Characterisation of Restrained Domination Number and Chromatic Number of a Fuzzy Graph

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Abstract: Let $G(V,\sigma,\mu)$ be a simple undirected fuzzy graph. A subset S of V is called a dominating set in G if every vertex in V-S is adjacent to at least one vertex in S. A subset S of V is said to be a restrained dominating set if every vertex in V-S is adjacent to atleast one vertex in S as well as adjacent to atleast one vertex in V-S. The restrained domination number of a fuzzy graph $G(V,\sigma,\mu)$ is denoted by γ_{fr}(G) which is the smallest cardinality of a restrained dominating set of G. The minimum number of colours required to colour all the vertices such that adjacent vertices do not receive the same colour is the chromatic number χ(G). For any fuzzy graph G a complete fuzzy sub graph of G is called a clique of G. In this paper we find an upper bound for the sum of the Restrained domination and chromatic number in fuzzy graphs and characterize the corresponding extremal fuzzy graphs.

Keywords: Fuzzy Restrained Domination Number, Chromatic Number, fuzzy graph, Clique

1.INTRODUCTION:

Let $G(V,\sigma,\mu)$ be simple undirected strong fuzzy graph. The degree of any vertex u in G is the number of edges incident with u and is denoted by d(u). The minimum and maximum degree of a vertex is denoted by δ(G) and Δ(G) respectively, P_n denotes the path on n vertices. The vertex connectivity κ(G) of a fuzzy graph G is the minimum number of vertices whose removal results in a disconnected fuzzy graph. The chromatic number χ is defined to be the minimum number of colours required to colour all the vertices such that adjacent vertices do not receive the same colour. For any fuzzy graph G a complete sub fuzzy graph of G is called a clique of G. The number of vertices in a largest clique of G is called the clique number of G.

Let G(V,E) be a simple undirected strong fuzzy graph. A subset S of V is called a dominating set in G if every vertex in V-S is adjacent to at least one vertex in S. A subset S of V is said to be a restrained dominating set if every vertex in V-S is adjacent to atleast one vertex in S as well as adjacent to atleast one vertex in V-S. The restrained domination number, denoted by γ_{fr}(G) is the smallest cardinality of a restrained dominating set of a fuzzy graph G. The minimum number of colours required to colour all the vertices such that adjacent vertices do not receive the same colour is the chromatic number χ(G). For any fuzzy graph G a complete sub fuzzy graph of G is called a clique of G.

If X is collection of objects denoted generically by x, then a Fuzzy set $\tilde{A}$ is X is a set of ordered pairs: $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) / x \in X\}$, $\mu_{\widetilde{A}}(x)$ is called the membership function of x in $\widetilde{A}$ that maps X to the membership space M (when M contains only the two points 0 and 1). Let E be the (crisp) set of nodes. A fuzzy graph is then defined by, $\widetilde{G}(x_i, x_j) = \{(x_i, x_j), \mu_{\widetilde{G}}(x_i, x_j) / (x_i, x_j) \in E \times E\}$. $\widetilde{H}(x_i, x_j)$ is a Fuzzy Sub graph of $\widetilde{G}(x_i, x_j)$ if $\mu_{\widetilde{H}}(x_i, x_j) \le \mu_{\widetilde{G}}(x_i, x_j)\ \forall (x_i, x_j) \in E \times E$, $\widetilde{H}(x_i, x_j)$ is a spanning fuzzy sub graph of $\widetilde{G}(x_i, x_j)$ if the node set of $\widetilde{H}(x_i, x_j)$ and $\widetilde{G}(x_i, x_j)$ are equal, that is if they differ only in their arc weights.

Several authors have studied the problem of obtaining an upper bound for the sum of a domination parameter and a fuzzy graph theoretic parameter and characterized the corresponding extremal fuzzy graphs. In [5], Paulraj Joseph J and Arumugam S proved that γ+k≤p. In[7], Paulraj Joseph J and Arumugam S proved that γ_c (G)+ $\chi \le$ p+1. They also characterized the class of fuzzy graphs for which the upper bound is attained. They also proved similar results for γ and γ_t . In[4], Mahadevan G introduced the concept the complementary perfect domination number γ_{cp} and proved that γ_{cp}(G)+ $\chi \le$ 2n-2, and characterized the corresponding extermal fuzzy graphs. In[9], S.Vimala and J.S.Sathya proved that γ_t (G)+χ(G)=2n-5. They also characterised the class of fuzzy graphs for which the upper bound is attained. In this paper we obtain sharp upper bound for the sum of the restrained domination number and chromatic number and characterize the corresponding extremal fuzzy graphs. We use the following previous results.

Theorem 1.1 [1]: For any connected fuzzy graph G, γ_{fr} (G) $\le n$

Theorem 1.2 [2]: For any connected fuzzy graph G, χ(G)≤ Δ(G)+1.

2.Main results

Theorem 2.1: For any connected fuzzy graph G, γ_{fr} (G)+ χ(G)≤2n and the equality holds if and only if G≅K_1

Proof: γ_{fr} (G)+χ(G) ≤ n+Δ+1=n+(n-1)+1≤2n. If γ_{fr} (G)+χ(G)=2n the only possible case is γ_{fr} (G)=n and χ(G)=n, Since χ(G)=n, G=K_n , But for K_n, γ_{fr} (G)=1, so that G≅K_1. Conversely if G is isomorphic to K_1, then for K_1, $\gamma_{fr}(G) = 1, and\ \chi(G) = 1 \gamma_{fr}$ (G)+χ(G)=2. Hence the proof.

Theorem 2.2: For any connected fuzzy graph G, γ_{fr} (G)+χ(G)=2n-1 and the equality holds if and only if G≅K_2

Proof: If G is isomorphic to K_2, then for K_2 , $\gamma_{fr}(G) = 1, and\ \chi(G) = 2$. γ_{fr} (G)+χ(G)=2n-1=3. Conversely assume that γ_{fr} (G)+χ(G)=2n-1. This is possible only if γ_{fr} (G)=n and χ(G)=n-1 (or) γ_{fr} (G) = n-1 and χ(G) = n.

Case (i) Let γ_{fr} (G)=n and χ(G)=n-1.

Since χ(G)= n-1, G contains a clique K on n-1 vertices. Let x be a vertex of G-K_{n-1}. Since G is connected the vertex x is adjacent to one vertex u_i for some i in K_{n-1} {x, u_i} is γ_{fr} – set, so that γ_{fr} (G)=2, we have n=2. Then $\chi = 1$, which is for totally disconnected fuzzy graph. Which is a contradiction. Hence no fuzzy graph exists.

Case (ii) Let γ_{fr} (G)=n-1 and χ(G)=n

Since χ(G)=n, G=K_n , But for K_n , γ_{fr} (G)=1, so that n=2, χ =2 Hence G≅K_2.

Theorem 2.3: For any connected fuzzy graph G, γ_{fr} (G)+χ(G)=2n-2 and the equality holds if and only if G≅K_3, P_3

Proof: Let G be isomorphic to K_3, then for K_3 , $\gamma_{fr}(G) = 1, and\ \chi(G) = 3$. γ_{fr} (G)+χ(G)=2n-2=4. And if G is isomorphic to P_3, then for P_3, $\gamma_{fr}(G) = 2, and\ \chi(G) = 2$. γ_{fr} (G)+χ(G)=2n-2=4. Conversely **a**ssume that γ_{fr} (G)+χ(G)=2n-2. This is possible only if γ_{fr} (G)=n and χ(G)=n-2 (or) γ_{fr} (G)=n-1 and χ(G)=n-1 (or) γ_{fr} (G)=n-2 and χ(G)=n.

Case (i) Let γ_{fr} (G)=n and χ(G)=n-2.

Since χ (G)= n-2, G contains a clique K on n-2 vertices. Let S={x,y}∈G-K_{n-2}. Then $< S >= K_2$ or $\overline{K_2}$

Subcase (a) Let $< S >= K_2$ Since G is connected, x is adjacent to some u_i of K_{n-2}. Then {y,u_j} for some i≠j is γ_{fr} - set, so that γ_{fr} (G)=2 and hence n=2. But χ(G)=n-2=0. Which is a contradiction. Hence no fuzzy graph exists.

Subcase (b) Let $< S >= \overline{K_2}$ Since G is connected, x is adjacent to some u_i of K_{n-2}. Then y is adjacent to the same u_i of K_{n-2}. Then {x,y,u_i} γ_{fr} - set, so that γ_{fr} (G)=3 and hence n=3. But χ(G)=n-2=1 which is for totally disconnected fuzzy graph. Which is a contradiction. Hence no fuzzy graph exists, (or) y is adjacent to u_j of K_{n-2} for i≠j. In this case {x,y,u_i} γ_{fr} - set, so that γ_{fr} (G)=3 and hence n=3. But χ(G)=n-2=1 which is for totally disconnected fuzzy graph. Which is a contradiction. Hence no fuzzy graph exists.

Case (ii) Let γ_{fr} (G)=n-1 and χ(G)=n-1.

Since χ (G)= n-1, G contains a clique K on n-1 vertices. Let x be a vertex of G-K_{n-1}. Since G is connected, x is adjacent to one vertex u_i for some i in K_{n-1}, so that {x, u_i} is γ_{fr} – set, so that γ_{fr} (G)=2, we have n=3. Then $\chi = 2$, Hence G≅P_3

Case (iii) Let γ_{fr} (G)=n-2 and χ(G)=n

Since χ(G)=n, G=K_n , But for K_n , γ_{fr} (G) = 1,so that n=3, χ = 3 Hence G≅K_3. Hence the proof.

Theorem 2.4: For any connected fuzzy graph G, γ_{fr} (G)+ χ(G)=2n-3 and the equality holds if and only if G≅$K_{1,3}$,$C_3(P_2)$, K_4

Proof: Let G be isomorphic to $K_{1,3}$, then for $K_{1,3}$, $\gamma_{fr}(G) = 3, and\ \chi(G) = 2$. γ_{fr} (G)+χ(G)=2n-3=5. Let G be isomorphic to $C_3(P_2)$, then for $C_3(P_2)$, $\gamma_{fr}(G) = 2, and\ \chi(G) = 3$$\gamma_{fr}$ (G)+χ(G)=2n-3=5. Let G be isomorphic to K_4 , then for K_4, $\gamma_{fr}(G) = 1, and\ \chi(G) =$

$4\gamma_{fr}$ (G)+χ(G)=2n-3=5. Conversely assume that γ_{fr} (G)+ χ(G)=2n-3. This is possible only if γ_{fr} (G)=n and χ(G)=n-3 (or) γ_{fr} (G)=n-1 and χ(G)=n-2 (or) γ_{fr} (G)=n-2 and χ(G)=n-1(or) γ_{fr} (G)=n-3 and χ(G)=n.

Case (i) Let γ_{fr} (G)=n and χ(G)=n-3.

Since χ (G)= n-3, G contains a clique K on n-3 vertices. Let S={x,y,z}∈ G-K_{n-3}. Then $< S >= K_3$, $\overline{K_3}$, $K_2 \cup K_1$,P_3

Subcase (i) Let $< S >= K_3$. Since G is connected, x is adjacent to some u_i of K_{n-3}. Then {x,u_i} is γ_{fr} - set, so that γ_{fr} (G)=2 and hence n=2. But χ(G)=n-3=negative value. Which is a contradiction. Hence no fuzzy graph exists.

Subcase (ii) Let $< S >= \overline{K_3}$ Since G is connected, one of the vertices of K_{n-3} say u_i is adjacent to all the vertices of S or two vertices of S or one vertex of S. If u_i for some i is adjacent to all the vertices of S, then {x,y,z,u_i} is a γ_{fr} -set of G, so thatγ_{fr} (G)=4 and hence n=4. But χ(G)=4-3=1which is for totally disconnected fuzzy graph. Which is a contradiction. Hence no fuzzy graph exists. Since G is connected u_i for some i is adjacent to two vertices of S say x and y and z is adjacent to u_j for i≠j in K_{n-3}, then {x,y,z,u_j} for i≠j in K_{n-3} is γ_{fr} -set of G, so that γ_{fr} (G)=4 and hence n=4. But χ(G)=n-3=1which is totally disconnected fuzzy graph. Which is a contradiction. Hence no fuzzy graph exists. If u_i for some i is adjacent to x and u_j is adjacent to y and u_k is adjacent to z, for i≠j≠k in K_{n-3} then {x,y,z,u_k} is a γ_{fr} -set of G. so that γ_{fr} (G)=4and hence n=4. But χ(G)=n-3=1 which is for totally disconnected fuzzy graph. Which is a contradiction. Hence no fuzzy graph exists.

Subcase (iii) Let $< S >= P_3 = \{x, y, z\}$. Since G is connected, x(or equivalently z) is adjacent to u_i for some i in K_{n-3}. Then {z,u_i} is a γ_{fr} -set of G. so that γ_{fr} (G)=2 and hence n=2. But χ(G)=n-3=negative value. Which is a contradiction. Hence no fuzzy graph exists. If u_i is adjacent to y then {x,z,u_j} for some i≠j is a γ_{fr} -set of G. so that γ_{fr} (G)=3 and hence n=3. But χ(G)=n-3=0. Which is a contradiction. Hence no fuzzy graph exists.

Subcase (iv) Let $< S >= K_2 \cup K_1$ Let xy be the edge and z be the isolated vertex of $K_2 \cup K_1$ Since G is connected, there exists a u_i in K_{n-3} is adjacent to x and z. Then { y,z,u_j} for some i≠j is a γ_{fr} -set of G, so that γ_{fr} (G)=3 and hence n=3. But χ(G)=n-3=0. Which is a contradiction. Hence no fuzzy graph exists. If z is adjacent to u_j for some i≠j then { y,z,u_j} for some i≠j is a γ_{fr} -set of G, so that γ_{fr} (G)=3 and hence n=3. But χ(G)=n-3=0. Which is a contradiction. Hence no fuzzy graph exists.

Case (ii) Let γ_{fr} (G)=n-1 and χ(G)=n-2.

Since χ(G)=n-2, G contains a clique K on n-2 vertices. Let S={x,y}∈G-K_{n-2}. Then $< S >= K_2$ or $\overline{K_2}$

Subcase (a) Let $< S >= K_2$ Since G is connected, x(or equivalently y) is adjacent to some u_i of K_{n-2}. Then {y,u_j} for some i≠j is γ_{fr} - set, so that γ_{fr} (G)=2 and hence n=3. But χ(G)=n-2=1 for which G is totally disconnected, which is a contradiction. Hence no fuzzy graph exists.

Subcase (b) Let $< S > = \overline{K_2}$ Since G is connected, x is adjacent to some u_i of K_{n-2}. Then y is adjacent to the same u_i of K_{n-2}. Then {x,y,u_j} for some i≠j is γ_{fr} - set, so that γ_{fr} (G)=3 and hence n=4. But χ(G)=n-2=2. Then G is isomorphic to $K_{1,3}$. Otherwise x is adjacent to u_i of K_{n-2} for some i and y is adjacent to u_j of K_{n-2} for i≠j. Then {x,y,u_k} for some i≠j≠k is γ_{fr} - set, so that γ_{fr} (G)=3 and hence n=4. But χ(G)=n-2=2. Then $K_{n-2}=K_2$ in K_2 the vertex u_k cannot be exist. Which is a contradiction. In this case also no fuzzy graph exists.

Case (iii) Let γ_{fr} (G)=n-2 and χ(G)=n-1.

Since χ(G)=n-1, G contains a clique K on n-1 vertices. Let x be a vertex of K_{n-1}. Since G is connected the vertex x is adjacent to one vertex u_i for some i in K_{n-1} so that {x,u_i} γ_{fr} -set of G γ_{fr} (G)=2, we have n=4 and $\chi = 3$ .Then K=K_3. If x is adjacent to u_i, then $G \cong C_3(P_2)$.

Case (iv) Let γ_{fr} (G)=n-3 and χ(G)=n

Since χ(G)=n, G=K_n, But for K_n, γ_{fr} (G)=1, so that n=4, $\chi = 4$ Hence G≅K_4. Hence the proof.

Theorem 2.5: For any connected fuzzy graph G, γ_{fr} (G)+χ(G)=2n-4 and the equality holds if and only if G≅$K_{1,4}$, S($K_{1,3}$), K_3(2P_2), K_3(P_2,P_2,0), K_4(P_1) P_4,K_5 .

Proof: If G is any one of the fuzzy graphs in the theorem, then it can be verified that γ_{fr} (G)+ χ(G)=2n-4.Conversely assume that γ_{fr} (G)+χ(G)=2n-4. This is possible only if γ_{fr} (G)=n and χ(G)=n-4 (or) γ_{fr} (G)=n-1 and χ(G)=n-3 (or) γ_{fr} (G)=n-2 and χ(G)=n-2 (or) γ_{fr} (G)=n-3 and χ(G)=n-1 (or) γ_{fr} (G)=n-4 and χ(G)=n.

Case (i) Let γ_{fr} (G)=n and χ(G)=n-4.

Since χ(G)=n-4, G contains a clique K on n-4 vertices. Let S = {v_1, v_2, v_3, v_4}∈G-K_{n-4}. Then the induced subfuzzy graph <s> has the following possible cases K_4,$\overline{K}_4$,P_4,C_4, P_3UK_1, K_2UK_2, K_3UK_1, $K_{1,3}$, K_4-e, C_3(1,0,0), K_2U$\overline{K}_2$

In all the above cases, it can be verified that no new fuzzy graphs exists.

Case(ii) Let γ_{fr} (G)=n-1 and χ(G)=n-3.

Since χ(G)=n-3, G contains a clique K on n-3 vertices. Let S={x,y,z}∈G-K_{n-3}. Then <S>=K_3 ,$\overline{K_3}$, K_2∪K_1,P_3

Subcase (i) Let $< S > = K_3$. Since G is connected, x is adjacent to some u_i of K_{n-3}. Then {z,u_i} is - set, so that γ_{fr} (G)=2 and hence n=3. But χ(G)=n-3=0. Which is a contradiction. Hence no fuzzy graph exists.

Subcase (ii) Let $< S > = \overline{K_3}$ Since G is connected, one of the vertices of K_{n-3} say u_i is adjacent to all the vertices of S or two vertices of S or one vertex of S. If u_i for some i is adjacent to all the vertices of S, then {x,y,z,u_j} for some i≠j in K_{n-3} is γ_{fr} -set of G. so that γ_{fr} (G)=4 and hence n=5. But χ(G)=n-3=2. Then $K_{n-3}=K_2$ so G is isomorphic to $K_{1,4}$. If u_i for some i is adjacent to two vertices of S say x and y then G is connected, z is adjacent to u_j for i≠j in K_{n-3}, then then {x,y,z,u_j} for some i≠j in K_{n-3} is γ_{fr} -set of G. so that γ_{fr} (G)=4 and hence n=5. But χ(G)=n-3=2. Then $K_{n-3}=K_2$ so G is isomorphic to S($K_{1,3}$). If u_i for some i is adjacent to x and u_j is adjacent to y and u_k is adjacent to z, then {x,y,z,u_l } for i≠j≠k≠l in K_{n-3} is γ_{fr} -set of G. so that γ_{fr} (G)=4 and hence n=5. χ(G)=2 Then $K_{n-2}=K_2$ in K_2 the vertex u_l cannot be exist. Which is a contradiction. In this case also no fuzzy graph exists.

Subcase (iii) Let $< S > = P_3 = \{x, y, z\}$. Since G is connected, x(or equivalently z) is adjacent to u_i for some i in K_{n-3}. Then {z,u_i} is γ_{fr} -set of G. so that γ_{fr} (G)=2 and hence n=3. But χ(G)=n-3=0. Which is a contradiction. Hence no fuzzy graph exists. If u_i is adjacent to y then {x,z,u_j,} is γ_{fr} -set of G. so that γ_{fr} (G)=3 and hence n=4. But χ(G)=n-3=1 which is for totally disconnected fuzzy graph. Which is a contradiction. Hence no fuzzy graph exists.

Subcase (iv) Let $< S > = K_2 \cup K_1$ Let xy be the edge and z be a isolated vertex of $K_2 \cup K_1$ Since G is connected, there exists a u_i in K_{n-3} is adjacent to x and z also adjacent to same u_i Then {y,z,u_k} is a γ_{fr} -set of G. So that γ_{fr} (G)=3 and hence n=4. But χ(G)=n-3=1 which is for totally disconnected fuzzy graph, Which is a contradiction. Hence no fuzzy graph exists. If z is adjacent to u_j for some i≠j then {y,z,u_k} is a γ_{fr} -set of G. So that γ_{fr} (G)=3 and hence n=4. But χ(G)=n-3=1 which is for totally disconnected fuzzy graph, Which is a contradiction. Hence no fuzzy graph exists.

Case (iii) Let γ_{fr} (G)=n-2 and χ(G)=n-2.

Since χ (G)= n-2, G contains a clique K on n-2 vertices. Let S={x,y}∈G-K_{n-2}. Then $< S > = K_2$ or $\overline{K_2}$

Subcase (a) Let $< S > = K_2$. Since G is connected, x(or equivalently y) is adjacent to some u_i of K_{n-2}. Then {y,u_j} is γ_{fr} - set, so that γ_{fr} (G)=2 and hence n=4. But χ(G)=n-2=2. Then G≅P_4.

Subcase (b) Let $< S > = \overline{K_2}$, since G is connected, x is adjacent to some u_i of K_{n-2}. Then y is adjacent to the same u_i of $K_{n-2.}$ Then {x,y,u_j} is γ_{fr} - set, so that γ_{fr} (G)=3 and hence n=5. But χ(G)=n-2=3. So that $K_{n-2}=K_3$ Then G≅$K_3(2P_2)$, or y is adjacent to u_j of K_{n-2} for i≠j. In this {x,u_j,u_k} is γ_{fr} - set, so that γ_{fr} (G)=3 and hence n=5. But χ(G)=3. So that $K_{n-2}=K_3$Then G≅$K_3(P_2,P_2,0)$

Case (iv) Let γ_{fr} (G)=n-3 and χ(G)=n-1.

Since χ (G)= n-1, G contains a clique K on n-1 vertices. Let x be a vertex of G-K_{n-1}. Since G is connected the vertex x is adjacent to one vertex u_i for some i in K_{n-1}, then {x,u_i} is γ_{fr} - set of G so that γ_{fr} (G) = 2, we have n=5 and $\chi = 4$. Then $K_{n-1}=K_4$ Let u_1,u_2,u_3,u_4 be the vertices of K_4. Then x must be adjacent to only one vertex of G-K_3. Without loss of generality let x be adjacent to u_1.If d(x)=1, then $G \cong K_4(P_2)$.

Case (v) Let γ_{fr} (G)=n-4 and χ(G)=n

Since χ(G)=n, G=K_n, But for K_n, γ_{fr}(G)=1, so that n=5, $\chi = 5$. Hence G≅K_5. Hence the proof.

CONCLUSION

In this paper, upper bound of the sum of fuzzy restrained domination and chromatic number is proved. In future this result can be extended to various domination parameters. The structure of the graphs had been given in this paper can be used in models and networks. The authors have obtained similar results with large cases of fuzzy graphs for which γ_{fr}(G)+χ(G)=2n-5, γ_{fr}(G)+χ(G)=2n-6, γ_{fr} (G)+χ(G)=2n-7,γ_{fr} (G)+χ(G)=2n-8

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