Some Results on Mean Cordial Graphs

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Abstract

Let G = (V, E) be a simple graph. G is said to be a mean cordial graph if f : V(G)→{0,1,2} such that for each edge uv the induced map f* defined by $f^*(uv) = \left\lfloor \frac{f(u)+f(v)}{2} \right\rfloor$ where ⌊x⌋ denote the least integer which is ≤ x and $|e_f(0) - e_f(1)| \le 1$ where $e_f(0)$ is no.of edges with zero label. $e_f(1)$ is no.of edges with one label.

The graph that admits a mean cordial labeling is called a mean cordial graph (MCG).

In this paper , we proved that $D_2[C_n]$, $D_2[K_{1,n}]$, $D_2[P_n + K_1]$, $D_2[P_n]$ are mean cordial graphs.

Key words : Mean cordial labeling , Mean cordial graph.

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1.INTRODUCTION:

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each e = {uv} of vertices in E is called an edge or a line of G. For graph theoretical Terminology we follow

2.PRELIMINARIES:

We define the concept of mean cordial labeling as follows.

Let G = (V, E) be a simple graph. G is said to be a mean cordial graph if f : V(G)→{0,1,2} such that for each edge uv the induced map f* defined by f*(uv) $= \left\lfloor \frac{f(u)+f(v)}{2} \right\rfloor$ where ⌊x⌋ denote the least integer which is ≤ x and $|e_f(0) - e_f(1)| \le 1$ where $e_f(0)$ is no.of edgeswith label 0. $e_f(1)$ is number of edges with label 1.

A graph that admits a mean cordial labeling is called a mean cordial graph. We proved that $D_2[C_n]$, $D_2[K_{1,n}]$, $D_2[P_n + K_1]$, $D_2[P_n]$ are mean cordial graphs.

DEFINITION 2.1 (SHADOW GRAPH)

Let G be a connected Graph. A Graph, constructed by taking two copies of G say G_1 and G_2 and joining each vertex u in G_1 to the neighbours of the corresponding vertex v in G_2 ,that is for every vertex u in G_1 there exists v in G_2 such that N(u) = N(v). The resulting Graph is known as shadow Graph and it is denoted by $D_2(G)$.

DEFINITION 2.2 (CYCLE)

A closed path is called a ***cycle*** and a cycle of length k is denoted by C_k.

DEFINITION 2.3(STAR)

Let $S_{m,n}$ (n > 2) is a ***star*** with n spokes in which each spoke is a path of length m.

DEFINITION 2.4(FAN)

The ***join*** $G_1 + G_2$ of G_1 and G_2 consists of $G_1 \cup G_2$ and all lines joining V_1 with V_2 as vertex set $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and edges $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$ [uv : u ∈V(G_1) and v ∈V(G_2)]. The graph $P_n + K_1$ is called a ***Fan*** and $P_n + 2K_1$ is called the ***Doublefan.***

DEFINITION 2.5(PATH)

If all the vertices in a walk are distinct, then it is called a ***path*** and a path of length k is denoted by P_{k+1}.

3. Main Results on Mean Cordial Graph

Theorem 3.1

$D_2(C_n)$ is a Mean Cordial Graph.

Proof:

Let G = (V, E)

Let G be $[D_2(C_n)]$

Let $V[D_2(C_n)] = \{u_i , v_i : 1 \le i \le n\}$

Let $E[D_2(C_n)] = \{[(u_i u_{i+1}) \cup (v_i v_{i+1}) : 1 \le i \le n-1] \cup [(u_1 u_n) \cup (v_1 v_n) \cup (u_1 v_2) \cup (v_1 u_2)] \cup [(u_i v_{i+1}) \cup (u_i v_{i-1}) \cup (v_i u_{i+1}) \cup (v_i u_{i-1}) : 2 \le i \le n-1]\}$

Define $f : V(G) \rightarrow \{0,1,2\}$ by

$f(u_1) = 1$

$f(v_1) = 1$

$$f(u_i) = \begin{cases} 0 & if\ i \equiv 0 \bmod 2 \\ 1 & if\ i \equiv 1 \bmod 2 \end{cases} , 2 \le i \le n$$

$$f(v_i) = \begin{cases} 1 & if\ i \equiv 0 \bmod 2 \\ 0 & if\ i \equiv 1 \bmod 2 \end{cases} , 2 \le i \le n$$

The induced edge labeling are

$$f^*(u_i u_{i+1}) = \begin{cases} 0 & if\ i \equiv 1 \bmod 2 \\ 1 & if\ i \equiv 0 \bmod 2 \end{cases} , 1 \le i \le n-1$$

$$f^*(v_i v_{i+1}) = \begin{cases} 1 & if\ i \equiv 1 \bmod 2 \\ 0 & if\ i \equiv 0 \bmod 2 \end{cases} , 1 \le i \le n-1$$

$$f^*(u_i v_{i+1}) = \begin{cases} 1 & if\ i \equiv 1 \bmod 2 \\ 0 & if\ i \equiv 0 \bmod 2 \end{cases} , 2 \le i \le n-1$$

$$f^*(u_i v_{i-1}) = \begin{cases} 1 & if\ i \equiv 1 \bmod 2 \\ 0 & if\ i \equiv 0 \bmod 2 \end{cases} , 2 \le i \le n-1$$

$$f^*(v_i u_{i+1}) = \begin{cases} 0 & if\ i \equiv 1 \bmod 2 \\ 1 & if\ i \equiv 0 \bmod 2 \end{cases} , 2 \le i \le n-1$$

$$f^*(v_i u_{i-1}) = \begin{cases} 0 & if\ i \equiv 1 \bmod 2 \\ 1 & if\ i \equiv 0 \bmod 2 \end{cases} , 2 \le i \le n-1$$

When n is even , $f^*(u_1 v_n) = 1$

$f^*(u_1 v_2) = 1$

$f^*(v_1 u_n) = 0$

$f^*(v_1 u_2) = 0$

When n is odd , $f^*(v_1 u_5) = 1$

$f^*(u_1 v_n) = 0$

$f^*(v_1 u_2) = 0$

$f^*(u_1 v_2) = 1$

Here $e_f(0) = e_f(1)$ for all n.

It satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence , $D_2(C_n)$ is a mean cordial graph.

For example the graph $D_2(C_4)$ and $D_2(C_5)$ are shown in the figure1 and figure 2.

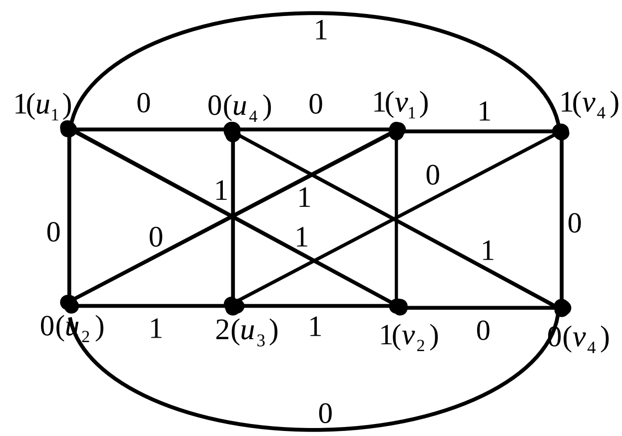

Figure 1

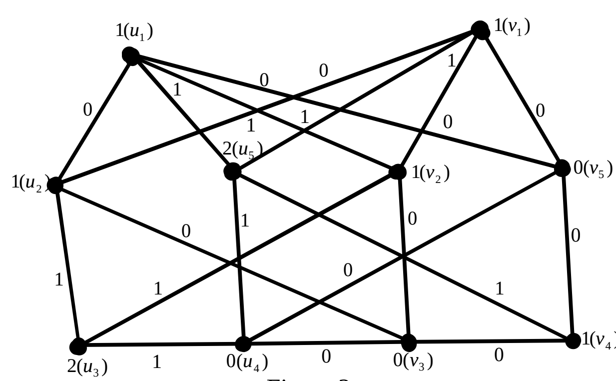

Figure 2

Theorem 3.2

$D_2[K_{1,n}]$ is a Mean cordial Graph.

Proof:

Let G = (V, E)

Let G be $D_2[K_{1,n}]$

Let $V[D_2(K_{1,n})] = \{u, v, (u_i\ v_i) : 1 \le i \le n\}$

Let $E[D_2(K_{1,n})] = \{[(u\ u_i) \cup (u\ v_i) \cup (v\ u_i) \cup (v\ v_i)] : 1 \le i \le n\}$

Define $f : V(G) \rightarrow \{0,1,2\}$ by

Case (i) : n is even

f(u) = 1

f(v) = 1

$$f(u_i) = \begin{cases} 0 & if\ i \equiv 1 \bmod 2 \\ 2 & if\ i \equiv 0 \bmod 2 \end{cases},\ 1 \le i \le n$$

$$f(v_i) = \begin{cases} 0 & if\ i \equiv 1 \bmod 2 \\ 2 & if\ i \equiv 0 \bmod 2 \end{cases},\ 1 \le i \le n$$

The induced edge labeling are

$$f^*(u\ u_i) = \begin{cases} 0 & if\ i \equiv 1 \bmod 2 \\ 1 & if\ i \equiv 0 \bmod 2 \end{cases},\ 1 \le i \le n$$

$$f^*(u\ v_i) = \begin{cases} 0 & if\ i \equiv 1 \bmod 2 \\ 1 & if\ i \equiv 0 \bmod 2 \end{cases},\ 1 \le i \le n$$

$$f^*(v\ u_i) = \begin{cases} 0 & if\ i \equiv 1 \bmod 2 \\ 1 & if\ i \equiv 0 \bmod 2 \end{cases},\ 1 \le i \le n$$

$$f^*(v\ v_i) = \begin{cases} 0 & if\ i \equiv 1 \bmod 2 \\ 1 & if\ i \equiv 0 \bmod 2 \end{cases},\ 1 \le i \le n$$

Here $e_f(0) = e_f(1)$

It satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence $[D_2(K_{1,n})]$ (n is even) is a mean cordial graph.

For example $D_2(K_{1,2})$ is shown in the figure 3.

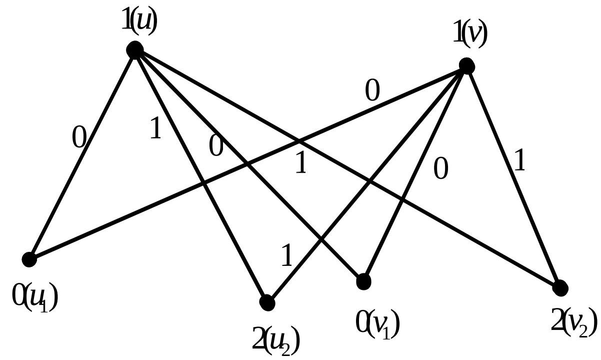

Figure 3

Case (ii): n is odd

f(u) = 1

f(v) = 1

$$f(u_i) = \begin{cases} 0 & if\ i \equiv 1 \bmod 2 \\ 2 & if\ i \equiv 0 \bmod 2 \end{cases},\ 1 \le i \le n$$

$$f(v_i) = \begin{cases} 0 & if\ i \equiv 0 \bmod 2 \\ 2 & if\ i \equiv 1 \bmod 2 \end{cases},\ 1 \le i \le n$$

The induced edge labeling are

$$f^*(u\ u_i) = \begin{cases} 0 & if\ i \equiv 1 \bmod 2 \\ 1 & if\ i \equiv 0 \bmod 2 \end{cases},\ 1 \le i \le n$$

$$f^*(u\ v_i) = \begin{cases} 0 & if\ i \equiv 0 \bmod 2 \\ 1 & if\ i \equiv 1 \bmod 2 \end{cases},\ 1 \le i \le n$$

$$f^*(v\ u_i) = \begin{cases} 0 & if\ i \equiv 1 \bmod 2 \\ 1 & if\ i \equiv 0 \bmod 2 \end{cases},\ 1 \le i \le n$$

$$f^*(v\ v_i) = \begin{cases} 0 & if\ i \equiv 1 \bmod 2 \\ 1 & if\ i \equiv 0 \bmod 2 \end{cases},\ 1 \le i \le n$$

Here $e_f(0) = e_f(1)$

It satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence $[D_2(K_{1,n})]$ (n is odd) is a mean cordial graph.

For example $D_2(K_{1,3})$ is shown in the figure.

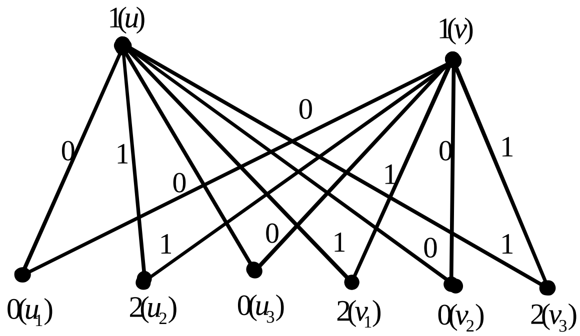

Figure 4

Theorem 3.3

Graph $D_2[P_n+K_1]$ is a Mean Cordial Graph.

Proof:

Let G = (V, E)

Let G be $D_2[P_n + K_1]$

Let $V[D_2(P_n + K_1)] = \{u, v, u_i, v_i : 1 \leq i \leq n\}$

Let $E[D_2(P_n+K_1)] = \{(u\, u_i) \cup (u\, v_i) \cup (v\, u_i) \cup$

$(v\, v_i) \cup (u_i\, u_{i+1}) \cup (v_i\, v_{i+1})$

$\cup (u_i v_{i+1}) \cup (v_i\, u_{i+1}) : 1 \leq i \leq n-1\}$

Define f: $V(G) \rightarrow \{0,1,2\}$ by

f(u) = 2

f(v) = 0

$$f(u_i) = \begin{Bmatrix} 1 & if\ \ i \equiv 1 \bmod 2 \\ 0 & if\ i \equiv 0 \bmod 2 \end{Bmatrix} \quad , 1 \leq i \leq n$$

$f(v_i) = 1$

The induced edge labeling are

$f^*(u\, u_i) = 1$

$f^*(u\, v_i) = 1$

$f^*(v\, u_i) = 0$

$f^*(v\, vi) = 0$

$f^*(u_i\, u_{i+1}) = 0$

$f^*(v_i\, v_{i+1}) = 1$

$$f^*(u_i\, v_{i+1}) = \begin{Bmatrix} 1 & if\ \ i \equiv 1 \bmod 2 \\ 0 & if\ i \equiv 0 \bmod 2 \end{Bmatrix} \quad , 1 \leq i \leq n-1$$

$$f^*(v_i\, u_{i+1}) = \begin{Bmatrix} 0 & if\ \ i \equiv 1 \bmod 2 \\ 1 & if\ i \equiv 0 \bmod 2 \end{Bmatrix} \quad , 1 \leq i \leq n-1$$

Hence $e_f(0) = e_f(1)$ for all n.

It satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence $D_2(P_n + K_1)$ is a mean cordial graph

For example, $D_2(P_2+ K_1)$ is shown in the figure 5.

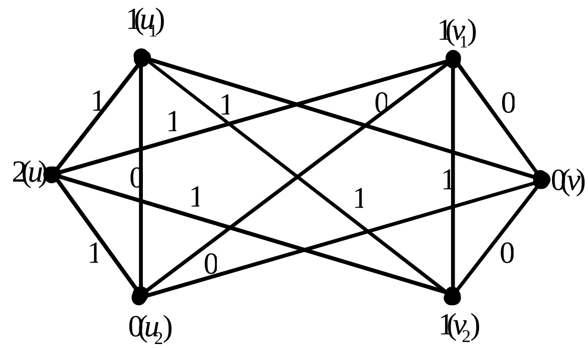

Figure 5

Theorem 3.4

Graph $D_2(P_n)$ is a Mean Cordial Graph.

Proof:

Let G= (V, E)

Let G be $D_2(P_n)$

Let $V[D_2(P_n)] = \{u_i, v_i : 1 \leq i \leq n\}$

Let $E[D_2(P_n)] = \{[(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(v_i\, v_{i+1}) : 1 \leq i \leq n-1]$

$\cup [(v_i\, u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i\, v_{i+1}) : 1 \leq i \leq n-1]\}$

Define f: $V(G) \rightarrow 0,1,2\}$ by

$$f(u_i) = \begin{Bmatrix} 2 & if\ \ i \equiv 1 \bmod 2 \\ 0 & if\ i \equiv 0 \bmod 2 \end{Bmatrix}, 1 \leq i \leq n$$

$$f(v_i) = \begin{Bmatrix} 0 & if\ \ i \equiv 0 \bmod 2 \\ 1 & if\ i \equiv 1 \bmod 2 \end{Bmatrix}, 1 \leq i \leq n$$

The induced edge labeling are

$f^*(u_i\, u_{i+1}) = 1, 1 \leq i \leq n-1$

$f^*(v_i\, v_{i+1}) = 0, 1 \leq i \leq n-1$

$$f^*(v_i\, u_{i+1}) = \begin{Bmatrix} 0 & if\ i \equiv 1 \bmod 2 \\ 1 & if\ i \equiv 0 \bmod 2 \end{Bmatrix},\ 1 \leq i \leq n\text{-}1$$

$$f^*(u_i\, v_{i+1}) = \begin{Bmatrix} 1 & if\ i \equiv 1 \bmod 2 \\ 0 & if\ i \equiv 0 \bmod 2 \end{Bmatrix},\ 1 \leq i \leq n\text{-}1$$

It satisfies the condition

$e_f(0) = e_f(1)$ for all n

Hence, $D_2(P_n)$ is a mean cordial graph.

For example, the mean cordial graph of $D_2(P_4)$ is shown in the figure 6.

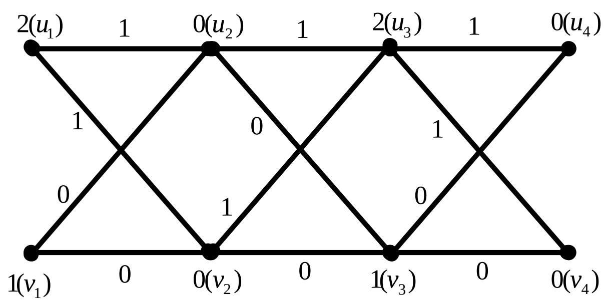

Figure 6

4. Conclusion

Graph labeling place a vital role not only in the theoretical aspect but also in many practical application problems. There are number of labeling such as magic labeling, graceful labeling, mean labeling and super-mean labeling.

Particularly cordial related labeling such as mean cordial, divisor cordial, mean-square cordial labeling etc. place an important role in digital technology

5. References

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