n-1\}$ as follows:

$f(u_i)=i-1$ if $1\le i\le m$;
$f(v_i)=m-1+i$ if $1\le i\le n$.

Let f* be the induced edge labeling of f.

The induced edge labels by f* as follows: $f^*(u_iu_{i+1})=2i-1$ if $1\le i\le m-1$;
$f^*(v_iv_{i+1})=2m-1+2i$ if $1\le i\le n-1$.

Theorem 2.2: Any graph $P_m\cup C_n$ admits a square difference labeling.

Proof: Let P_m be the path graph with m vertices and $m-1$ edges. Let C_n be the cycle graph with n vertices and n edges. Let $V(P_m)=\{u_i : 1\le i\le m\}$.

Let $V(C_n)=\{v_i : 1\le i\le n\}$. Therefore $V(P_m\cup C_n)=\{u_i : 1\le i\le m\ ;\ v_i : 1\le i\le n\}$
Let $E(P_m)=\{u_iu_{i+1} : 1\le i\le m-1\}$.

Let $E(C_n)=\{v_iv_{i+1} : 1\le i\le n-1 ; v_1v_n\}$.

Therefore
$E(P_m\cup C_n)=\begin{cases} u_iu_{i+1} : 1\le i\le m-1 \\ v_iv_{i+1} : 1\le i\le n-1 ; v_1v_n \end{cases}$

Then we have $|V(P_m\cup C_n)|=m+n$ and $|E(P_m\cup C_n)|=m+n-1$. Define a bijection from the vertices of $P_m\cup C_n$ to $\{0,1,2,....,m+n-1\}$

as follows: $f(u_i)=i-1$ if $1\le i\le m$;
$f(v_i)=m-1+i$ if $1\le i\le n$.

Let f* be the induced edge labeling of f.

The induced edge labels by f* as follows:

$f^*(u_iu_{i+1})=2i-1$ if $1\le i\le m-1$;
$f^*(v_iv_{i+1})=2m-1+2i$ if $1\le i\le n-1$;

$f^*(v_1v_n)=(n-1)(2m+n-1)$.

Theorem 2.3: All the graph $P_m\cup S_n$ is a square difference graph.

Proof: Let P_m be the path graph with m vertices and $m-1$ edges. Let S_n be the star graph with $n+1$ vertices and n edges. Let $V(P_m)=\{u_i : 1\le i\le m\}$.

Let $V(S_n)=\{v_i : 1\le i\le n+1\}$.Therefore $V(P_m\cup S_n)=\{u_i : 1\le i\le m\ ;\ v_i : 1\le i\le n+1\}$
Let $E(P_m)=\{u_iu_{i+1} : 1\le i\le m-1\}$.

Let $E(S_n)=\{v_iv_{n+1} : 1\le i\le n\}$.Therefore $E(P_m\cup S_n)=\{u_iu_{i+1} : 1\le i\le m-1 ; v_iv_{n+1} : 1\le i\le n\}$
Then we have $|V(P_m\cup S_n)|=m+n+1$ and $|E(P_m\cup S_n)|=m+n-1$. Define a bijection from the vertices of $P_m\cup S_n$ to $\{0,1,2,....,m+n\}$

as follows: $f(u_i)=i-1$ if $1\le i\le m$;
$f(v_i)=m-1+i$ if $1\le i\le n+1$.

Let f* be the induced edge labeling of f. The induced edge labels by f* as follows:

$f^*(u_iu_{i+1})=2i-1$ if $1\le i\le m-1$;

$f^*(v_iv_{n+1})=(2m+n-1+i)(n+1-i)$ if $1\le i\le n$

Theorem 2.4: Every graph $P_m \cup (C_n \Theta K_1)$ is a square difference graph.

Proof: Let P_m be the path graph with m vertices and $m-1$ edges. Let $C_n \Theta K_1$ be the crown graph with $2n$ vertices and $2n$ edges.

Let $V(P_m)=\{u_i : 1 \le i \le m\}$.

Let $V(C_n \Theta K_1)=\{v_i , w_i : 1 \le i \le n\}$.

Therefore $V(P_m \cup (C_n \Theta K_1)) = \begin{cases} u_i : 1 \le i \le m \\ v_i , w_i : 1 \le i \le n \end{cases}$

Let $E(P_m)=\{u_i u_{i+1} : 1 \le i \le m-1\}$.

Let $E(C_n \Theta K_1) = \begin{cases} v_1 v_n \; ; v_i v_{i+1} : 1 \le i \le n-1 \\ v_i w_i : 1 \le i \le n \end{cases}$

Therefore

$$E(P_m \cup (C_n \Theta K_1)) = \begin{cases} u_i u_{i+1} : 1 \le i \le m-1 \\ v_i v_{i+1} : 1 \le i \le n-1 \\ v_1 v_n \; ; \; v_i w_i : 1 \le i \le n \end{cases}$$

Then we have $|V(P_m \cup (C_n \Theta K_1))| = m+2n$ and $|E(P_m \cup (C_n \Theta K_1))| = m+2n-1$.

Define a bijection from the vertices of $P_m \cup (C_n \Theta K_1)$ to $\{0,1,2,....,m+2n-1\}$ as follows:

$f(u_i) = i-1 \quad if \; 1 \le i \le m$;
$f(v_i) = m-1+i \quad if \; 1 \le i \le n$;

$f(w_i) = m+n-1+i \; if \; 1 \le i \le n$.

Let f* be the induced edge labeling of f.

The induced edge labels by f* as follows:

$f^*(u_i u_{i+1}) = 2i-1 \;\; if \; 1 \le i \le m-1$;

$f^*(v_i v_{i+1}) = (2m-1+2i) \;\; if \; 1 \le i \le n-1$;

$f^*(v_1 v_n) = (n-1)(2m+n-1)$;
$f^*(v_i w_i) = n(2m+n-2+2i) \;\; if \; 1 \le i \le n$.

Example 2.5: A square difference labeling of $P_7 \cup (C_{10} \Theta K_1)$ is shown in the Figure 2.1.

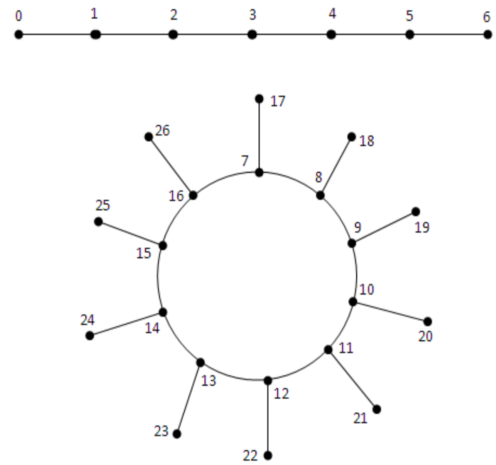

Figure 2.1

Theorem 2.6: The graph $(P_m \Theta K_1) \cup P_n$ is a square difference graph.

Proof: Let $P_m \Theta K_1$ be the comb graph with $2m$ vertices and $2m-1$ edges. Let P_n be the path graph with n vertices and $n-1$ edges.

Let $V(P_m \Theta K_1) = \{u_i \; , \; w_i \; : 1 \le i \le m\}$.

Let $V(P_n) = \{v_i \; : 1 \le i \le n\}$. Therefore

$$V((P_m \Theta K_1) \cup P_n) = \begin{cases} u_i \; , \; w_i \; : 1 \le i \le m \\ v_i \; : 1 \le i \le n \end{cases}$$

Let $E(P_m \Theta K_1)) = \begin{cases} u_i u_{i+1} \; : 1 \le i \le m-1 \\ u_i w_i \; : 1 \le i \le m \end{cases}$.

Let $E(P_n) = \{v_i v_{i+1} : 1 \le i \le n-1\}$. Therefore

$$E((P_m \Theta K_1) \cup P_n) = \begin{cases} u_i u_{i+1} \; : 1 \le i \le m-1 \\ u_i w_i \; : 1 \le i \le m \\ v_i v_{i+1} \; : 1 \le i \le n-1 \end{cases}$$

Then $|V((P_m \Theta K_1) \cup P_n)| = 2m+n$ and $|E((P_m \Theta K_1) \cup P_n)| = 2m+n-2$.

Define a bijection from the vertices of $(P_m \Theta K_1) \cup P_n$ to $\{0,1,2,....,2m+n-1\}$ as follows:

$$f(u_i) = 2i-2 \text{ if } 1 \le i \le m \text{ ;}$$
$$f(v_i) = 2m-1+2i \text{ if } 1 \le i \le n \text{ ;}$$
$$f(w_i) = 2i-1 \text{ if } 1 \le i \le m .$$

Let f* be the induced edge labeling of f.

The induced edge labels by f* as follows:

$$f^*(u_i u_{i+1}) = 8i-4 \text{ if } 1 \le i \le m-1 \text{ ;}$$
$$f^*(u_i w_i) = 4i-3 \text{ if } 1 \le i \le m;$$
$$f^*(v_i v_{i+1}) = 4m-1+2i \text{ if } 1 \le i \le n-1.$$

Theorem 2.7: Any graph $(P_m \Theta K_1) \cup C_n$ admits a square difference labeling.

Proof: Let $P_m \Theta K_1$ be the comb graph with $2m$ vertices and $2m-1$ edges. Let C_n be the cycle graph with n vertices and n edges.

Let $V(P_m \Theta K_1) = \{u_i \ , \ w_i \ : 1 \le i \le m\}$.

Let $V(C_n) = \{v_i \ : 1 \le i \le n\}$. Therefore

$$V((P_m \Theta K_1) \cup C_n) = \begin{cases} u_i \ , \ w_i \ : 1 \le i \le m \\ v_i \ : 1 \le i \le n \end{cases}$$

Let $E(P_m \Theta K_1)) = \begin{cases} u_i u_{i+1} \ : 1 \le i \le m-1 \\ u_i w_i \ : 1 \le i \le m \end{cases}$.

Let $E(C_n) = \{v_i v_{i+1} : 1 \le i \le n-1 ; \ v_1 v_n \}$.

Therefore

$$E((P_m \Theta K_1) \cup C_n) = \begin{cases} u_i u_{i+1} \ : 1 \le i \le m-1 \\ u_i w_i \ : 1 \le i \le m \\ v_i v_{i+1} \ : 1 \le i \le n-1 ; \ v_1 v_n \end{cases}$$

Then $|V((P_m \Theta K_1) \cup C_n)| = 2m+n$ and $|E((P_m \Theta K_1) \cup C_n)| = 2m+n-1$.

Define a bijection from the vertices of $(P_m \Theta K_1) \cup C_n$ to $\{0,1,2,....,2m+n-1\}$ as follows:

$$f(u_i) = 2i-2 \text{ if } 1 \le i \le m \text{ ;}$$
$$f(v_i) = 2m-1+2i \text{ if } 1 \le i \le n \text{ ;}$$
$$f(w_i) = 2i-1 \text{ if } 1 \le i \le m .$$

Let f* be the induced edge labeling of f.

The induced edge labels by f* as follows:

$$f^*(u_i u_{i+1}) = 8i-4 \text{ if } 1 \le i \le m-1 \text{ ;}$$
$$f^*(u_i w_i) = 4i-3 \text{ if } 1 \le i \le m;$$
$$f^*(v_i v_{i+1}) = 4m-1+2i \text{ if } 1 \le i \le n-1 \text{ ;}$$
$$f^*(v_1 v_n) = (n-1)(4m-1+n).$$

Theorem 2.8: The graph $(P_m \Theta K_1) \cup S_n$ is a square difference graph.

Proof: Let $P_m \Theta K_1$ be the comb graph with $2m$ vertices and $2m-1$ edges. Let S_n be the star graph with $n+1$ vertices and n edges.

Let $V(P_m \Theta K_1) = \{u_i \ , \ w_i \ : 1 \le i \le m\}$.

Let $V(S_n) = \{v_i \ : 1 \le i \le n+1\}$.Therefore

$$V((P_m \Theta K_1) \cup S_n) = \begin{cases} u_i \ , \ w_i \ : 1 \le i \le m \\ v_i \ : 1 \le i \le n+1 \end{cases}.$$

Let $E(P_m \Theta K_1)) = \begin{cases} u_i u_{i+1} \ : 1 \le i \le m-1 \\ u_i w_i \ : 1 \le i \le m \end{cases}$.

Let $E(S_n) = \{v_i v_{n+1} : 1 \le i \le n \}$.Therefore

$$E((P_m \Theta K_1) \cup S_n) = \begin{cases} u_i u_{i+1} \ : 1 \le i \le m-1 \\ u_i w_i \ : 1 \le i \le m \\ v_i v_{n+1} \ : 1 \le i \le n \end{cases}$$

Then $|V((P_m \Theta K_1) \cup S_n)| = 2m+n+1$ and $|E((P_m \Theta K_1) \cup S_n)| = 2m+n-1$. Define a bijection from the vertices of $(P_m \Theta K_1) \cup S_n$ to $\{0,1,2,....,2m+n\}$ as follows:

$$f(u_i) = 2i-2 \text{ if } 1 \le i \le m \text{ ;}$$
$$f(v_i) = 2m-1+2i \text{ if } 1 \le i \le n+1 \text{ ;}$$

$f(w_i) = 2i-1 \quad if \ 1 \le i \le m$. Let f* be the induced edge labeling of f. The induced edge labels by f* as follows: $f^*(u_i u_{i+1}) = 8i-4 \quad if \ 1 \le i \le m-1$;

$f^*(u_i w_i) = 4i-3 \quad if \ 1 \le i \le m$

$f^*(v_i v_{n+1}) = (4m+n-1+i)(n+1-i) \ if \ 1 \le i \le n$

Example 2.9: A square difference labeling of $(P_8 \Theta K_1) \cup S_{13}$ is shown in the Figure 2.2.

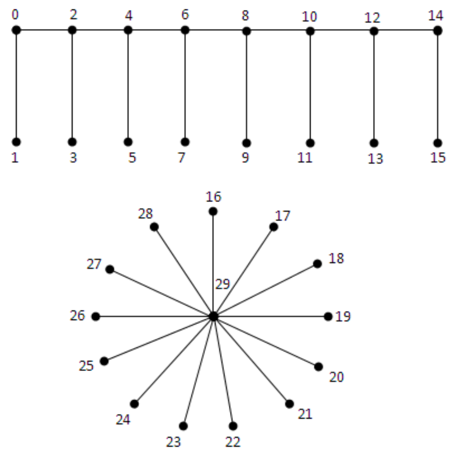

Figure 2.2

Theorem 2.10: The graph $(P_m \Theta K_1) \cup (P_n \Theta K_1)$ is a square difference graph.

Proof: Let $P_m \Theta K_1$ be a comb graph with $2m$ vertices and $2m-1$ edges. Let $P_n \Theta K_1$ be another comb graph with $2n$ vertices and $2n-1$ edges.

Let $V(P_m \Theta K_1)) = \{u_i \ , \ w_i \ :1 \le i \le m\}$.

Let $V(P_n \Theta K_1)) = \{v_i \ , \ z_i \ :1 \le i \le n\}$

Therefore

$$V((P_m \Theta K_1) \cup (P_n \Theta K_1)) = \begin{cases} u_i \ , \ w_i \ :1 \le i \le m \\ v_i \ , \ z_i \ :1 \le i \le n \end{cases} .$$

Let $E(P_m \Theta K_1)) = \begin{cases} u_i u_{i+1} \ :1 \le i \le m-1 \\ u_i w_i \ :1 \le i \le m \end{cases}$

Let $E(P_n \Theta K_1)) = \begin{cases} v_i v_{i+1} \ :1 \le i \le n-1 \\ v_i z_i \ :1 \le i \le n \end{cases}$

Therefore

$$E((P_m \Theta K_1) \cup (P_n \Theta K_1)) = \begin{cases} u_i u_{i+1} \ : 1 \le i \le m-1 \\ u_i w_i \ : 1 \le i \le m \\ v_i v_{i+1} \ : 1 \le i \le n-1 \\ v_i z_i \ : 1 \le i \le n \end{cases}$$

Then $|V((P_m \Theta K_1) \cup (P_n \Theta K_1))| = 2m + 2n$ and

$|E((P_m \Theta K_1) \cup (P_n \Theta K_1))| = 2m + 2n - 2$.

Define a bijection from the vertices of $(P_m \Theta K_1) \cup (P_n \Theta K_1)$ to $\{0,1,2,....,2m+2n-1\}$ as follows:

$f(u_i) = 2i-2 \quad if \ 1 \le i \le m$;

$f(v_i) = 2m-2+2i \quad if \ 1 \le i \le n$;

$f(w_i) = 2i-1 \quad if \ 1 \le i \le m$;

$f(z_i) = 2m-1+2i \quad if \ 1 \le i \le n$.

Let f* be the induced edge labeling of f.

The induced edge labels by f* as follows:

$f^*(u_i u_{i+1}) = 8i-4 \quad if \ 1 \le i \le m-1$;

$f^*(u_i w_i) = 4i-3 \ if \ 1 \le i \le m$;

$f^*(v_i v_{i+1}) = 4(2m-1+2i) \ if \ 1 \le i \le n-1$;

$f^*(v_i z_i) = 4m-3+4i \ if \ 1 \le i \le n$.

Example 2.11: A square difference labeling of $(P_7 \Theta K_1) \cup (P_{10} \Theta K_1)$ is shown in the Figure 2.3.

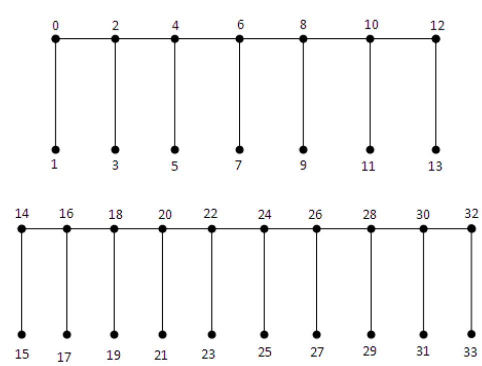

Figure 2.3

Theorem 2.12: The graph $(P_m \Theta K_1) \cup (C_n \Theta K_1)$ is a square difference graph.

Proof: Let $P_m \Theta K_1$ be a comb graph with $2m$ vertices and $2m-1$ edges.

Let $C_n \Theta K_1$ be the crown graph with $2n$ vertices and $2n$ edges.

Let $V(P_m \Theta K_1)) = \{u_i \ , \ w_i \ :1 \le i \le m\}$.

Let $V(C_n \Theta K_1)) = \{v_i \ , \ z_i \ :1 \le i \le n\}$.

Therefore

$$V((P_m \Theta K_1) \cup (C_n \Theta K_1)) = \begin{cases} u_i \ , \ w_i \ :1 \le i \le m \\ v_i \ , \ z_i \ :1 \le i \le n \end{cases}$$

Let $E(P_m \Theta K_1)) = \begin{cases} u_i u_{i+1} \ :1 \le i \le m-1 \\ u_i w_i \ :1 \le i \le m \end{cases}$

Let $E(C_n \Theta K_1)) = \begin{cases} v_i v_{i+1} \ :1 \le i \le n-1 \\ v_1 v_n \, ; \, v_i z_i \ :1 \le i \le n \end{cases}$ .Then

$$E((P_m \Theta K_1) \cup (C_n \Theta K_1)) = \begin{cases} u_i u_{i+1} \ :1 \le i \le m-1 \\ u_i w_i \ \ :1 \le i \le m \\ v_i v_{i+1} \ :1 \le i \le n-1 \\ v_1 v_n \, ; \, v_i z_i : 1 \le i \le n \end{cases}$$

Then $|V((P_m \Theta K_1) \cup (C_n \Theta K_1))| = 2m + 2n$ and $|E((P_m \Theta K_1) \cup (C_n \Theta K_1))| = 2m + 2n - 1$.

Define a bijection from the vertices of $(P_m \Theta K_1) \cup (C_n \Theta K_1)$ to $\{0,1,2,....,2m+2n-1\}$ as follows:

$f(u_i) = 2i-2 \ \ if \ 1 \le i \le m$;
$f(v_i) = 2m-1+2i \ \ if \ 1 \le i \le n$;

$f(w_i) = 2i-1 \ \ if \ 1 \le i \le m$;
$f(z_i) = 2m+n-1+i \ \ if \ 1 \le i \le n$.

Let f* be the induced edge labeling of f.

The induced edge labels by f* as follows:

$f^*(u_i u_{i+1}) = 8i - 4 \ \ if \ 1 \le i \le m-1$;
$f^*(u_i w_i) = 4i - 3 \ \ if \ 1 \le i \le m$;
$f^*(v_1 v_n) = (n-1)(4m+n-1)$;
$f^*(v_i v_{i+1}) = 4m - 1 + 2i \ \ if \ 1 \le i \le n-1$;
$f^*(v_i z_i) = n(4m+n-2+2i) \ if \ 1 \le i \le n$.

Theorem 2.13: The graph $(P_m \Theta K_1) \cup L_n$ is a square difference graph.

Proof: Let $P_m \Theta K_1$ be the comb graph with $2m$ vertices and $2m-1$ edges. Let L_n be the ladder with $2n$ vertices and $3n-2$ edges.

Let $V(P_m \Theta K_1) = \{u_i \ , \ z_i \ :1 \le i \le m\}$.

Let $V(L_n) = \{v_i \ , \ w_i \ :1 \le i \le n\}$. Therefore

$$V((P_m \Theta K_1) \cup L_n) = \begin{cases} u_i \ , z_i \ : 1 \le i \le m \\ v_i \, , w_i \ : \ 1 \le i \le n \end{cases}$$

Let $E(P_m \Theta K_1) = \begin{cases} u_i u_{i+1} \ :1 \le i \le m-1 \\ u_i \ z_i \ :1 \le i \le m \end{cases}$

Let $E(L_n) = \begin{cases} v_i v_{i+1} \ , \ w_i w_{i+1} \ :1 \le i \le n-1 \\ v_i w_i \, :1 \le i \le n \end{cases}$.

Therefore

$$E((P_m \Theta K_1) \cup L_n) = \begin{cases} u_i u_{i+1} & :1 \le i \le m-1 \\ u_i z_i & :1 \le i \le m \\ v_i v_{i+1} & :1 \le i \le n-1 \\ w_i w_{i+1} & :1 \le i \le n-1 \\ v_i w_i & :1 \le i \le n \end{cases}$$

Then $|V((P_m \Theta K_1) \cup L_n)| = 2m + 2n$ and $|E((P_m \Theta K_1) \cup L_n)| = 2m + 3n - 3$. Define a bijection from the vertices of $(P_m \Theta K_1) \cup L_n$ to $\{0,1,2,....,2m+2n-1\}$ as follows:

$f(u_i) = 2i-2 \ \ if \ \ 1 \le i \le m$;
$f(z_i) = 2i-1 \ \ if \ 1 \le i \le m$;

$f(v_i) = 2m-1+i \ \ if \ 1 \le i \le n$;
$f(w_i) = 2m+2n-i \ \ if \ \ 1 \le i \le n$.

Let f* be the induced edge labeling of f .The induced edge labels by f* as follows:

$f^*(u_iu_{i+1}) = 8i-4 \quad if \quad 1 \le i \le m-1$;
$f^*(u_iz_i) = 4i-3 \quad if \quad 1 \le i \le m$;

$f^*(v_iv_{i+1}) = 4n-1+2i \ if \ 1 \le i \le n$;
$f^*(w_iw_{i+1}) = 4m+4n-1-2i \quad if \ 1 \le i \le n$;

$f^*(v_iw_i) = (4m+2n-1)(2n+1-2i) \ if 1 \le i \le n$

Theorem 2.14: Every graph $(P_m \Theta K_1) \cup (L_n \Theta K_1)$ is a square difference graph.

Proof: Let $P_m \Theta K_1$ be the comb graph with $2m$ vertices and $2m-1$ edges.

Let $L_n \Theta K_1$ be the graph with $4n$ vertices and $5n-2$ edges.

Let $V(P_m \Theta K_1) = \{u_i , x_i : 1 \le i \le m\}$ .Let $V(L_n \Theta K_1) = \{v_i , w_i , y_i , z_i : 1 \le i \le n\}$ .Then

$$V((P_m \Theta K_1) \cup (L_n \Theta K_1)) = \begin{cases} u_i , x_i : 1 \le i \le m \\ v_i , w_i : 1 \le i \le n \\ y_i , z_i : 1 \le i \le n \end{cases}$$

Let $E(P_m \Theta K_1) = \begin{cases} u_iu_{i+1} : 1 \le i \le m-1 \\ u_i x_i : 1 \le i \le m \end{cases}$.

Let $E(L_n \Theta K_1) = \begin{cases} v_iv_{i+1} & : 1 \le i \le n-1 \\ w_iw_{i+1} & : 1 \le i \le n-1 \\ v_iw_i & : 1 \le i \le n \\ v_iy_i & : 1 \le i \le n \\ w_iz_i & : 1 \le i \le n \end{cases}$

Then

$$E((P_m \Theta K_1) \cup (L_n \Theta K_1)) = \begin{cases} u_iu_{i+1} & : 1 \le i \le m-1 \\ u_ix_i & : 1 \le i \le m \\ v_iv_{i+1} & : 1 \le i \le n-1 \\ w_iw_{i+1} & : 1 \le i \le n-1 \\ v_iw_i & : 1 \le i \le n \\ v_iy_i & : 1 \le i \le n \\ w_iz_i & : 1 \le i \le n \end{cases}$$

Then $|V((P_m \Theta K_1) \cup (L_n \Theta K_1))| = 2m+4n$ and

$|E((P_m \Theta K_1) \cup (L_n \Theta K_1))| = 2m+5n-3$.

Define a bijection from the vertices of $(P_m \Theta K_1) \cup (L_n \Theta K_1)$ to $\{0,1,2,....,2m+4n-1\}$ as follows:

$f(u_i) = 2i-2 \quad if \quad 1 \le i \le m$;

$f(x_i) = 2i-1 \quad if \ 1 \le i \le m$;
$f(v_i) = 2m-1+i \quad if \ 1 \le i \le n$;

$f(w_i) = 2m+2n-i \quad if \quad 1 \le i \le n$;
$f(y_i) = 2m+2n-1+i \quad if \quad 1 \le i \le n$;

$f(z_i) = 2m+4n-i \quad if \quad 1 \le i \le n$.

Let f* be the induced edge labeling of f.The induced edge labels by f* as follows:

$f^*(u_iu_{i+1}) = 8i-4 \quad if \quad 1 \le i \le m-1$;

$f^*(u_ix_i) = 4i-3 \quad if \quad 1 \le i \le m$;
$f^*(v_iv_{i+1}) = 4n-1+2i \ if \ 1 \le i \le n$;

$f^*(w_iw_{i+1}) = 4n+4n-1-2i \quad if \ 1 \le i \le n$;

$f^*(v_iw_i) = (4m+2n-1)(2n+1-2i) if 1 \le i \le n$
$f^*(v_iy_i) = (4n)(2m+n-1+i) \ if \ 1 \le i \le n$;

$f^*(w_iz_i) = (4n)(2m+3n-i) \ if \ 1 \le i \le n$.

Example 2.15: A square difference labeling of $(P_{10} \Theta K_1) \cup (L_9 \Theta K_1)$ is shown in the Figure 2.4.

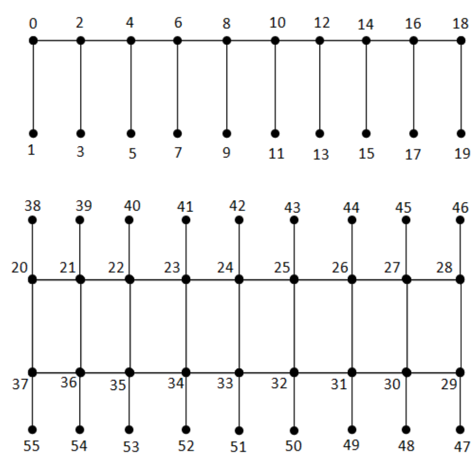

Figure 2.4

3.Conclusion

In this paper, we investigated the square difference labeling behavior of some union related graphs. We have planned to investigate the square difference labeling of some more special graphs in the next paper.

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