Neighbourly Regular Strength of Bipartite Graphs

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Abstract — **A graph is said to be a neighbourly irregular graph (or simply an NI graph) if every pair of its adjacent vertices have distinct degrees. Let G be a simple graph of order n. Let NI(G) denote the set of all NI graphs in which G is an induced subgraph. The neighbourly regular strength of G is denoted by NRS(G) and is defined as the minimum positive integer k for which there is an NI graph in NI(G) of order n+k. In this paper, we show that NRS(G) ≤ 2 for any bipartite graph G. In addition, we show that NRS(T) is either 0 or 1 for any tree T.**

Keywords — **Regular graph, irregular graph, neighbourly irregular graph and neighbourly regular strength of a graph.**

I. Introduction

Throughout this paper we consider only undirected, finite and simple graphs. Notations and terminology that we do not define here can be found in [10]. Let G be a graph of order n. For $1 \leq i \leq n-1$, the subset $V_i(G)$ (or simply V_i) is defined as the set of all vertices of degree i in G. That is, $V_i(G) = \{v \in V(G) \mid d(v) = i\}$. Note that $|V_i| \leq n$ for all i, $0 \leq i \leq n-1$. If G is r-regular, then $V_r(G) = V(G)$ and $V_i(G)$ is empty for all $i \neq r$. In fact, $|V_r| = n$ if and only if G is r-regular. A graph which is not regular is called irregular [3].

For a vertex v, let $N_i(v)$ be the set of all vertices adjacent to v with degree i, that is, $N_i(v) = \{w \in N(v) \mid d(w) = i\}$. In other words, $N_i(v) = N(v) \cap V_i(G)$. If $d(v) = i$, then we define $N_i[v] = N_i(v) \cup \{v\}$. Any vertex of even (odd) degree is called an even (odd) vertex. A vertex of degree one is called a pendant vertex. For any two graphs G and H, the join $G \vee H$ is the graph obtained by joining each vertex in G to each vertex in H. A spanning 1-regular subgraph of G is called a 1-factor of G and is denoted by F. P_n denotes the path of order n, C_n denotes the cycle of length n and W_n denotes the wheel of order n.

We know that in any graph, all the degrees cannot be distinct, that is, any graph has at least two vertices of the same degree. Let I_n denote a graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$ and the edge set $E = \{v_{n+1-i}v_j, 1 \leq i \leq \lfloor \frac{n}{2} \rfloor, \ i \leq j \leq n-i\}$, (where $\lfloor x \rfloor$ denotes the largest integer which is less than or equal to x) which has precisely two vertices with same degree [7]. In [13], the graph I_n is referred as a pairlone graph and is denoted by PL_n. It has been proved in [13] that, for any $n \geq 2$, there exists a unique pairlone graph of order n.

In [12], S. Gnaana Bhragsam and Ayyaswamy introduced a new concept of neighbourly irregular graph. A simple graph G is said to be a *neighbourly irregular graph* (or simply an NI graph) if no two adjacent vertices of G have the same degree. For example, the complete bipartite graph $K_{m,n}$, $m \neq n$, is an NI graph. For more types of irregular graphs, one can refer [1], [2], [3], [6], [8], [9], [11], [14] and [15].

II Neighbourly regular strength of a graph

In [12], it has been proved that any graph of order n is an induced subgraph of an NI graph of order at most n(n+1)/2. But in [4], it has been proved that any graph of order n is an induced subgraph of an NI graph of order at most 2n-1. Based on this result, a new concept called neighbourly regular strength of a graph has been introduced and studied in [4]. More results on NRS have been obtained in [5].

For a simple graph G of order n, the *neighbourly regular strength* NRS(G) of G is the minimum number k for which there is an NI graph NI(G) in **NI(G)** of order n+k, where **NI(G)** denotes the set of all NI graphs in which G is an induced subgraph. For example, $NRS(P_4) = 1$, $NRS(C_5) = 2$ and $NRS(W_4) = 2$. The respective NI graphs in **NI(P_4)**, **NI(C_5)** and **NI(W_4)** are shown in Fig 1.

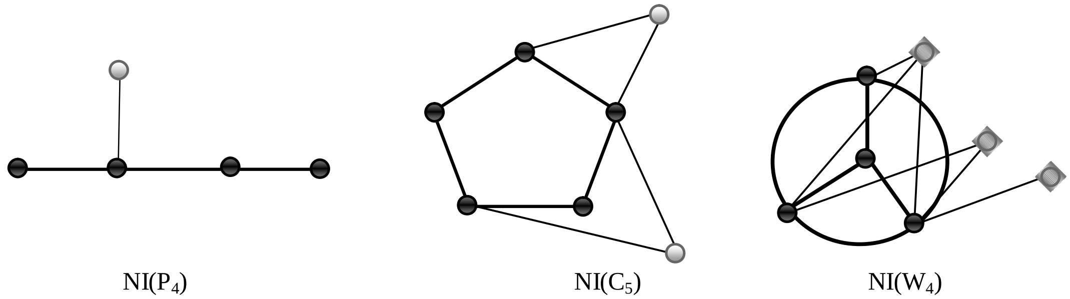

Fig 1

The following facts on NRS can be verified easily [4]:

Fact 1 NRS(G) = 0 for any NI graph G.

Fact 2 NRS(G – v) = 0 or 1 for any NI graph G. For, if G – v is NI, then NRS(G – v) = 0. Otherwise since G – v is an induced subgraph of G, NRS(G – v) = 1. For example, consider the path P_3 which is NI. $NRS(P_3 - v) = \begin{cases} 0 \text{ if } d(v) = 2 \\ 1 \text{ otherwise} \end{cases}$.

Fact 3 Let G be the disjoint union of graphs $G_1, G_2, \ldots, G_m$. Then $NRS(G) \leq \sum_{i=1}^{m} NRS(G_i)$.

Fact 4 NRS of a graph does not have the hereditary property. That is, H is a subgraph of G does not imply that NRS(H) ≤ NRS(G). For example, $NRS(K_{2,3}) = 0$ where as $NRS(K_{3,3}) = 1$.

Fact 5 $NRS(K_{n,m}) = \begin{cases} 0 \text{ if } n \neq m \\ 1 \text{ otherwise} \end{cases}$.

Fact 6 $NRS(P_n) = \begin{cases} 0 \text{ if } n = 1 \text{ or } 3 \\ 1 \text{ otherwise} \end{cases}$.

Fact 7 $NRS(C_n) = \begin{cases} 1 \text{ if } n \text{ is even and } n \neq 6 \\ 2 \text{ otherwise} \end{cases}$.

Fact 8 $NRS(W_n) = \begin{cases} 1 \text{ for any odd } n \geq 5 \\ 2 \text{ for any even } n \geq 6. \\ 3 \text{ for } n = 4 \end{cases}$

Fact 9 $NRS(I_n) = \begin{cases} 0 \text{ if } n \text{ is odd} \\ 1 \text{ otherwise} \end{cases}$.

Let G be any NI graph with clique number ω(G) = k. Since the k vertices in the clique must have distinct degrees in G, Δ(G) ≥ 2k-2. This forces that

Fact 10 Any NI graph with clique number k has at least 2k-1 vertices.

Since $\omega(K_n) = n$, $\omega(NI(K_n)) \geq n$. Thus by Fact 10, it is easy to observe that

Fact 11 $NRS(K_n) \geq n-1$ for any $n \geq 1$.

Fact 12 Let G be an NI graph of order n. For any $m \geq 1$, $G \vee K_m^c$ is NI if and only if $V_{n-m}(G)$ is empty.

Fact 13 If G is NI, then G is the only NI graph in **NI(G)** of order n+NRS(G). If G is not an NI graph, then the number of NI graphs of order n+NRS(G) in **NI(G)** need not be unique. For example, consider the cycle of order 6. Clearly $NRS(C_6) = 2$. Two NI graphs of order 8 in **NI(C_6)** are shown in Fig 2.

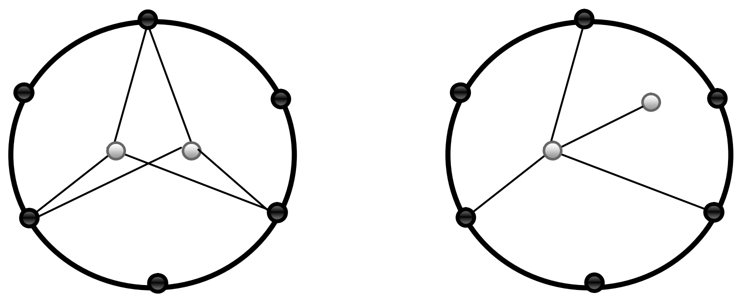

Fig 2

The following results on NRS have been proved:

Theorem A [4] For any graph G of order n, NRS(G) $\leq$ n-1.

Theorem B [4] For any graph G of order n, NRS(G) = n-1 if and only if G $\cong K_n$.

Theorem C [4] NRS(G) $\leq$ n-3, for any connected irregular graph G of order n.

Theorem D [4] For any non-negative integer s and for any n $\geq$ s+3, there exists a graph G of order n with NRS(G) = s.

Theorem E [4] For any graph G, NRS(G) $\geq$ max $\{\omega(< V_i >)\}$-1, where maximum runs over i, $2 \leq i \leq n-1$.

Theorem F [5] For any connected regular graph G, $NRS(G) = \begin{cases} \chi(G) & \text{if } G \cong K_{m,m} - F \text{ where } m \geq 2 \\ \chi(G) - 1 & \text{otherwise} \end{cases}$.

Theorem G [5] Let G be any graph of order n, n $\geq$ 6. NRS(G) = n-2 if and only if G $\cong K_1 \cup K_{n-1}$.

In this paper, the bound on NRS of any bipartite graph has been obtained. It is well known that, trees are special class in bipartite graphs. It has been proved that the NRS of any non- NI tree is one.

III NRS of bipartite graphs

While determining the neighbourly regular strength of a graph, it is easy to observe from the Fact 10 and the Theorem E that the clique number of the particular graph plays a vital role. The clique number of a bipartite graph is always two. The following theorem shows that NRS of any bipartite graph is at most two.

Theorem 1 For any bipartite graph G, NRS(G) $\leq$ 2.

Proof Let G be a bipartite graph with bipartition (X, Y). If G is an NI graph, then NRS(G) = 0 and hence the result follows. Note that by Theorem F, the result holds when G is regular. So assume that G is irregular and not NI. Let E_x and E_y denote the set of all even vertices in X and Y respectively and let O_x and O_y denote the set of all odd vertices in X and Y respectively. Since G is not an NI graph, among the four sets at least two must be non-empty.

Suppose exactly two sets in $\{E_x, E_y, O_x, O_y\}$ are non-empty. Then either E_x and E_y are non-empty or O_x and O_y are non-empty. Assume that E_x and E_y are non-empty. Now construct a new graph G_1 from G by introducing a new vertex u and joining u with the vertices in E_x. If G_1 is NI, then the result follows. Otherwise, construct a graph G_2 from G_1 by introducing a new vertex w and joining it with u. Clearly $G_2 \in$ **NI(G)**. Hence NRS(G) $\leq$ 2. A similar construction can be used to prove the case when O_x and O_y are non-empty.

Suppose exactly three sets in $\{E_x, E_y, O_x, O_y\}$ are non-empty. Consider the case that E_x, E_y and O_x are non-empty. Construct a new graph G_1 from G by introducing a new vertex u and joining u with the vertices in E_x. Note that in G_1, the vertices in X are all odd and the vertices in Y are all even. Hence for any edge xy in G, we have $d(x) \neq d(y)$ in G_1. If G_1 is NI, then the result follows. Otherwise, add a new vertex w and a new edge uw in G_1. The resulting graph is in **NI(G)** and hence NRS(G) $\leq$ 2. One can use a similar construction for the remaining three possibilities of three non-empty sets.

Suppose E_x, E_y, O_x and O_y are all non-empty. Construct a new graph G_1 from G by introducing two new vertices u_1 and u_2, and joining u_1 with all the vertices in E_x and u_2 with all the vertices in $O_y - V_1(G)$. Note that in G_1, the vertices in X are all odd and any vertex in Y is either an even vertex or a pendant vertex. Hence for any edge xy in G, $d(x) \neq d(y)$ in G_1. If G_1 is NI, then NRS(G) $\leq$ 2. Otherwise

(i) $d(u_1) = d(u)$ for some $u \in N(u_1)$ or

(ii) $d(u_2) = d(v)$ for some $v \in N(u_2)$ in G_1.

If both (i) and (ii) holds, then u_1 is odd and u_2 is even. In such a case, $G_1+u_1u_2$ is in **NI(G)** of order n+2 and hence NRS(G) $\leq$ 2.

If only (i) holds, then clearly u_1 is odd and $d(u_1) \geq 3$. If u_2 is even or $d(u_2) = 1$, then $G_1+u_1u_2$ is the required NI graph. Otherwise, u_2 is odd and $d(u_2) \geq 3$.

Now construct a new graph G_2 from G_1 by joining the vertices in X with u_2 and the vertices in Y with u_1 respectively. If G_2 is an NI graph, then the result follows. Suppose G_2 is not an NI graph. Clearly in G_2, the vertices in X are even and the vertices in Y are odd. Also note that in G_2, $d(u_1) \geq |Y|+3$, $d(u_2) \geq |X|+3$ and $d(v) = d_G(v)+1$ or $d_G(v)+2$ for any vertex v in V(G). Hence in G_2, $d(u) < d(u_1)$ for any $u \in X$ and $d(v) < d(u_2)$ for any $v \in Y$. Thus $\Delta(G_2) = d(u_1)$ or $d(u_2)$. Clearly, if $\Delta(G_2) = d(u_1) = d(u_2)$, then G_2 is an NI graph. Without loss of generality, assume that $\Delta(G_2) = d(u_1)$ and so $d(u_1) > d(u_2)$. Since G_2 is not an NI graph, $d(u_2) = d(v)$ for some v in $N(u_2)$. Clearly $G_2+u_1u_2$ is in **NI(G)** of order n+2 and hence NRS(G) $\leq$ 2. Using the similar arguments, one can easily prove that NRS(G) $\leq$ 2 if $\Delta(G_2) = d(u_2)$.

Suppose only (ii) holds. Then clearly u_2 is even and $d(u_2) \geq 4$. If u_1 is odd, or $d(u_1) = 2$ and $N_3(u_1)$ (The set of all vertices adjacent to u_1 of degree 3) = ϕ, then $G_1+u_1u_2$ is a required NI graph. If $d(u_1) = 2$ and $N_3(u_1) \neq \phi$, then $(G_1 - u_1w)+u_2w$ where $w \in N_3(u_1)$ is in **NI(G)**. Otherwise, u_1 is even and $d(u_1) \geq 4$. Now construct G_2 as in above. Note that in G_2, $d(u_1) \geq |Y|+4$, $d(u_2) \geq |X|+4$ and $d(v) = d_G(v)+1$ or $d_G(v)+2$ for any v in V(G). Hence as discussed above, NRS(G) $\leq$ 2. ■

Theorem 1 means that, for any bipartite graph G, NRS(G) can be 0, 1 or 2. For example, $NRS(K_{1,n}) = 0$ for all n $\geq$ 2, $NRS(P_n) = 1$ for all n $\geq$ 4, and $NRS(K_{n,n} - F) = 2$ for all n $\geq$ 2. This means that the bound attained in Theorem 1 is sharp.

As an illustration, consider the graph G shown in Fig 3. Clearly G is non-NI in which E_x and E_y are non-empty and O_x and O_y are non-empty. Thus a new graph G_1 shown in Fig 3 is constructed form G by introducing two new vertices u_1 and u_2. Note that in G_1, $d(u_1) = d(u)$ for some $u \in N(u_2)$. Therefore, the graph G_2 shown in Fig 3 is constructed form G_1 as in the proof of Theorem 1. Clearly $\Delta(G_2) = d(u_1) = d(u_2)$. Hence G_2 is an NI graph in **NI(G)**.

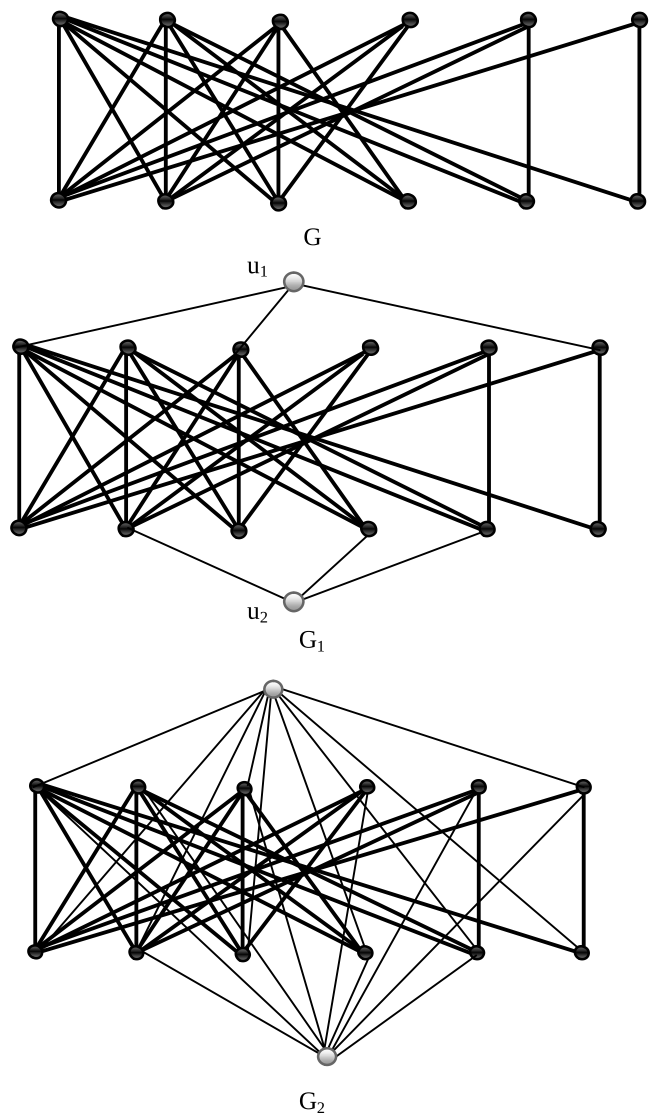

Fig 3

Theorem 2 Any bipartite graph of order n is an induced subgraph of a bipartite NI graph of order at most n+3.

Proof If G is an NI graph then the result holds. Suppose G is not an NI graph. Let G be a bipartite graph of order n with bipartition (X, Y). Let E_x and E_y denote the set of all even vertices in X and Y respectively and let O_x and O_y denote the set of all odd vertices in X and Y respectively. Suppose it is not the case that E_x, E_y, O_x and O_y are all non-empty in G. Then for such graphs the NI graphs constructed in Theorem 1 are bipartite and of order at most n+2.

Suppose E_x, E_y, O_x and O_y are all non-empty. Construct G_1 from G by introducing two new vertices u_1 and u_2, and joining u_1 with all the vertices in E_x and u_2 with all the vertices in $O_y - V_1(G)$. Clearly G_1 is bipartite. If G_1 is an NI graph, then the proof is complete. Otherwise, as discussed in Theorem 1, $d(u_1) = d(u)$ for some $u \in N(u_1)$ or $d(u_2) = d(v)$ for some $v \in N(u_2)$ in G_1. If $d(u_1) = d(u)$ for some $u \in N(u_1)$ and $d(u_2) = d(v)$ for some $v \in N(u_2)$, then u_1 is odd and u_2 is even. In this case, $G_1+u_1u_2$ is a required bipartite NI graph. If $d(u_1) = d(u)$ for some $u \in N(u_1)$, then construct a new graph G_2 from G_1 by introducing a new vertex u_3 and joining it with u_1. Clearly G_2 is a bipartite NI graph of order n+3 in which G is an induced subgraph. Similarly the same process can be followed if $d(u_2) = d(u)$ for some $u \in N(u_2)$. This completes the proof. ■

For example, consider the graph G shown in Fig 3. A bipartite NI graph in which G is an induced subgraph obtained by the process as in the proof of Theorem 2 is shown in Fig 4.

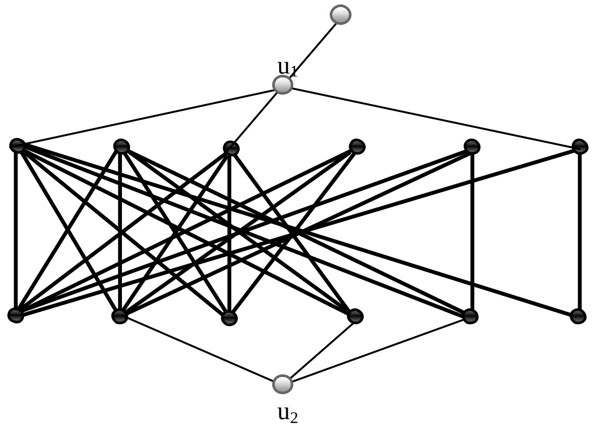

Fig 4

IV NRS of trees

It is well known that in a tree T, the number of pendant vertices is greater than or equal to Δ. That is, $|V_1(T)| \geq \Delta$. The number of pendant vertices in a tree T is exactly two if and only if T is a path. That is, $|V_1(T)| = 2$ if and only if $T \cong P_n$, for some $n \geq 2$. For $\Delta \geq 3$, the following lemma characterizes the trees with exactly Δ pendant vertices.

Lemma 1 Let T be a tree with $\Delta \geq 3$. The number of pendant vertices in T is exactly Δ if and only if T has exactly one vertex of degree Δ and the remaining vertices are of degree either one or two, that is, if and only if T is isomorphic to $K_{1,\Delta}$ or to a subdivision of $K_{1,\Delta}$.

Proof Let T be a tree of order n. Let $|V_i| = n_i$ for each i. $1 \leq i \leq \Delta$. It is well known that for any graph G, $\sum_{v \in V} d(v) = 2m$, where m is the number of edges in G. Therefore, in T, $\sum_{i=1}^{\Delta} i n_i = 2(n-1)$. Clearly $n_1 = 2 + \sum_{i=3}^{\Delta}(i-2)n_i$.

If $n_1 = \Delta$, then $\Delta - 2 = \sum_{i=3}^{\Delta-1}(i-2)n_i + (\Delta-2)n_\Delta$ and so $n_\Delta = 1$ and $n_i = 0$ for each i, $3 \leq i \leq \Delta-1$. Conversely if $n_\Delta = 1$ and $n_i = 0$ for $3 \leq i \leq \Delta-1$, then $n_1 = \Delta$. ■

For a given set of k integers $m_1, m_2, \ldots m_k$ where $m_i \geq 2$ for each i, construct a tree of order $m_1 + m_2 + \ldots + m_k + 1 - k$ which is obtained by identifying a pendant vertex in each of the paths $P_{m_1}, P_{m_2}, \ldots P_{m_k}$ and denote it by $P(m_1, m_2, \ldots m_k)$. For example, P(2, 3, 3, 3, 4) is shown in Fig 5.

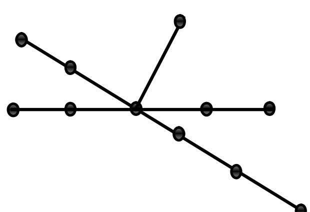

Fig 5

Clearly any path P_n, $n \geq 3$ is P(n-1, 2). Note that the number of pendant vertices in any tree T of order $n \geq 3$ is exactly Δ if and only if T is isomorphic to $P(m_1, m_2, \ldots, m_\Delta)$. It is easy to observe that $P(m_1, m_2, \ldots m_k)$ is NI if and only if $m_i \leq 3$, for each i, $1 \leq i \leq k$ and $k \geq 3$. Thus,

Lemma 2 For any tree T with $\Delta \geq 3$ and $|V_1(T)| = \Delta$, T is NI if and only if its radius is at most 2. ■

The bound attained in Lemma 2 is sharp. For example, the radius of the NI tree P(3, 3, 3) shown in Fig 6 is 2.

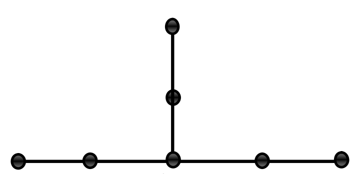

Fig 6

Lemma 3 Let T be any tree of order n. If $|V_1(T)| = \Delta$, then NRS(T) = 0 or 1.

Proof Let T be any tree of order n with $|V_1(T)| = \Delta$. If T itself is NI, then NRS(T) = 0. Assume that NRS(T) ≥ 1. If $\Delta = 2$, then T is a path and so the results holds trivially. Suppose $\Delta \geq 3$. Then by Lemma 1, T has exactly one vertex of degree Δ and the remaining vertices are of degree either one or two. Let u be the vertex of degree Δ in T. As T is bipartite, let (X, Y) be the bipartition of T. Without loss of generality, assume that u is in X. Since T is not NI, $V_2(T)$ is neither empty nor independent in T. Hence both $V_2(T) \cap X$ and $V_2(T) \cap Y$ are non-empty.

Construct a new graph G from T by introducing a new vertex w and joining it with the vertices in $V_2(T) \cap X$. If G is NI, then the result holds. Otherwise, $d(w) = |V_2(T) \cap X| = 3$.

If d(u) > 3, then G+uw is in **NI(T)** of order n+1. Suppose $d(u) = 3 = \Delta$. Then $|V_1(T)| = 3$. If $V_1(T) \cap Y \neq \phi$, let $v \in V_1(T) \cap Y$. Then G+wv is in **NI(T)**. Otherwise, $V_1(T) \subseteq X$. But $|V_2(T) \cap X| = 3$. Clearly $|V_2(T) \cap Y| \geq 4$. In this case, construct a graph G from T by introducing a new vertex w and joining it with the vertex u and the vertices in Y. Clearly G is in **NI(T)** of order n+1. Thus NRS(T) = 1. ■

For example, an NI graph in **NI(P(4, 4, 4, 4))** is shown in Fig 7

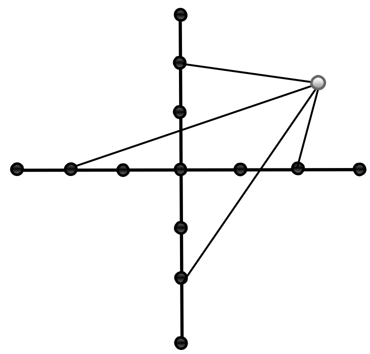

Fig 7

Lemma 4 Let T be any tree of order n. If $|V_1(T)| > \Delta$, then NRS(T) = 0 or 1.

Proof If T is an NI tree, then the result is obvious.

Suppose T is not an NI tree. Let (X, Y) be the bipartition of T. Without loss of generality, assume that $|V_1(T) \cap X| \geq |V_1(T) \cap Y|$. Now construct a new graph G from T by introducing a new vertex w and joining it with the even vertices in Y and the odd vertices in $X – V_1(T)$.

In G, the vertices in Y are odd and any vertex in X is either an even vertex or a pendant vertex. Hence for any edge xy in T, $d_G(x) \neq d_G(y)$. Note that for any vertex $v \in V(T)$, $d_G(v) = d_T(v)$ or $d_T(v)+1$. Clearly in G, $N_i(w) = \phi$ for each $i > \Delta(T)+1$.

Let $d_G(w) = d$. If $N_d(w) = \phi$ in G, then G is NI. Suppose $N_d(w) \neq \phi$. Note that by the construction of G, $d \geq 3$. Since the vertices in Y are odd, any vertex v in $V_1(T) \cap X$ is not adjacent to the vertex of degree 2. Since $|V_1(T)| \geq \Delta(T)+1$ and $|V_1(T) \cap X| \geq |V_1(T) \cap Y|$, we have $|V_1(T) \cap X| \geq \left\lceil \frac{\Delta(T)+1}{2} \right\rceil$.

Case 1 Suppose $d \leq \left\lceil \frac{\Delta(T)+1}{2} \right\rceil$.

Let k be the least positive integer such that $N_{d+k}(w) = \phi$. If $1 \leq k \leq d-1$, then join k vertices from $V_1(T) \cap X$ with w in G. Otherwise, w is adjacent to the vertices of degree d+i for each i, $0 \leq i \leq d-1$. Now join d vertices from $V_1(T) \cap X$ with w in G. The above constructions are possible, since $d \leq \left\lceil \frac{\Delta(T)+1}{2} \right\rceil \leq |V_1(T) \cap X|$.

Case 2 Suppose $d > \left\lceil \frac{\Delta(T)+1}{2} \right\rceil$.

Now join all the vertices of $V_1(T) \cap X$ to w in G. Note that in the resulting graph, $d(w) \neq d(v)$ for each vertex v in N(w), as $d(w) > \left\lceil \frac{\Delta(T)+1}{2} \right\rceil + \left\lceil \frac{\Delta(T)+1}{2} \right\rceil \geq \Delta(T)+1$.

Clearly in both the cases the resultant graph is in **NI(T)** of order n+1 and hence NRS(T) = 1. ■

Combining Lemma 3 and Lemma 4, we have

Theorem 3 For any tree T, NRS(T) = 0 or 1. ■

Theorem 4 Any tree of order n is an induced subgraph of an NI tree of order at most $n + \left\lfloor \frac{n-|V_1(T)|}{2} \right\rfloor$.

Proof Let T be any tree of order n. If T is NI, then the result is obvious. Suppose T is not an NI tree. Let (X, Y) be the bipartition of T. Let E_x and E_y denote the set of all even vertices in X and Y respectively and let O_x and O_y denote the set of all odd vertices in X and Y respectively. Let $p = |E_x|+|O_y – V_1(T)|$ and $q = |E_y|+|O_x – V_1(T)|$. Without loss of generality, assume that

$p \le q$. Note that $p \le \left\lfloor \frac{n-|V_1(T)|}{2} \right\rfloor$. Now for each vertex v in $E_x \cup (O_y - V_1(T))$ introduce a new vertex and join it with v. Let the resultant tree be T_1. Clearly in T_1, all vertices in X are odd with degree at least 3 and the vertices in Y are either even or a pendant vertex. Thus T_1 is an NI tree in which T is an induced subgraph and hence the result. ■

The bound obtained in the above theorem is sharp. For example, consider the path P_n, $n \ge 3$. P_n is an induced subgraph of an NI tree of order $n+\left\lfloor \frac{n-|V_1(T)|}{2} \right\rfloor$. As an illustration, the NI trees in which P_6 and P_7 are induced subgraphs are respectively shown in Fig 8.

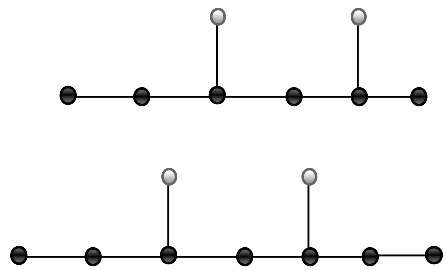

Fig 8

V Conclusion

- NRS(G) ≤ 2 for any bipartite graph G.
- NRS(T) = 0 or 1 for any tree T.

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