Connectedness and Compactness via Semi-Star-Alpha-Open Sets

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Abstract: In this paper, we introduce new concepts, namely semi*α-connectedness and semi*α-compactness using semi*α-open sets. We investigate their basic properties. We also discuss their relationships with already existing concepts of connectedness and compactness.

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I. INTRODUCTION

In 1974, Das [1] defined the concept of semi-connectedness in topological spaces and investigated its properties. Compactness is one of the most important, useful and fundamental concepts in topology. In 1981, Dorsett [2] introduced and studied the concept of semi-compact spaces. Since then, Hanna and Dorsett [3], Ganster [4] investigated the properties of semi-compact spaces. S. Pasunkili Pandian[5] introduced semi*-pre-compact spaces and investigated their properties. The authors have recently introduced and studied semi*-connectedness and semi*-compactness [6] in topological spaces. They have also defined semi*α-open sets [7] and semi*α-closed sets [8] and investigated their properties.

In this paper, we introduce the concept of semi*α-connected spaces. We investigate their basic properties. We also discuss their relationship with already existing concepts namely connectedness, semi-connectedness, α-connectedness, semi-pre-connectedness and semi*-connectedness. Further we define semi*α-compact spaces and investigate their properties. We also show that the concept of semi*α-compactness is stronger than each of the concepts of compactness, semi*-compactness and α-compactness but weaker than the concepts of semi*-pre-compactness, semi-compactness and semi*-compactness.

II. PRELIMINARIES

Throughout this paper X will always denote a topological space. If A is a subset of the space X, *Cl*(A) and *Int*(A) denote the closure and the interior of A respectively.

Definition 2.1:
A subset A of a topological space (X, τ) is called
(i) ***generalized closed*** (briefly g-closed)[9] if *Cl*(A)⊆U whenever A⊆U and U is open in X.
(ii) ***generalized open*** (briefly g-open)[9] if X\A is g-closed in X.

Definition 2.2:
A be a subset of X. The ***generalized closure*** [10] of A is defined as the intersection of all g-closed sets containing A and is denoted by *Cl**(A).

Definition 2.3:
A subset A of a topological space (X, τ) is called
(i) ***semi-open*** [11] (resp. ***α-open***[12], ***semi α-open***[13] ***semi-preopen***[14] ***semi*-open***[15], ***semi*α-open***[7], ***semi*-preopen***[5]) if A⊆*Cl*(*Int*(A)) (resp. A⊆*Int*(*Cl*(*Int*(A))), A⊆*Cl*(*Int*(*Cl*(*Int*(A)))), A⊆*Cl*(*Int*(*Cl*(A))) A⊆*Cl**(*Int*(A)), A⊆*Cl**(α*Int*(A)), A⊆*Cl**(*pInt*(A))).
(ii) ***semi-closed*** [1] (resp. ***semi α-closed***[16], ***semi-preclosed***[14], ***semi*-closed***[17], ***semi*α-closed***[8],

semi*-preclosed[5]**)** if equivalently if *Int*(*Cl*(A))⊆A (resp. *Int*(*Cl*(*Int*(*Cl*(A))))⊆A, *Int*(*Cl*(*Int*(A)))⊆A, *Int**(*Cl*(A))⊆A), *Int**(α*Cl*(A))⊆A, *Int**(*pCl*(A))⊆A.

(iii) ***semi*α-regular*** [8] if it is both semi*α-open and semi*α-closed.

Definition 2.4:

Let A be a subset of X. Then the ***semi*α-closure*** [8] of A is defined as the intersection of all semi*α-closed sets in X containing A and is denoted by *s*αCl*(A).

Theorem 2.5[7]**:**

(i) Every α-open set is semi*α-open.
(ii) Every open set is semi*α-open.
(iii) Every semi*-open set is semi*α-open.
(iv) Every semi*α-open set is semi α-open.
(v) Every semi*α-open set is semi*-preopen.
(vi) Every semi*α-open set is semi-preopen.
(vii) Every semi*α-open set is semi-open.

Definition 2.6:

If A is a subset of X, the semi*α-frontier [18] of A is defined by s*αFr(A)=*s*αCl*(A)*s*αInt*(A)**.**

Theorem 2.7[18]**:**

Let A be a subset of a space X. Then A is semi*α-regular if and only if s*αFr(A)=ϕ.

Theorem 2.8[8]**:**

If A is a subset of X, then

(i) *s*αCl*(X\A)=X*s*αInt*(A).
(ii) *s*αInt*(X\A)=X*s*αCl*(A).
(iii) *A* is semi*α-closed if and only if *s*αCl(A)=A*.

Definition 2.9:

A topological space X is said to be connected [19] (resp. semi-connected [1], α-connected, semi*-connected [6], semi*-pre-connected [5]) if X cannot be expressed as the union of two disjoint nonempty open (resp. semi-open, α-open, semi*-open, semi*-preopen) sets in X.

Definition 2.10:[19]

A subset A of a topological space (X, τ) is called clopen if it is both open and closed in X.

Theorem 2.11 [19]**:**

A topological space X is connected if and only if the only clopen subsets of X are ϕ and X.

Definition 2.12:

A collection **B** of open (resp. semi-open) sets in X is called an open (resp. semi-open) cover of A⊆X if A⊆∪{U_α : U_α∈**B** } holds.

Definition 2.13:

A space X is said to be compact [19] (resp. semi-compact [2]) if every open (resp. semi-open) cover of X has a finite subcover.

Definition 2.14: [20]

A function *f* :X⟶Y is said to be

(i) semi*α-continuous if f^{-1}(V) is semi*α-open in X for every open set V in Y.
(ii) semi*α-irresolute if f^{-1}(V) is semi*α-open in X for every semi*α-open set V in Y.
(iii) semi*α-open if *f*(V) is semi*α-open in Y for every open set V in X.
(iv) semi*α-closed if *f*(V) is semi*α-closed in Y for every closed set V in X.
(v) pre-semi*α-open if *f*(V) is semi*α-open in Y for every semi*α-open set V in X.
(vi) pre-semi*α-closed if *f*(V) is semi*α-closed in Y for every semi*α-closed set V in X.
(vii) totally semi*α-continuous if f^{-1}(V) is semi*α-regular in X for every open set V in Y.
(vii) semi*α-totally continuous if f^{-1}(V) is clopen in X for every semi*α-open set V in Y.
(vii) strongly semi*α-continuous if f^{-1}(V) is semi*α-regular in X for every subset V in Y.
(viii) contra-semi*α-continuous if f^{-1}(V) is semi*α-closed in X for every open set V in Y.
(ix) contra-semi*α-irresolute if f^{-1}(V) is semi*α-closed in X for every semi*α-open set V in Y.

Theorem 2.15:[20]

Let *f* :X⟶Y be a function. Then

(i) *f* is semi*α-continuous if and only if f^{-1}(F) is semi*α-closed in X for every closed set F in Y.
(ii) *f* is semi*α-irresolute if and only if f^{-1}(F) is semi*α-closed in X for every semi*α-closed set F in Y.
(iii) *f* is contra-semi*α-continuous if and only if f^{-1}(F) is semi*α-open in X for every closed set F in Y.
(iv) *f* is contra-semi*α-irresolute if and only if f^{1}(F)is semi*α-open in X for every semi*α-closed set F in Y.
(iv) *f* is semi*α-totally continuous if f^{-1}(F) is clopen in X for every semi*α-closed set F in Y.
(iv) *f* is totally semi*α-continuous if f^{-1}(F) is semi*α-regular in X for every closed set F in Y.

Remark 2.16:[7]

If (*X*, τ) is a locally indiscrete space, τ =αO(*X*, τ)= S*αO(*X*, τ) = SαO(*X*, τ)=S*O(X,τ)=SO(*X*, τ).

Theorem 2.17:[7]

A subset A of X is semi*α-open if and only if every point of A is a semi*α-interior point of A.

III. SEMI*α-CONNECTED SPACES

In this section we introduce semi*α-connected spaces. We give characterizations for semi*α-connected spaces and also investigate their basic properties.

Definition 3.1:

A topological space X is said to be ***semi*α-connected*** if X cannot be expressed as the union of two disjoint nonempty semi*α-open sets in X.

Theorem 3.2:

(i) Every semi*α-connected space is α-connected.

(ii) Every semi*α-connected space is connected.

(iii) Every semi*α-connected space is semi*-connected.

(iv) Every semi α-connected space is semi*α-connected.

(v) Every semi*-pre-connected space is semi*α-connected.

(vi) Every semi-pre-connected space is semi*α-connected.

(vii) Every semi-connected space is semi*α-connected.

Proof: Follows from Theorem 2.5 and definitions.

Remark 3.3:

It can be seen that the converse of each of the statements in Theorem 3.2 is not true.

Definition 3.4:

The sets A and B in a topological space X are said to be ***semi*α-separated*** if $A\cap s^*\alpha Cl(B)=s^*\alpha Cl(A)\cap B=\phi$.

Theorem 3.5:

For a topological space X, the following statements are equivalent:

(i) X is semi*α-connected.

(ii) X cannot be expressed as the union of two disjoint nonempty semi*α-closed sets in X.

(iii) The only semi*α- regular subsets of X are ϕ and X itself.

(iv) Every semi*α-continuous function of X into a discrete space Y is constant.

(v) Every nonempty proper subset of X has non-empty semi*α-frontier.

(vi) X cannot be expressed as the union of two non-empty semi*α-separated sets.

Proof: (i)⇒(ii): Let X be a semi*α-connected space. Suppose $X=A\cup B$ where A and B are disjoint non-empty semi*α-closed sets. Then $A=X\backslash B$ and $B=X\backslash A$ are disjoint non-empty semi*α-open sets in X. This is a contradiction to X is semi*α-connected. This proves (ii).

(ii)⇒(i): Assume that X cannot be expressed as the union of two disjoint nonempty semi*α-closed sets in X. Suppose $X=A\cup B$ where A and B are disjoint non-empty semi*α-open sets. Then $A=X\backslash B$ and $B=X\backslash A$ are disjoint non-empty semi*α-closed sets in X. This is a contradiction to (ii).

(i)⇒(iii): Suppose X is a semi*α-connected space. Let A be non-empty proper subset of X that is semi*α-regular. Then $X\backslash A$ is a non-empty semi*α-open and $X=A\cup(X\backslash A)$. This is a contradiction to X is semi*α-connected.

(iii)⇒(i): Suppose $X=A\cup B$ where A and B are disjoint non-empty semi*α-open sets. Then $A=X\backslash B$ is semi*α-closed. Thus A is a non-empty proper subset that is semi*α-regular. This is a contradiction to (iii)

(iii)⇒(iv): Let f be a semi*α-continuous function of the semi*α-connected space X into the discrete space Y. Then for each $y\in Y$, $f^{-1}(\{y\})$ is a semi*α-regular set of X. Since X is semi*α-connected, $f^{-1}(\{y\})=\phi$ or X. If $f^{-1}(\{y\})=\phi$ for all $y\in Y$, then f fails to be a function. Therefore $f^{-1}(\{y_0\})=X$ for a unique $y_0\in Y$. This implies $f(X)=\{y_0\}$ and hence f is a constant function.

(iv)⇒(iii): Let U be a semi*α- regular set in X. Suppose $U\neq\phi$. We claim that $U=X$. Otherwise, choose two fixed points y_1 and y_2 in Y. Define $f:X\longrightarrow Y$ by

$$f(x)=\begin{cases} y_1 \text{ if } x\in U \\ y_2 \text{ otherwise}\end{cases}$$

Then for any open set V in Y, $f^{-1}(V)$ equals U if V contains y_1 but not y_2, equals $X\backslash U$ if V contains y_2 but not y_1, equals X if V contains both y_1 and y_2 and equals ϕ otherwise. In all the cases $f^{-1}(V)$ is semi*α-open in X. Hence f is a non-constant semi*α-continuous function of X into Y. This is a contradiction to our assumption. This proves that the only semi*α- regular subsets of X are ϕ and X.

(iii)⇒(v): Suppose that a space X is semi*α-connected. If possible, let A be a non-empty proper subset of X. We claim that $s^*\alpha Fr(A)\neq\phi$. If possible, let $s^*\alpha Fr(A)=\phi$. Then by Theorem 2.7, A is semi*α-regular. This is a contradiction.

(v)⇒(iii): Suppose that every non-empty proper subset of X has a non-empty semi*α-frontier. The only semi*α- regular subsets of X are ϕ and X itself. On the contrary, suppose that X has a non-empty proper subset A which is semi*α-regular. By Theorem 2.7, s*α$Fr(A)$=ϕ. This contradiction proves (iii).

(i)⇒(vi): Suppose $X=A\cup B$ where A and B are disjoint non-empty semi*α-separated sets in X. Since $A\cap$s*α$Cl(B)$=ϕ, s*α$Cl(B)\subseteq X\backslash A=B$ and hence s*α$Cl(B)=B$ and so by Theorem 2.8(iii), B is semi*α-closed. Therefore A is semi*α-open. Similarly, B is semi*α-open. Hence X is not semi*α-connected. This is contradiction to (i).

(vi)⇒(i): Suppose X is not semi*α-connected. Then X can be written as $X=A\cup B$ where A and B are disjoint non-empty semi*α-open sets. Now $A=X\backslash B$ is semi*α-closed and hence by Theorem 2.8(iii), s*α$Cl(A)=A$ and so s*α$Cl(A)\cap B$=ϕ. Similarly $A\cap$s*α$Cl(B)$=ϕ. Thus A and B are semi*α-separated. This is a contradiction to (vi).

Theorem 3.6:
Let $f:X\longrightarrow Y$ be a semi*α-continuous bijection and X be semi*α-connected. Then Y is connected.

Proof: Let $f:X\longrightarrow Y$ be semi*α-continuous surjection and X be semi*α-connected. Let V be a clopen subset of Y. By Definition 2.14(i) $f^{1}(V)$ is semi*α-open and by Theorem 2.15(i), $f^{1}(V)$ is semi*α-closed and hence $f^{1}(V)$ is semi*α-regular in X. Since X is semi*α-connected, by Theorem 3.5 $f^{-1}(V)$=ϕ or X. Hence V=ϕ or Y. This proves that Y is connected.

Theorem 3.7:
Let $f:X\longrightarrow Y$ be a semi*α-irresolute bijection. If X is semi*α-connected, so is Y.

Proof: Let $f:X\longrightarrow Y$ be a semi*α-irresolute surjection and let X be semi*α-connected. Let V be a subset of Y that is semi*α-regular in Y. By Definition 2.14(ii) and by Theorem 2.15(ii), $f^{-1}(V)$ is semi*α-regular in X. Since X is semi*α-connected, $f^{-1}(V)$=ϕ or X. Hence V=ϕ or Y. This proves that Y is semi*α-connected.

Theorem 3.8:
Let f:X⟶Y be a pre-semi*α-open and pre-semi*α-closed bijection. If Y is semi*α-connected, so is X.

Proof: Let A be subset of X that is semi*α- regular in X. Since f is both pre-semi*α-open and pre-semi*α-closed, $f(A)$ is semi*α- regular in Y. Since Y is semi*α-connected, $f(A)$=ϕ or Y. Hence A=ϕ or X. Therefore by Theorem 3.5, X is semi*α-connected.

Theorem 3.9:
If $f:X\longrightarrow Y$ is a semi*α-open and semi*α-closed bijection and Y is semi*α-connected, then X is connected.

Proof: Let A be a clopen subset of X. Since f is semi*α-open, $f(A)$ is semi*α-open in Y. Since f is a semi*α-closed map, $f(A)$ is semi*α-closed in Y. Hence $f(A)$ is semi*α-regular in Y. Since Y is semi*α-connected, by Theorem 3.5, $f(A)$=ϕ or Y. Hence A=ϕ or X. By Theorem 2.11, X is connected.

Theorem 3.10:
If there is a semi*α-totally continuous function from a connected space X onto Y, then the only semi*α-open sets in Y are ϕ and Y.

Proof: Let f be a semi*α-totally continuous function from a connected space X onto Y. Let V be any open set in Y. Then by Theorem 2.5(ii), V is semi*α-open in Y. Since f is semi*α-totally continuous, $f^{-1}(V)$ is clopen in X. Since X is connected, by Theorem 2.11, $f^{-1}(V)$=ϕ or X. This implies V=ϕ or Y.

Theorem 3.11:
If $f:X\longrightarrow Y$ is a strongly semi*α-continuous bijection and Y is a space with at least two points, then X is not semi*α-connected.

Proof: Let $y\in Y$. Then $f^{-1}(\{y\})$ is a non-empty proper subset that is semi*α-regular in X. Hence by Theorem 3.5, X is not semi*α-connected.

Theorem 3.12:
Let $f:X\longrightarrow Y$ be a contra-semi*α-continuous surjection and X be semi*α-connected. Then Y is connected.

Proof: Let $f:X\longrightarrow Y$ be a contra-semi*α-continuous surjection and X be semi*α-connected. Let V be a clopen subset of Y. By Definition 2.14(viii) and by Theorem 2.15(iii), $f^{-1}(V)$ is semi*α-regular in X. Since X is semi*α-connected, $f^{-1}(V)$=ϕ or X. Hence V=ϕ or Y. This proves that Y is connected.

Theorem 3.13:
Let $f:X\longrightarrow Y$ be a semi*α-irresolute bijection. If X is semi*α-connected, so is Y.

Proof: Let $f:X\longrightarrow Y$ be a semi*α-irresolute bijection and let X be semi*α-connected. Let V be a subset of Y that is semi*α-regular in Y. By Definition 2.14(ii) and by Theorem 2.15(ii), $f^{-1}(V)$ is semi*α-regular in X. Since X is semi*α-connected, $f^{-1}(V)=\phi$ or X. Hence $V=\phi$ or Y. This proves that Y is semi*α-connected.

Theorem 3.14:
Every contra-semi*α-continuous function from a semi*α-connected space into a T_1 space is necessarily constant.

Proof: Let $f:X\longrightarrow Y$ be a contra-semi*α-continuous function and X be semi*α-connected and Y be T_1. Since Y is T_1, for each $y\in Y$, $\{y\}$ is closed in Y. Since f is contra-semi*α-continuous, by Theorem 2.15(iii), $f^{-1}(\{y\})$ is semi*α-open in X. Therefore $\{f^{-1}(\{y\}): y\in Y\}$ is a collection of pair wise disjoint semi*α-open sets in X. Since X is semi*α-connected,
$f^{-1}(\{y_0\})=X$ for some fixed $y_0\in Y$. Hence $f(X)=y_0$. Thus f is a constant function.

Theorem 3.15:
Every contra-semi*α-irresolute function from a semi*α-connected space into a semi*α-T_1 space is necessarily constant.

Proof: Let $f:X\longrightarrow Y$ be a contra-semi*α-irresolute function and X be semi*α-connected and Y be semi*α-T_1. Since Y is semi*α-T_1, for each $y\in Y$, $\{y\}$ is semi*α-closed in Y. Since f is contra-semi*α-continuous, $f^{-1}(\{y\})$ is semi*α-open in X. Therefore $\{f^{-1}(\{y\}): y\in Y\}$ is a collection of pair wise disjoint semi*α-open sets in X. Since X is semi*α-connected, $f^{-1}(\{y_0\})=X$ for some fixed $y_0\in Y$. Hence $f(X)=y_0$. Thus f is constant.

IV. SEMI*α-COMPACT SPACES

In this section we introduce semi*α-compact spaces and study their properties. We give characterizations for these spaces.

Definition 4.1:
A collection $\mathcal{A}$ of semi*α-open sets in X is called a ***semi*α-open cover*** of a subset B of X if
$B\subseteq\cup\{U_\alpha : U_\alpha\in\mathcal{A}\}$ holds.

Definition 4.2:
A space X is said to be ***semi*α-compact*** if every semi*α-open cover of X has a finite subcover.

Definition 4.3:
A subset B of X is said to be ***semi*α-compact relative to*** X if for every semi*α-open cover $\mathcal{A}$ of B**,** there is a finite subcollection of $\mathcal{A}$ that covers B.

Remark 4.5:
Every finite topological space is semi*α-compact.

Theorem 4.6:
(i) Every semi*α-compact space is α-compact.
(ii) Every semi*α-compact space is compact.
(iii) Every semi*α-compact space is semi*-compact.
(iv) Every semi-α-compact space is semi*α-compact.
(v) Every semi*-pre-compact space is semi*α-compact.
(vi) Every semi-pre-compact space is semi*α-compact.
(vii) Every semi-compact space is semi*α-compact.

Proof: Follows from Theorem 2.5 and Definitions.

Theorem 4.7:
Every semi*α-closed subset of a semi*α-compact space X is semi*α-compact relative to X.

Proof: Let A be a semi*α-closed subset of a semi*α-compact space X. Let $\mathbf{B}$ be a semi*α-open cover of A. Then $\mathbf{B}\cup\{X\backslash A\}$ is a semi*α-open cover of X. Since X is semi*α-compact, this cover contains a finite subcover of X and hence contains a finite subcollection of $\mathbf{B}$ that covers A. This shows that A is semi*α-compact relative to X.

Theorem 4.8:
A space X is semi*α-compact if and only if for every family of semi*α-closed sets in X which has empty intersection has a finite subfamily with empty intersection.

Proof: Suppose X is semi*α-compact and $\{F_\alpha : \alpha\in\Delta\}$ is a family of semi*α-closed sets in X such that $\cap\{F_\alpha : \alpha\in\Delta\}=\phi$. Then $\cup\{X\backslash F_\alpha : \alpha\in\Delta\}$ is a semi*α-open cover for X. Since X is semi*α-compact, this cover has a finite subcover $\{X\setminus F_{\alpha_1}, X\setminus F_{\alpha_2},..., X\setminus F_{\alpha_n}\}$. That is, $X=\cup\{X\setminus F_{\alpha_i} : i=1,2,...,n\}$.

On taking the complements on both sides we get $\bigcap_{i=1}^{n} F_{\alpha_i}$ =ϕ. Conversely, suppose that every family of semi*α-closed sets in X which has empty intersection has a finite subfamily with empty intersection. Let $\{U_\alpha : \alpha\in\Delta\}$ be a semi*α-open cover for X. Then $\cup\{U_\alpha : \alpha\in\Delta\}=X$. Taking the complements, we get $\cap\{X\backslash U_\alpha : \alpha\in\Delta\}=\phi$. Since $X\backslash U_\alpha$ is semi*α-closed for each α∈Δ, by the assumption, there is a finite sub family, $\{X\backslash U_{\alpha_1}, X\backslash U_{\alpha_2},..., X\backslash U_{\alpha_n}\}$ with empty intersection. That is $\bigcap_{i=1}^{n}(X\backslash U_{\alpha_i})$ =ϕ. Taking the complements on both sides, we get $\bigcup_{i=1}^{n} U_{\alpha_i}$ =X. Hence X is semi*α-compact.

Theorem 4.9:
Let $f:X\longrightarrow Y$ be a semi*α-irresolute bijection. If X is semi*α-compact, then so is Y.
Proof: Let $f:X\longrightarrow Y$ be a semi*α-irresolute bijection and X be semi*α-compact. Let $\{V_\alpha\}$ be a semi*α-open cover for Y. Then $\{f^{-1}(V_\alpha)\}$ is a cover of X by semi*α-open sets. Since X is semi*α-compact, $\{f^{-1}(V_\alpha)\}$ contains a finite subcover, namely $\{f^{-1}(V_{\alpha_1}), f^{-1}(V_{\alpha_2}),..., f^{-1}(V_{\alpha_n})\}$. Then $\{V_{\alpha_1}, V_{\alpha_2},...,V_{\alpha_n}\}$is a finite subcover for Y. Thus Y is semi*α-compact.

Theorem 4.10:
Let $f:X\longrightarrow Y$ be a semi*α-continuous bijection and X be semi*α-compact. Then Y is compact.
Proof: Let $f:X\longrightarrow Y$ be a semi*α-continuous bijection and X be semi*α-compact. Let $\{V_\alpha\}$ be an open cover for Y. Then $\{f^{-1}(V_\alpha)\}$ is a cover of X by semi*α-open sets. Since X is semi*α-compact, $\{f^{-1}(V_\alpha)\}$ contains a finite sub cover, namely $\{f^{-1}(V_{\alpha_1}), f^{-1}(V_{\alpha_2}),..., f^{-1}(V_{\alpha_n})\}$. Then $\{V_{\alpha_1}, V_{\alpha_2},...,V_{\alpha_n}\}$ is a cover for Y. Thus Y is compact.

Theorem 4.11:
Let $f:X\longrightarrow Y$ be a pre-semi*α-open injection. If Y is semi*α-compact, then so is X .
Proof: Let $\{V_\alpha\}$ be a semi*α-open cover for X. Then $\{f(V_\alpha)\}$ is a cover of Y by semi*. Since Y is semi*α-compact, $\{f(V_\alpha)\}$contains a finite subcover, namely $\{f(V_{\alpha_1}), f(V_{\alpha_2}),..., f(V_{\alpha_n})\}$. Then $\{V_{\alpha_1}, V_{\alpha_2},..., V_{\alpha_n}\}$ is a finite subcover for X. Therefore X is semi*α-compact.

Theorem 4.12:
If $f:X\longrightarrow Y$ is a semi*α-open injection and Y is semi*α-compact, then X is compact.
Proof: Let $\{V_\alpha\}$ be an open cover for X. Then $\{f(V_\alpha)\}$is a cover of Y by semi*α-open sets. Since Y is semi*α-compact, $\{f(V_\alpha)\}$contains a finite sub cover, namely $\{f(V_{\alpha_1}), f(V_{\alpha_2}),..., f(V_{\alpha_n})\}$. Then $\{V_{\alpha_1}, V_{\alpha_2},..., V_{\alpha_n}\}$ is a finite sub cover for X. Thus X is compact.

Theorem 4.13:
Let $f:X\longrightarrow Y$ be a contra-semi*α-continuous function and Y be T_1. If X is semi*α-compact, then the range of f is finite. Further if Y is infinite, f cannot be onto.
Proof: Since Y is T_1, for each $y\in Y$, $\{y\}$ is closed in Y. Since f is contra-semi*α-continuous, by Theorem 2.15(iii), $f^{-1}(\{y\})$ is semi*α-open in X. Therefore $\{f^{-1}(\{y\}): y\in Y\}$ is a semi*α-open cover for X. Since X is semi*α-compact, there are $y_1, y_2,..., y_n$ in Y such that $\{f^{-1}(\{y_i\}): i=1,2,...n\}$ is a cover of X by semi*α-open sets . Therefore $\cup\{f^{-1}(\{y_i\}): i=1,2,...n\}=X$. That is, $f^{-1}(\{y_1, y_2, ..., y_n\})=X$. This implies $f(X)=\{y_1, y_2, ..., y_n\}$. Thus the range of f is finite. If Y is infinite, $f(X)\neq Y$. Hence f cannot be onto.

Theorem 4.14:
Let $f:X\longrightarrow Y$ be a contra-semi*α-irresolute function and Y be semi*α-T_1. If X is semi*α-compact, then the range of f is finite. Further if Y is infinite, f cannot be onto.
Proof: Since Y is semi*α-T_1, for each $y\in Y$, $\{y\}$ is semi*α-closed in Y. Since f is contra-semi*α-continuous, by Theorem 2.15(iv), $f^{-1}(\{y\})$ is semi*α-open in X. Therefore $\{f^{-1}(\{y\}): y\in Y\}$ is a semi*α-open cover for X. Since X is semi*α-compact, there are $y_1, y_2,..., y_n$ in Y such that $\{f^{-1}(\{y_i\}): i=1,2,...n\}$ is a cover of X by semi*α-open sets . Therefore $\cup\{f^{-1}(\{y_i\}): i=1,2,...n\}=X$. That is, $f^{-1}(\{y_1, y_2, ..., y_n\})=X$. This implies $f(X)=\{y_1, y_2, ..., y_n\}$. Thus the range of f is finite. If Y is infinite, $f(X)\neq Y$. Hence f cannot be onto.

V. CONCLUSION

We have introduced the concepts of semi*α-connectedness and semi*α-compactness in topological spaces using semi*α-open sets. We have also found characterizations for semi*α-connected spaces and semi*α-compact spaces. We have shown that the concept of semi*α-connectedness is stronger than each of the concepts of α-connectedness and semi*-connectedness but weaker than semi α-connectedness and semi*-pre-connectedness.

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