Product Cost Method to Find Solution of Transportation Problem

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Abstract

This paper is an attempt to find a new method (Product Cost Method) for finding initial feasible solution of transportation problems. This method gives better initial solution of a transportation problem and sometimes equal to optimal solution. I have given two examples in which this method gives solution equal to optimal solution.

Keywords: Transportation Problem, Initial Basic Feasible Solution, Optimal Solution

1. Introduction

A certain class of linear programming problem known as transportation problems arises very frequently in practical applications. The classical transportation problem received its name because it arises naturally in the contacts of determining optimum shipping pattern. For example: A product may be transported from factories to retail stores. The factories are the sources and the store are the destinations. The amount of products that is available is known and the demands are also known. The problem is that different legs of the network joining the sources to the destination have different costs associated with them. The aim is to find the minimum cost routing of products from the supply point to the destination. The general transportation problem can be formulated as: A product is available at each of m origin and it is required that the given quantities of the product be shipped to each of n destinations. The minimum cost of shipping a unit of the product from any origin to any destination is known. The shipping schedule which minimizes the total cost of shipment is to be determined. The problem can be formulated as:

$$\text{Min Z}=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1,2,...m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1,2,...n$$

$x_{ij} \geq 0$, for all i, j

For each supply point $i, (i = 1,2,...m)$ and demand point $j, (j = 1,2,...n)$
c_{ij}=unit transportation cost from i^{th} source to j^{th} destination
x_{ij}=amount of homogeneous product transported from i^{th} source to j^{th} destination
a_i=amount of supply at i^{th} source.
b_j=amount of demand at j^{th} destination.
where a_i and b_j are given non-negative numbers and assumed that total supply is equal to total demand, i.e. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, then transportation problem is called balanced otherwise it is called unbalanced. The aim is to minimize the objective function satisfying the above mentioned constraints. In the classical transportation problem of linear programming, the traditional objective is one of minimizing the total cost.

Because of the special structure of the transportation model, the problem can also be represented as Table 1.

Table 1: Tabular representation of model (α)

Destination $\rightarrow$ source $\downarrow$	D_1	D_2	$\dots$	D_n	supply(a_i)
S_1	c_{11}	c_{12}	$\dots$	c_{1n}	a_1
S_2	c_{21}	c_{22}	$\dots$	c_{2n}	a_2
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\vdots$
S_m	c_{m1}	c_{m2}	$\dots$	c_{mn}	a_m
Demand (b_j)	b_1	b_2	$\dots$	b_n	

1.1. Algorithm of Proposed Method

To proceed with proposed method the given steps are followed:

step 1. Represent the given TP into the form of cost matrix as Table 1.

step 2. Balance the given TP, if it is not balanced by adding dummy row/column according to requirement of supply/demand.

step 3. Find product of costs of each row and each column of the given TP.

step 4. Select the row or column which has minimum product and make maximum possible allocation in the cell having minimum cost in the selectd row or column.

step 5. Cross the row or column for which demand or supply is satisfied

step 6. Repeat the step 4 and step 5 until whole demand and supply not satisfied.

step 7. Calculate the transportation cost by adding the multiplication of all allocations and corresponding cost.

2. Numerical Examples

Numerical example: Input data and solution obtained by applying product cost method for example is given in table 2-3

Table 2: Input data and optimal solution

Ex.	Input Data	Obtained Allocations by product cost Method	Obtained Cost	optimal solution
1	$[c_{ij}]_{3\times3}$=[50 100 150 ; 100 150 200; 150 200 250;]; $[a_i]_{3\times1}$=[10, 15, 25]; $[b_j]_{1\times3}$=[20, 15, 15]	$x_{11} = 10$ $x_{21} = 10$, $x_{22} = 5$, $x_{32} = 10$, $x_{33} = 15$,	8000	8000

Table 3: Input data and optimal solution

Ex.	Input Data	Obtained Allocations by product cost Method	Obtained Cost	optimal solution
1	$[c_{ij}]_{3\times3}$=[12 8 6 ; 4 2 12; 8 16 6;]; $[a_i]_{3\times1}$=[30, 40, 50]; $[b_j]_{1\times3}$=[60, 25, 35]	$x_{13} = 30$ $x_{21} = 15$, $x_{22} = 25$, $x_{31} = 45$, $x_{33} = 5$,	680	680

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