Cycle Related Divisor Cordial Graphs

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Abstract

A divisor cordial labeling of graph G with vertex set V is bijection from V to {1,2,……..V(G)} such that if each edge uv is assigned the label 1 if f(u)/ f(v) or f(v)/f(u) and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

A graph which admits divisor cordial labeling is the divisor cordial graph.

In this paper, it is proved that Shell(F_n),Umberlla(U(n,3)),Wheel(W_n),Globe(Gl(n))are divisor cordial graphs.

Keywords: Divisor cordial labeling, Divisor cordial graph 2010 Mathematics subject classification Number: 05C78

1. Introduction:

A graph G is a finite non empty set of objects called vertices together with a set of pairs of distinct vertices of G which is called edges. Each e = {uv} of vertices in E is called an edge or a line of G.

2. Preliminaries:

Definition:2.1

Let Gbe a graph and we define the concept of divisor cordial labeling as follows:

A divisor cordial labeling of a graph G with vertex set V is a bijection from V to {1,2,…..V(G)} such that if each edge uv is assigned the label 1 if f(u)/f(v) or f(v)/f(u) and 0 otherwise , then

the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

A graph which admits divisor cordial labeling is the divisor cordial graph.

Definition:2.2

A $\mathbf{F_n}$ (***shell***) is a graph obtained by taking n-3 chords in cycle $\mathbf{C_n}$ .The vertex at which all chords are concurrent is called the apex vertex.

Definition:2.3

A graph obtained from a path $\mathbf{P_n}$ by joining each vertex of $\mathbf{P_n}$ to a pendent vertex is called a fan $\mathbf{F_n}$.

A graph obtained from a fan by joining a path of length m , $\mathbf{P_m}$ to a middle vertex of a path $\mathbf{P_n}$in fan $\mathbf{F_n}$. It is denoted by **U(m,n)**and it is called an***Umberlla***.

Definition:2.4

A ***wheel*** on n ($n \geq 4$) vertices is a graph obtained from a cycle $\mathbf{C_n}$ by adding a new vertex and edges joining it to all the vertices of the cycle; the new edges are called the ***spokes of the wheel***. Also, $\mathbf{W_n = C_n + u}$, ($n \geq 4$).

Definition:2.5

A ***globe*** is a graph obtained from two isolated vertex are joined by n paths of length two. It is denoted by **Gl(n).**

3. Main Results

THEOREM:3.1

Shell (F_n), $n \geq 4$ is a divisor cordial graph.

Proof:

Let $V(F_n) = \{u,[u_i:1\leq i \leq n]\}$

Let $E(F_n) = \{(uu_1)U[(u_iu_{i+1}):1\leq i < n-1]U(u_{n-1}u)U[(uu_i):2\leq i < n-1]$

Define f: $V(F_n)\rightarrow\{1,2,3\ldots\ldots\ldots n\}$

The vertex labeling are

$f(u)=1$

$f(u_i)=i+1$, $1\leq i < n$

The induced edge labeling are,

$f^*(uu_1)=1$

$f^*(u_iu_{i+1})=0$, $1 \leq i < n-1$

$f^*(u_{n-1}u)=1$

$f^*(uu_i)=1$, $2 \leq i \leq n-2$

Here, $e_f(1)=e_f(0)+1$

Clearly, it satisfies the conditions $|e_f(1)-e_f(0)| \leq 1$

Hence, the induced edge labeling shows that Shell (F_n) is a divisor cordial graph.

For Example, F_5 is a divisor cordial graph as shown in the figure 3.2

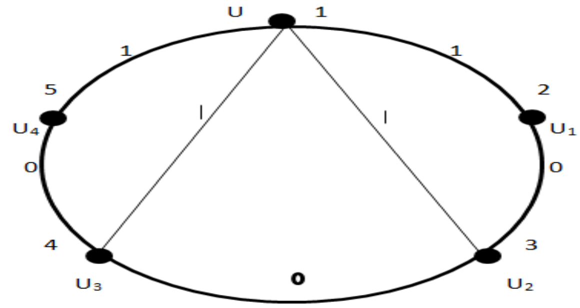

Figure 3.2

THEOREM:3.3

Umbrella U(n,3) is a divisor cordial graph.

Proof:

Case 1: when n is odd.

Let V(U(n,3)) = [u,v,w,u_i: $1\leq i\leq n$]

Let E(U(n,3)) = [{(uu_i): $1\leq i\leq n$}U($u_{(n+1)/2}$v)U(vw)U{(u_iu_{i+1}): $1\leq i<n$}]

Define f: V(U(n,3))→{1,2,3…………..n+3}

The vertex labeling are

f(u)=1

$f(u_i)=i+1$, $1\leq i\leq n$

f(v)=n+2

f(w)=n+3

The induced edge labeling are

$f^*(uu_i)=1$, $1\leq i\leq n$

$f^*(u_{(n+1)/2}v)=0$

f*(vw)=0

$f^*(u_iu_{i+1})=0$, $1\leq i<n$

Here, $e_f(1)=e_f(0)-1$

Clearly, it satisfies the conditions $|e_f(1)-e_f(0)|\leq 1$.

Case 2 : when n is even.

Let V(U(n,3)) = [u,v,w,u_i:$1\leq i\leq n$]

Let $E(U(n,3)) = [\{(uu_i):1 \le i \le n\}U(u_{n/2}v)U(vw)U\{(u_iu_{i+1}):1\le i \le n\}]$

Define f: $V(U(n,3))\rightarrow\{1,2,3,\ldots\ldots\ldots..n+3\}$

The vertex labeling are

$f(u)=1$

$f(u_i)=i+1, \quad 1\le i \le n$

$f(v)=n+2$

$f(w)=n+3$

The induced edge labeling are

$f^*(uu_i)=1, \quad 1 \le i \le n$

$f^* (u_iu_{i+1})=0, \quad 1 \le i < n$

$f^*(u_{n/2}v)=1$

$f^*(vw)=0$

Here, $e_f(1)=e_f(0)+1$

Clearly, it satisfies the conditions $|e_f(1)-e_f(0)| \le 1$.

Hence, the induced edge labeling shows that UmberllaU(n,3) is a divisor cordial graph.

For Example, when n is odd.

U(5,3) is a divisor cordial graph as shown in the figure 3.4 and

When n is even ,U(6,3) is a divisor cordial graph as shown in the figure 3.5

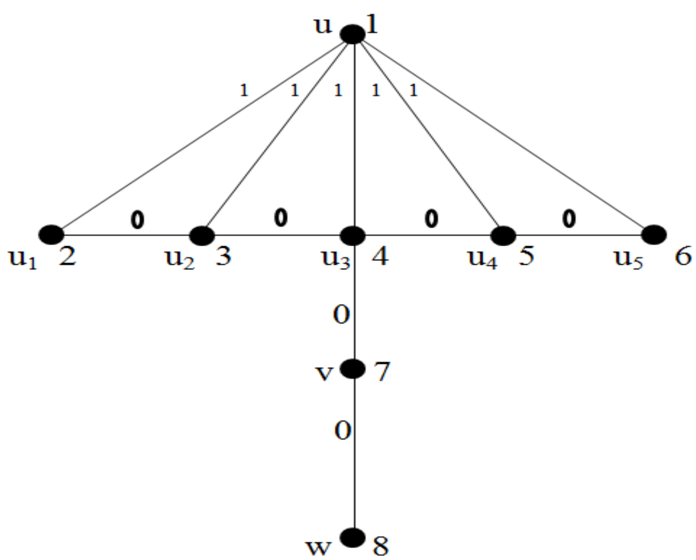

Figure 3.4

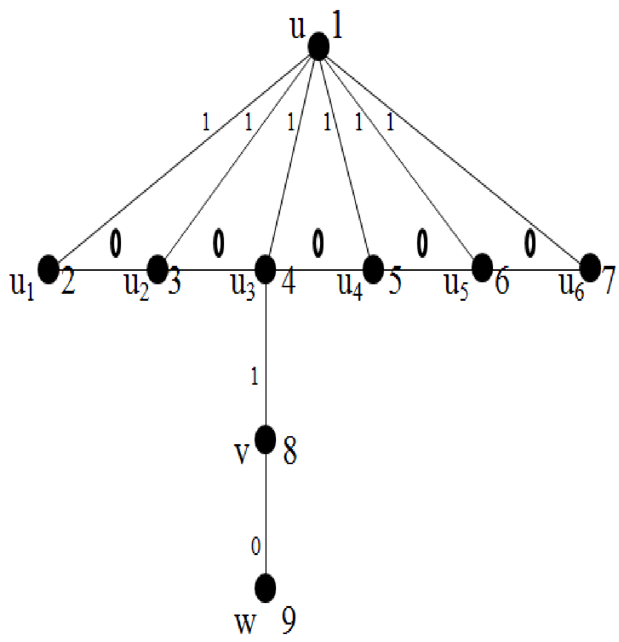

Figure 3.5.

THEOREM: 3.6

Wheel $W_n = C_n +\ u$ is a divisor cordial graph.

Proof:

Let $V(W_n) = [u, u_i: 1 \leq i \leq n]$

Let $E(W_n) = [(u_1u_n)U\{(uu_i): 1 \leq i \leq n\}U\{(u_iu_{i+1}): 1 \leq i < n\}]$

Define f: $V(W_n) \rightarrow \{1,2,3 \ldots\ldots\ldots. n+1\}$

The vertex labeling are

Case 1: when n is even

f(u)=1

$f(u_1)=n$

$f(u_i)=i, \qquad 1< i < n$

$f(u_n)=n+1$

Case 2: when n is odd.

f(u)=1

$f(u_i)=i+1, \qquad 1 \leq i \leq n$

The induced edge labeling in both cases are

$f^*(uu_i)=1, \quad 1 \leq i \leq n$

$f^*(u_iu_{i+1})=0, \quad 1 \leq i < n$

$f^*(u_1u_n)=0$

Here, $e_f(1) = e_f(0)$

Clearly, it satisfies the conditions $|e_f(1) - e_f(0)| \leq 1$.

Hence, the induced edge labeling shows that wheel ($W_n=C_n+u$) is a divisor cordial graph.

For Example, when n is odd , W_5 is a divisor cordial graph as shown in the figure 3.7 and

When n is even, W_6 is a divisor cordial graph as shown in the figure 3.8

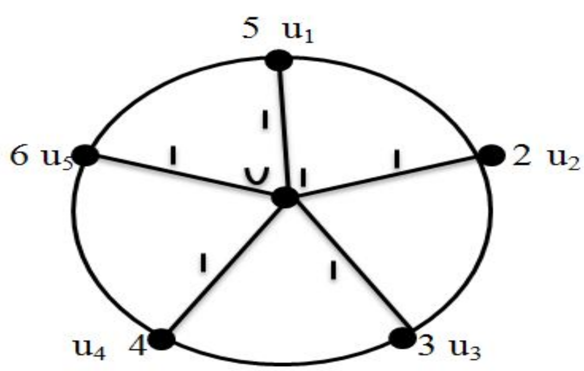

Figure 3.7

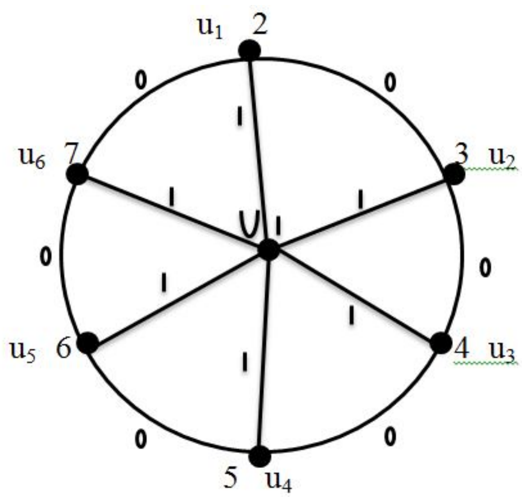

Figure 3.8

THEOREM: 3.9

Globe Gl(n) is a divisor cordial graph.

Proof:

Let $V(Gl(n)) = [u,v, w_i:1\leq i \leq n]$

Let $E(Gl(n)) = [\{(uw_i):1\leq i \leq n\}U\{(vw_i):1\leq i \leq n\}]$

Define f: $V(Gl(n))\rightarrow\{1,2,3\ldots\ldots\ldots n+2\}$

The vertex labeling are

Case 1: $n \equiv 1 \ mod \ 2$

Subcase 1a: when $n \neq 6k+1 , k \in N$

$f(u)=1$

$f(v)=n+2$

$f(w_i)=i+1 , \quad 1\leq i \leq n$

Subcase 1b : when $n = 6k+1 , k \in N$

$f(u)=1$

$f(v)=n$

$$f(w_i) = \begin{cases} i+1 , & 1\leq i \leq n-2 \\ i+2 , & n-1\leq i \leq n \end{cases}$$

Case 2: $n \equiv 0 \ mod \ 2$

Subcase 2a: when $n \neq 6k+2 , k \in N$

$f(u)=1$

$f(v)=n+1$

$$f(w_i) = \begin{cases} i+1 , & 1\leq i < n \\ i+2 , & i=n \end{cases}$$

Subcase 2b: If $n = 6k+2 , k \in N$

$f(u)=1$

$f(v)=n-1$

$$f(w_i) = \begin{cases} i+1 \ , \ 1 \leq i \leq n-3 \\ i+2 \ , \ n-2 \leq i \leq n \end{cases}$$

The induced edge labeling in both cases are

$f^*(uw_i)=1$, $1\leq i \leq n$

$f^*(vw_i)=0$, $1\leq i \leq n$

Here , $e_f(1)=e_f(0)$

Clearly, it satisfies the conditions $|e_f(1) - e_f(0)| \leq 1$.

Hence, the induced edge labeling shows that Globe Gl(n) is a divisor cordial graph.

For Example, Gl(5) is a divisor cordial graph as shown in the figure 3.10

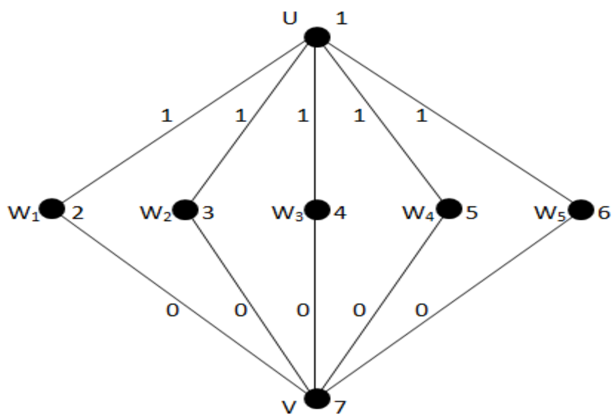

Figure 3.10

Gl(7) is a divisor cordial graph as shown in the figure 3.11

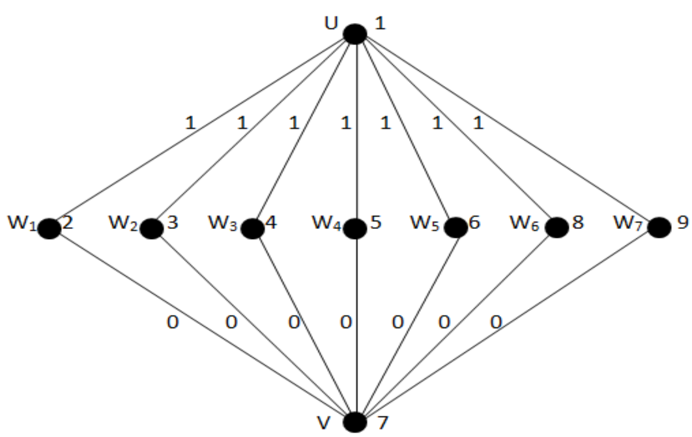

Figure 3.11

Gl(4) is a divisor cordial graph as shown in the figure 3.12

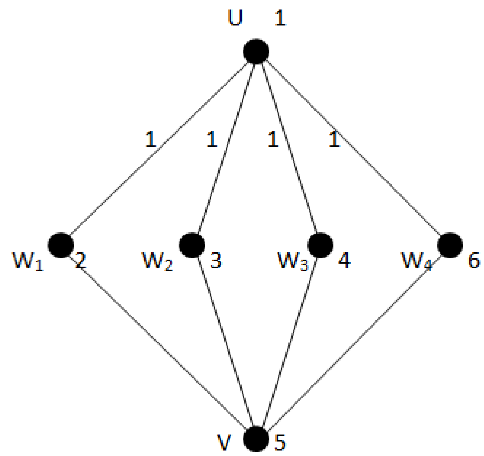

Figure 3.12

Gl(8) is a divisor cordial graph as shown in the figure 3.13

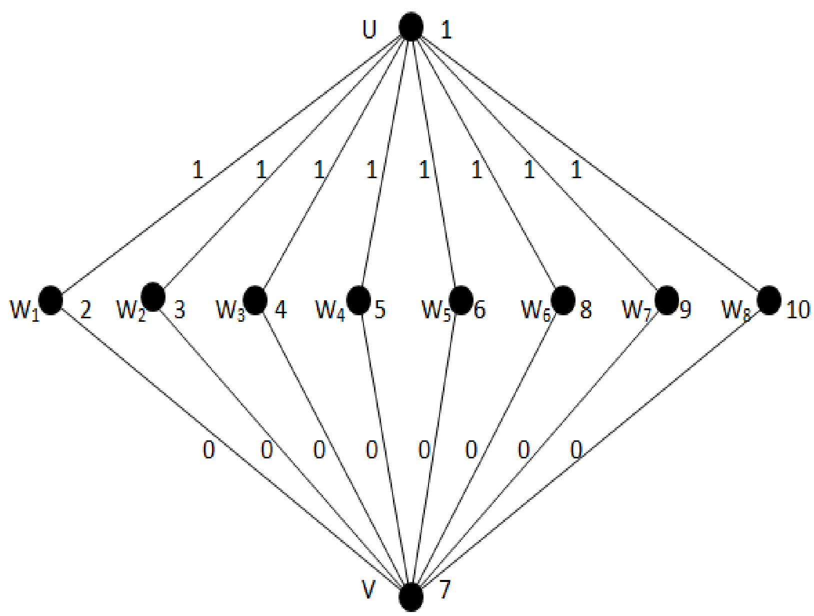

Figure 3.13

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