

# Effect of Stenosis on Bingham Plastic Flow of Blood through an Arterial Tube

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**Abstract-** A mathematical model for blood flow through an axially symmetric but radially non- symmetric stenosed artery has been considered. The effect of non-Newtonian nature of blood has been taken into account by modelling blood as a Bingham plastic fluid. Variation of flux, wall shear stress , resistance to flow with different stenosis height has been incorporated. The results are shown graphically and discussed.

**Key words:** Bingham plastic fluid , resistance to flow , wall shear stress , flow rate.

## I.INTRODUCTION

Nowadays healthcare problems are apparently concerned by the people. For over centuries cardiovascular diseases have been noticed as one of the major illness, where numerous people suffer from them. Disease of the heart and circulatory system are still a major cause of death in the Industrialized World. Blood flow characteristics in arteries can be altered significantly by arterial disease , such as stenosis and aneurysm. It is known that stenosis is a serious cardiovascular disease that may be caused due to abnormal growth along the lumen of the artery (Young and Tsai[1]). Due to the presence of a stenosis in an artery, bore of the vessel is reduced and as a result normal blood flow is disturbed apparently. The altered hemodynamics may further influence the development of the disease and arterial deformity and change the regional blood rheology (Smedby[2]). The study of physiologically realistic pulsatile flow through stenosis has profound implications for the diagnosis and treatment of vascular disease.

In view of this, several authors have considered various mathematical models for blood flow through stenosed artery (Young [3] , Lee and Fung [4], Sukla et.al. [5], Chaturani and Samy [6], Radhakrishnamacharya et. al. [7]). They have considered the blood as a Newtonian fluid. But since blood consists of a suspension of cells in an aqueous solution, Majhi and Nair [8] suggested that blood behaves like a non-newtonian fluid under certain conditions. Some authors have presented mathematical models ( Maruthiprasad and Radhakrishnamacharya [9], Maruthiprasad et. al. [10] , Siddiqui et. A. [11], Biswas et. al.[12])by considering blood as a Herschel-Bulkley type non-Newtonian fluid . Sanyal and Maiti [13] have investigated two layered mathematical model by taking the blood as Herschel-Bulkley type non-Newtonian fluid. Many researchers have used the casson fluid model for mathematical modelling of blood flow in narrow

arteries at low shear rates. Aroesty and Gross [14] have developed a Casson fluid theory for pulsatile blood flow through narrow uniform arteries. Chaturani and Samy [15] have analysed the pulsatile flow of Casson fluid through stenosed arteries using the perturbation method.

Dechant [16] have presented a perturbation model for the oscillatory flow of a Bingham plastic in rigid and periodically displaced tubes. Biswas et. al. [17] have studied two layered pulsatile flow of blood through an arterial tube by considering the core layer as Bingham plastic fluid and the peripheral layer as Newtonian fluid.

In a recent paper Parmar et. al.[18] have presented a mathematical model to show the effect of stenosis on Casson flow of blood.

In the present study I propose to discuss the effects of stenosis on Bingham plastic flow of blood through an arterial tube.

## II. MATHEMATICAL FORMULATION

Let us consider the steady flow of blood through an axially symmetric but radially non-symmetric stenosed artery.

The geometry of stenosis is given by

$$\frac{R}{R_0} = 1 - \varepsilon \left[ l_0^{s-1} (z - d) - (z - d)^s \right]; d < z \leq d + l_0$$

$$= 1; \text{otherwise} \tag{1}$$

Where  $R_0$ , the radius of the tube;  $R(z)$ , the radius in the stenotic region;  $s \geq 2$  is a shape parameter;  $l_0$ , the stenosis length and  $d$  indicates its location,

$$\varepsilon = \frac{\delta}{R_0} \frac{s^{s-1}}{l_0^s (s-1)}$$

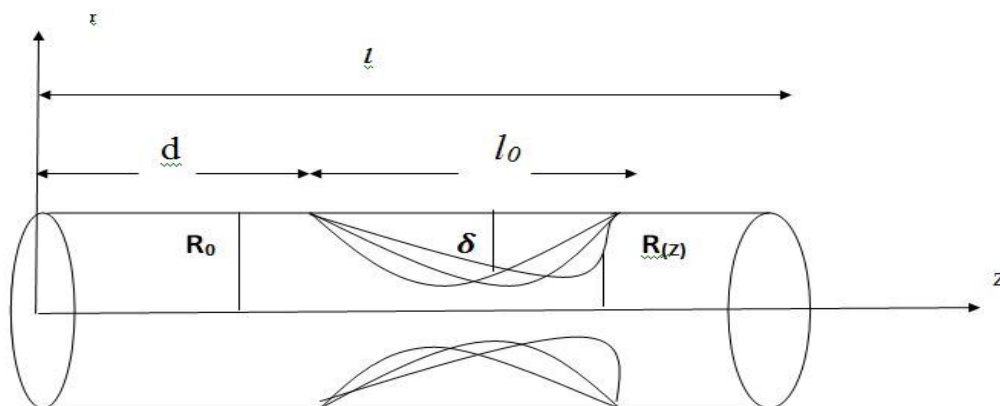


Fig.-1

$\delta$  be the maximum height of the stenosis located at

$$z = d + \frac{l_0}{s^{\frac{1}{s-1}}}$$

In case of Bingham plastic flow, the relationship between shear stress and shear rate is given by

$$\begin{aligned} \tau &= \mu \left( -\frac{\partial w}{\partial r} \right) + \tau_0, \tau \geq \tau_0 \\ -\frac{\partial w}{\partial r} &= 0, \tau \leq \tau_0 \end{aligned} \quad (2)$$

### III. SOLUTION

Equation (2) can be written as

$$-\frac{\partial w}{\partial r} = \frac{\tau - \tau_0}{\mu} \quad (3)$$

The volumetric flow rate ie, the flux is given by

$$Q = \int_0^R 2\pi r w dr \quad (4)$$

Integrating (4) and using no slip boundary condition i.e,  $u = 0$  at  $r = R$  we get

$$\begin{aligned} Q &= \int_0^R \left( -\frac{\partial w}{\partial r} \right) \frac{r^2}{2} dr \\ &= \pi \int_0^R \frac{\tau - \tau_0}{\mu} r^2 dr \end{aligned} \quad (5)$$

Since

$$\tau = \frac{-r}{2} \frac{dp}{dz} \text{ and } \tau_w = \frac{-R}{2} \frac{dp}{dz}$$

We get

$$\begin{aligned} \frac{\tau}{\tau_w} &= \frac{r}{R} \Rightarrow r = \frac{R}{\tau_w} \tau \\ &\Rightarrow dr = \frac{R}{\tau_w} d\tau \end{aligned}$$

Now limit of (5) is

$$\tau = 0 \text{ at } r = 0 \text{ and } \tau = \tau_w \text{ at } r = R$$

From (5) we get

$$Q = \frac{\pi R^3}{\mu \tau_w^3} \int_0^{\tau_w} (\tau^3 - \tau_0 \tau^2) d\tau$$

$$= \frac{\pi R^3}{\mu} \left[ \frac{\tau_w^4}{4} - \frac{\tau_0 \tau_w^3}{3} \right]$$

From which we get

$$\tau_w = \frac{4\tau_0}{3} + \frac{4\mu Q}{\pi R^3} \quad (6)$$

Since

$$\tau_w = \frac{-R}{2} \frac{dp}{dz}$$

We get

$$\frac{dp}{dz} = \frac{-8\tau_0}{3R} - \frac{8\mu Q}{\pi R^4}$$

Now

$$p = p_1 \text{ at } z = 0 \text{ and } p = p_2 \text{ and at } z = l$$

So we have

$$p_2 - p_1 = -\frac{8\tau_0}{3R_0} \int_0^l \left(\frac{R}{R_0}\right)^{-1} dz - \frac{8\mu Q}{\pi R_0^4} \int_0^l \left(\frac{R}{R_0}\right)^{-4} dz$$

Therefore,

$$\lambda = \frac{p_2 - p_1}{Q}$$

$$= -\frac{8\tau_0}{3R_0 Q} \int_0^l \left(\frac{R}{R_0}\right)^{-1} dz - \frac{8\mu}{\pi R_0^4} \int_0^l \left(\frac{R}{R_0}\right)^{-4} dz. \quad (7)$$

Suppose

$$f_1 = \frac{8\tau_0}{3R_0 Q}, f_2 = \frac{8\mu}{\pi R_0^4}$$

Thus

$$\lambda = -f_1 \left\{ \int_0^d \left(\frac{R}{R_0}\right)^{-1} dz + \int_d^{d+l_0} \left(\frac{R}{R_0}\right)^{-1} dz + \int_{d+l_0}^l \left(\frac{R}{R_0}\right)^{-1} dz \right\}$$

$$- f_2 \left\{ \int_0^d \left(\frac{R}{R_0}\right)^{-4} dz + \int_d^{d+l_0} \left(\frac{R}{R_0}\right)^{-4} dz + \int_{d+l_0}^l \left(\frac{R}{R_0}\right)^{-4} dz \right\}$$

So we get

$$\lambda = -(l - l_0)(f_1 + f_2) - (f_1 I_1 + f_2 I_2), \quad (8)$$

Where,

$$I_1 = \int_d^{d+l_0} \left(\frac{R}{R_0}\right)^{-1} dz, \quad I_2 = \int_d^{d+l_0} \left(\frac{R}{R_0}\right)^{-4} dz. \quad (9)$$

If there is no stenosis i.e., in the normal condition we have

$$\lambda_N = -(f_1 + f_2)l \quad (10)$$

The resistance to flow is given by

$$\bar{\lambda} = 1 - \frac{l_0}{l} + \frac{f_1 I_1 + f_2 I_2}{(f_1 + f_2)l} \quad (11)$$

The wall stress in normal in normal situation is written as

$$\tau_N = \frac{4\mu Q}{\pi R_0^3} \quad (12)$$

The wall shear stress ratio can be obtained as

$$\begin{aligned} \frac{\tau_w}{\tau_N} &= \frac{\tau_w}{\tau_N} \\ &= \frac{\pi R_0^3 \tau_0}{3\mu Q} + \left(\frac{R}{R_0}\right)^{-3} \end{aligned} \quad (13)$$

The wall shear stress ratio at the mid-point of the stenosis is

$$\frac{\tau_{wm}}{\tau_N} = \frac{\pi R_0^3 \tau_0}{3\mu Q} + \left(1 - \frac{\delta}{R_0}\right)^{-3} \quad (14)$$

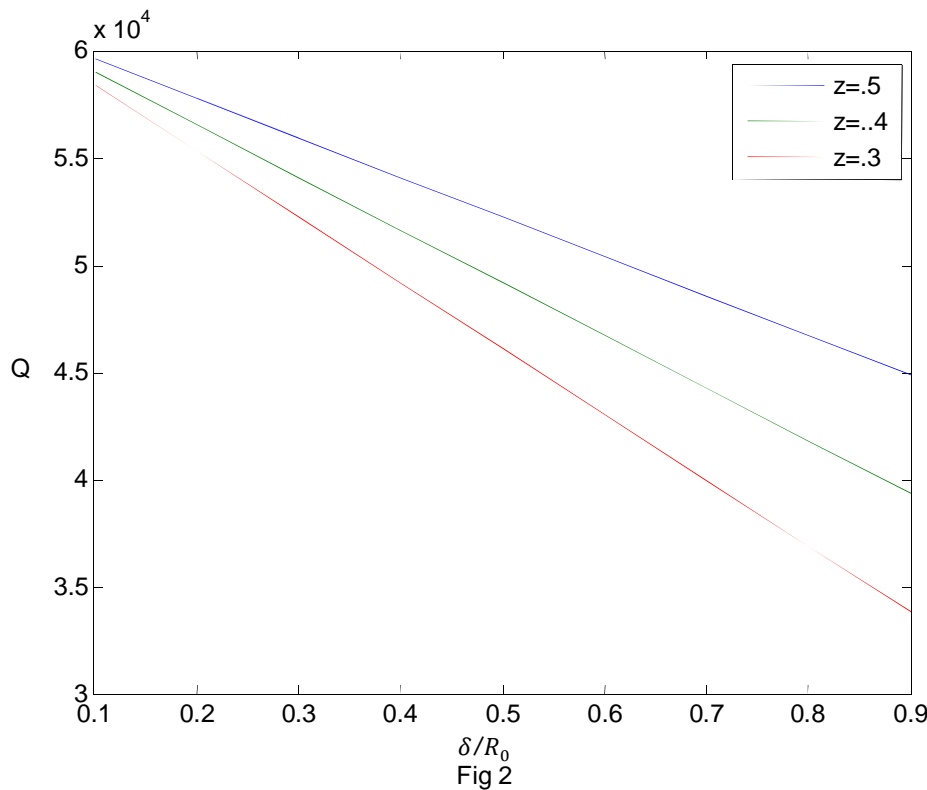
IV. NUMERICAL DISCUSSIONS

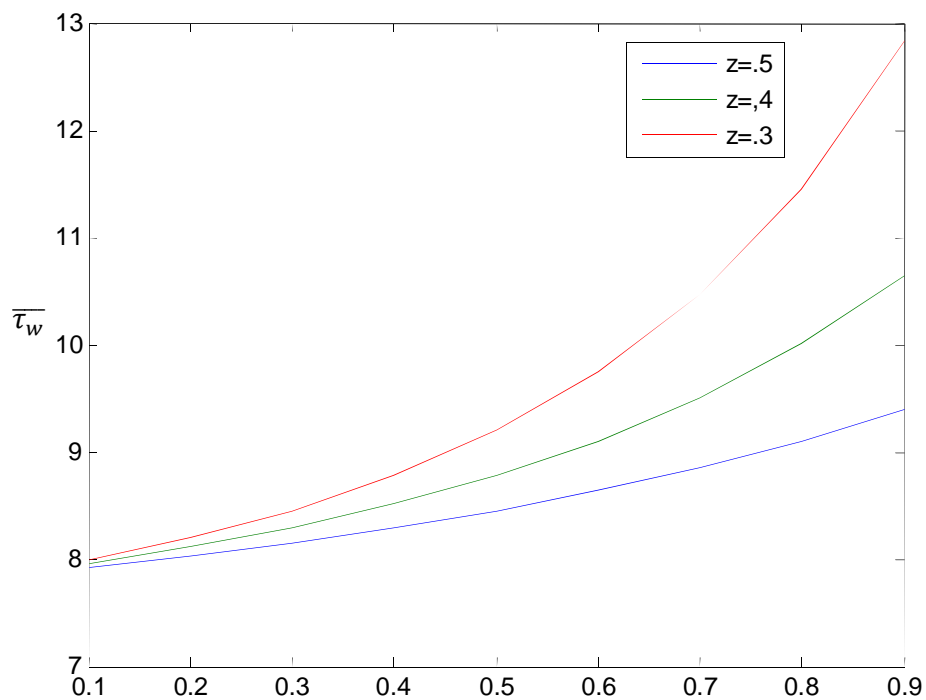
To illustrate the flow analysis the results are shown graphically with the help of MATLAB-7.6. To attain the numerical results for flow rate, wall shear stress, resistance to flow, some parameters have been taken constant

$$R_0 = 1.5, l = 5, \mu = 0.000345 \text{ pa.s}, \tau_0 = 0.02 \text{ n / m}^2.$$

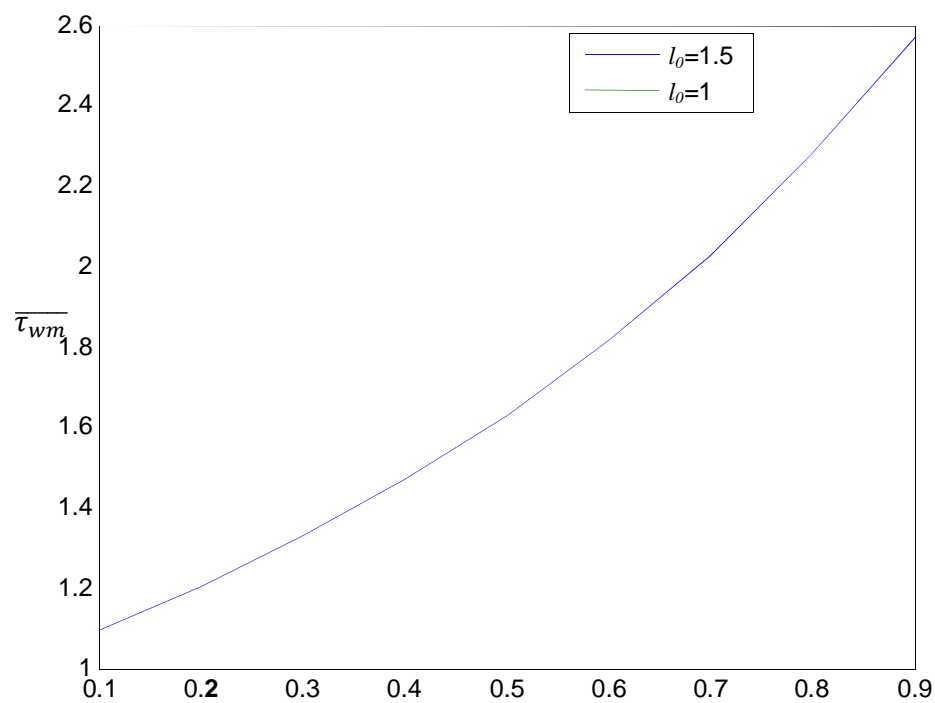
with the values,

Fig.2 shows the variation of flow rate for different values of z. It is found that Q decreases with the increase of  $\delta / R_0$  but increases for increasing z. Fig.3 shows the variation of wall shear stress for various values of z and for fixed s. It is observed that  $\tau_w$  increases with the increase of  $\delta / R_0$ , but the reverse effects exit when z increases for fixed  $\delta / R_0$ . Fig.4 depicts the variation of wall shear stress at middle point of the stenosis and it is found that  $\tau_{wm}$  increases for both the values of increasing  $\delta / R_0$  and  $l_0$ . Fig.5 illustrates the resistance to flow for different yield stress  $\tau_0$  and it is observed that it increases for increasing  $\delta / R_0$  and decreases for increasing  $\tau_0$ .





$\delta/R_0$   
Fig :3



$\delta/R_0$   
Fig:4

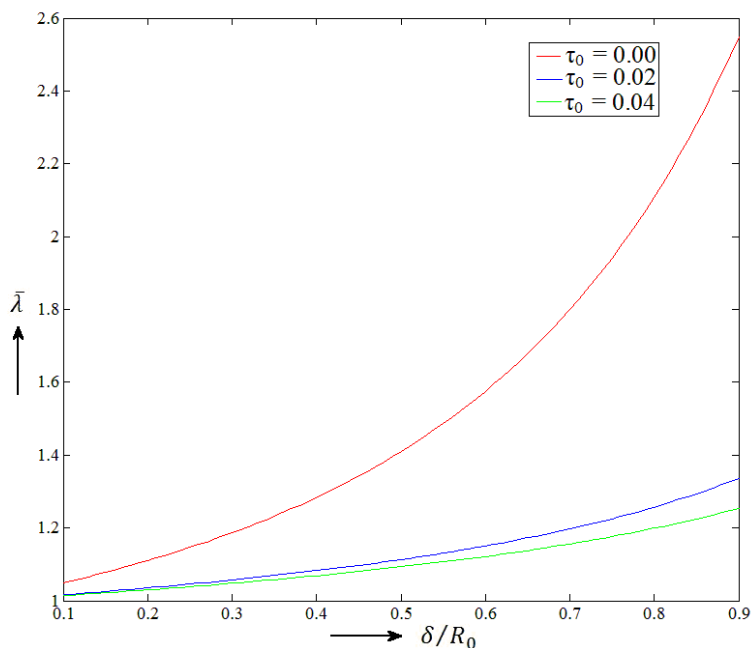


Figure 5

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