Proving the Beal Conjecture

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***Abstract*— Fermat's Last Theorem stated, without proof, that the equation, $X^n + Y^n = Z^n$, where X, Y, Z and n are integers greater than 2, had no solution for X, Y and Z co-primes. This Theorem was proven by Andrew Wiles in 1994 using mathematical techniques unknown to Fermat 350 years ago. Andrew Beal posed a related conjecture that the equation $X^a + Y^b = Z^c$ had no solution for X, Y, Z, a, b, and c, where they are all integers greater than 2, and X, Y and Z are co-primes. A simple mathematical proof available to Fermat is used here to prove the Beal conjecture.**

***Keywords*— Fermat, Wiles, Beal, Pythagoras.**

I. Introduction

In 1993 Beal [1] conjectured that if $X^a + Y^b = Z^c$ (1) where X, Y, Z, a, b, c are all integers greater than 2, then X, Y and Z have a common prime factor [2]. The impossibility of the situation where integers a = b = c = n > 2 was first postulated by Fermat who claimed a simple proof thereof that was lost to history [3]. Perhaps it might have run as follows:

For any right triangle with Base = X (any odd integer > 1), Height = Y = $(X^2 - 1)/2$ (an even integer) and Hypotenuse = Z = Y + 1 (an odd integer), we can write the Pythagorean relationship $X^2 + Y^2 = Z^2$ (2). [N.B. X cannot be even because Y would be an odd integer divided by 2.]

Then multiplying each term by finite X^{n-2} we obtain, $X^n + X^{n-2} Y^2 = X^{n-2} Z^2$ (3) and we can then write it as $X^n + Y^n (X^{n-2}/Y^{n-2}) = Z^n (X^{n-2}/ Z^{n-2})$ (4).

We now compare this to Fermat's target relationship for n > 2, $X^n + Y^n = Z^n$ (5). From (4) and (5) if both are true we get the following equality by eliminating X^n: $Z^n - Y^n = Z^n (X^{n-2}/ Z^{n-2}) - Y^n (X^{n-2}/Y^{n-2})$ (6).
But (6) can only be true if the parenthetical multipliers are both equal to 1 and that is only possible if n = 2. Therefore Fermat's last Theorem is proven, that there is no solution possible for X, Y, Z and n if they are all integers greater than 2 with no common primes.

II. PROVING THE BEAL CONJECTURE

The Beal conjecture (1) is also proven by the same technique, from (2) as follows: $X^2 + Y^2 = Z^2$ (2). Multiplying by X^{a-2} we obtain $X^a + X^{a-2} Y^2 = X^{a-2} Z^2$ (7), which can be rewritten as $X^a + Y^b (X^{a-2}/Y^{b-2}) = Z^c (X^{a-2}/Z^{c-2})$ (8). Eliminating X^a from (1) and (8) we obtain
$Z^c - Y^b = Z^c (X^{a-2}/Z^{c-2}) - Y^b (X^{a-2}/Y^{b-2})$ (9). But (9) can only be true if the parenthetical multipliers are both equal to 1 and that is only possible if a = b = c = 2.

III. Conclusion

Beal's Conjecture is proven, that there is no solution possible for (1) with X, Y, Z and a, b, c if they are all integers greater than 2, if X, Y, and Z are co-primes.

References

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