A Numerical Study on Effect of Magnetic field on Stenosed Artery

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***Abstract*— In this paper the flow of blood in a stenosed artery under the influence of external induced magnetic field is studied numerically . The artery is considered as circular tube. The effects of external induced magnetic field on axial velocity, flow rate and wall shear rate are studied. The variation of the axial velocity and magnetic field distribution has been illustrated graphically for variation of different parameters such as Reynolds number, Hartmann number , Magnetic Reynolds number etc. The effects of all the parameters are quite significant on flow characteristics.**

***Keywords*—Stenosed artery, External induced Magnetic field, Reynolds number**

I. INTRODUCTION

Blood is a concentrated suspension of formed cellular elements that includes red blood cells or erythrocytes, white blood cells or leukocytes and platelets or thrombocytes. These cells are suspended in an aqueous polymer solution, the plasma, containing erythrocytes or organic molecules. The study of blood flow in stenosed human arteries has been an object of scientific research nowadays. Stenosis means localized narrowing of an artery. With the help of magneto-hydrodynamics the flow of blood through such type of arteries can be controlled. The blood flow under the application of appropriate magnetic field may help in the treatment of certain cardiovascular diseases and in the disease with accelerated blood circulation such as hypertension, hemorrhages etc. The application of magnetohydrodynamics principles in medicine, engineering is of growing interest. Lee and Fung [1] solved the problem of blood flow in constricted tube numerically. The flow of blood in stenosed artery has been studied by Shukla [2, 3] with slip at the boundary. Barnothy [4] has studied that the biological system is greatly affected by the application of external magnetic field. Bhuyan *et al.* [5] have investigated the problem of blood flow with the effects of slip in the arterial stenosis in presence of transverse magnetic field. Singh *et al.* [6] have studied the blood flow through an artery having radially non-symmetric mild stenosis. Sarma [7] has studied the flow of blood through stenosed artery with the effect of slip at the boundary in presence of transverse magnetic field. Varshney *et al.* [8] studied the influence of transverse magnetic field and multi-stenosis blood flow problem. The applied magnetic field reduces the shear stress parameter and changes the speed of blood. So it is essential to study the flow in circulatory system in presence of magnetic field.

In this paper an attempt has been made to study the effect of external induced magnetic field on axial velocity , flow rate and wall shear rate.

II. MATHEMATICAL FORMULATION

Consider a steady-state flow of blood through a stenosed artery. The flow is also considered as axially symmetric, incompressible and fully developed. It is entirely reasonable to consider the flow in one direction as the velocity does not change in the direction of flow, except near the entrance and exit regions [10]. Consider an external magnetic field of strength B_0 which makes an angle θ with the z^*–axis which induced a magnetic field B makes also an angle θ to the free stream velocity. The component of velocities and magnetic field are taken as $\left(0,0,w^*(r^*)\right)$ and $\left(B_0\sqrt{1-\lambda^2},0,\lambda B(r^*)\right)$ where $\lambda = Cos\theta$

A. Governing Equations

Under the above assumptions the governing equations reduces to

$$0=-\frac{\partial p^*}{\partial z^*}+\mu\left(\frac{\partial^2 w^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial w^*}{\partial r^*}\right)-\sigma B_0{}^2 w^*\left(1-\lambda^2\right) \qquad (1)$$

$$0 = B_0\sqrt{\frac{1}{\lambda^2}-1}\left(\frac{\partial w^*}{\partial r^*}+\frac{w^*}{r^*}\right)+\eta_m\left(\frac{\partial^2 B}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial B}{\partial r^*}\right) \qquad (2)$$

where μ is the coefficient of viscosity and η_m is the magnetic diffusivity.

The boundary conditions appropriate to the problem are

$$\left.\begin{array}{l} w^* = 0,\ B = B_s \text{ at } r^* = R^*(z) \\ \dfrac{\partial w^*}{\partial r^*} = 0,\ B = B_0 \text{ at } r^* = 0 \end{array}\right\} \qquad (3)$$

B. Flow Geometry

It is assumed that the stenosis is developed in an axially symmetric manner. The radius of the artery $R(z)$ in stenosed region can be taken as [9]

$$R(z) = R_0 - \frac{\delta}{2}\left\{1 + Cos\frac{2\pi}{L_0}(z-d-L_0)\right\},\ \left(d \le z \le d+L_0\right)$$
$$= R_0 \quad (\text{otherwise})$$

where R_0 and $R(z)$ are the radius of the uniform and constricted region. L is the length of the artery , L_0 ,d ,δ are the length, location and maximum height of the stenosis. And r and z are the radial and axial co-ordinates.

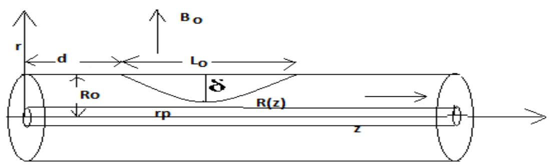

Fig a: flow geometry and co-ordinate system

C. Solution of the Problem:

Introducing the non dimensional scheme

$$p = \frac{p^*}{\rho W_0^{\ 2}},\ w = \frac{w^*}{W_0},\ r = \frac{r^*}{R_0},\ z = \frac{z^*}{R_0},\ b = \frac{B}{B_0} \qquad (4)$$

Using the non dimensional scheme the governing equations (1) and (2) reduces to

$$0 = -\frac{\partial p}{\partial z}+\frac{1}{\text{Re}}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)-\frac{M^2}{\text{Re}}\left(1-\lambda^2\right)w \qquad (5)$$

$$0 = R_m\sqrt{\frac{1}{\lambda^2}-1}\left(\frac{\partial w}{\partial r}+\frac{w}{r}\right)+\left(\frac{\partial^2 B}{\partial r^2}+\frac{1}{r}\frac{\partial B}{\partial r}\right) \qquad (6)$$

Where $M = \sqrt{\frac{\sigma}{\rho\gamma}}B_0R_0$, $\text{Re} = \frac{R_0W_0}{\gamma}$, $\text{R}_\text{m} = \frac{\eta_m}{\gamma}$ are Hartmann number, Reynolds number and magnetic Reynolds number respectively.

The boundary conditions (3) reduces to

$$\left.\begin{array}{l} w=0,\ \mathrm{b}=b_s \ \text{ at } r=R(z) \\ \dfrac{\partial w}{\partial r}=0,\ \mathrm{b}=b_0 \ \text{ at } r=0 \end{array}\right\} \quad (7)$$

Now taking $\eta=\dfrac{r}{R}$

The governing equation reduces to

$$\frac{\partial^2 w}{\partial \eta^2}+\frac{1}{\eta}\frac{\partial w}{\partial \eta}=R^2\left\{M^2\left(1-\lambda^2\right)w+\mathrm{Re}\left(\frac{\partial p}{\partial z}\right)\right\} \quad (8)$$

$$\frac{\partial^2 b}{\partial \eta^2}+\frac{1}{\eta}\frac{\partial b}{\partial \eta}=-RR_m\sqrt{\frac{1}{\lambda^2}-1}\left(\frac{\partial w}{\partial \eta}+\frac{w}{\eta}\right) \quad (9)$$

The boundary conditions (7) reduces to

$$\left.\begin{array}{l} w=0,\ \mathrm{b}=b_s \ \text{ at } \eta=1 \\ \dfrac{\partial w}{\partial r}=0,\ \mathrm{b}=b_0 \ \text{ at } \eta=0 \end{array}\right\} \quad (10)$$

The volumetric rate of flow can be defined as

$$\begin{aligned} Q &= 2\pi\int_0^{R(z)} rwdr \\ &= 2\pi R^2\int_0^1 \eta w d\eta \end{aligned} \quad (11)$$

The wall shear stress can be defined as

$$\begin{aligned} \tau_w &= \mu\left(\frac{dw}{dr}\right)_{r=R(z)} \\ &= \frac{\mu}{R}\left(\frac{dw}{d\eta}\right)_{\eta=1} \end{aligned} \quad (12)$$

D. Results and Discussions

The problem under investigation is solved numerically and the expressions derived have been computed for different parameters such as for Hartmann number, Reynolds number and Magnetic Reynolds number. Here the pressure gradient is function of z only so it is assumed as constant. In this paper the effects of these parameters on axial velocity, volumetric flow rate and wall shear stress in presence of external induced magnetic field is studied. The present analysis corresponds to the flow of Newtonian fluid. The results are presented graphically.

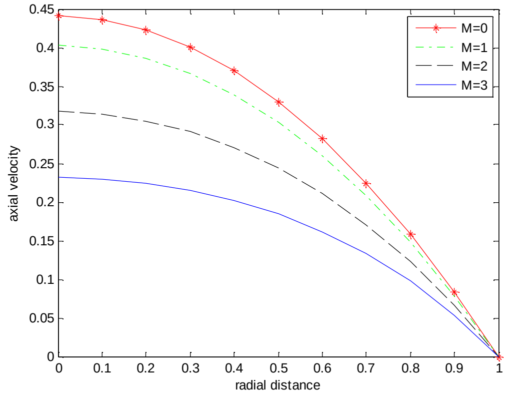

Fig 1: Variation of axial velocity with radial distance

for different values of Hartmann Number

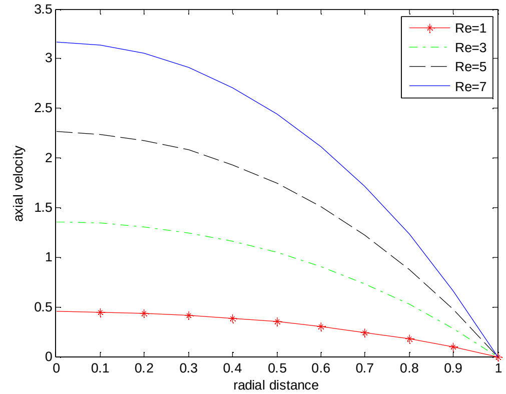

Fig 2: Variation of axial velocity with radial distance

for different values of Reynolds Number

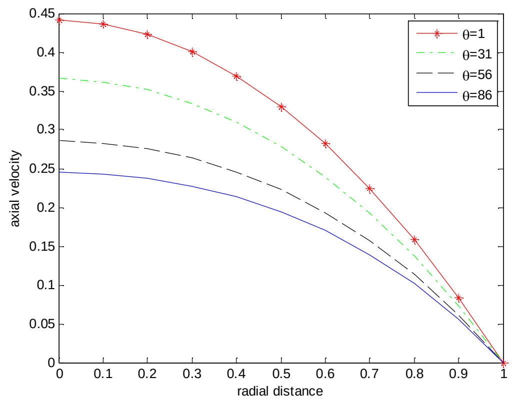

Fig 3: Variation of axial velocity with radial distance

for different values of theta

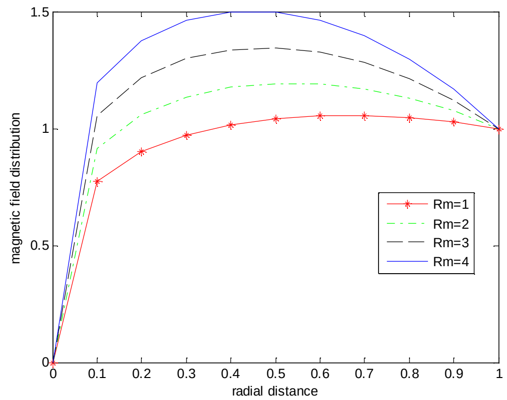

Fig 4: Variation of magnetic field distribution for

different values of magnetic Reynolds Number

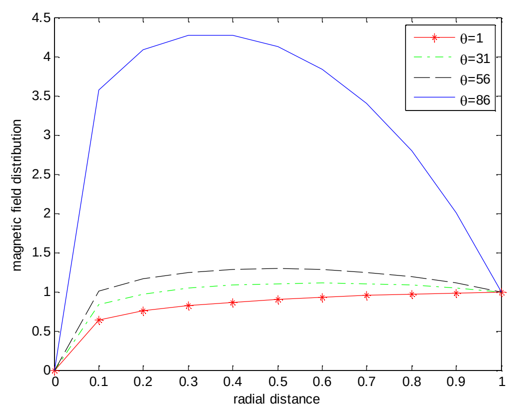

Fig 5: Variation of magnetic field distribution with radial

distance for different values of theta

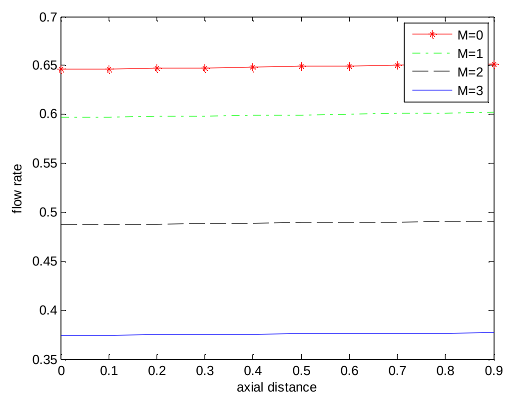

Fig 6: Variation of flow rate with axial distance

for different values of Hartmann Number

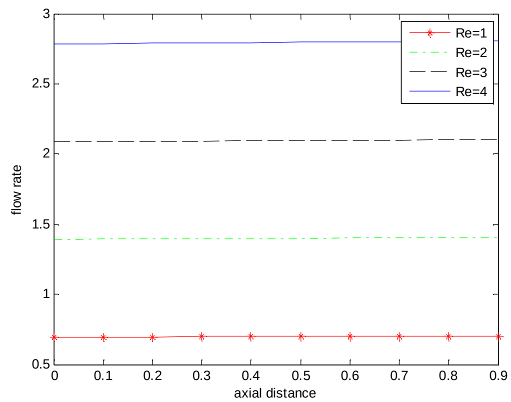

Fig 7: Variation of flow rate with axial distance for different values of Reynolds Number

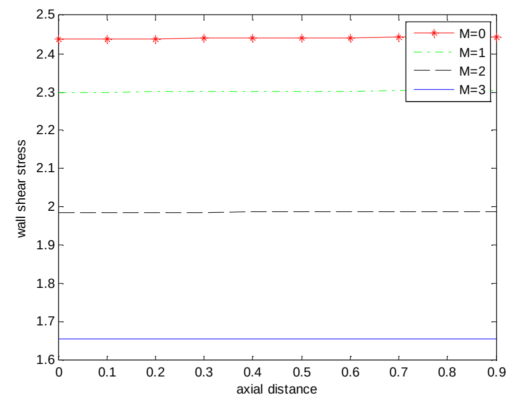

Fig 8: Variation of wall shear stress with axial distance for different values of Hartmann Number

In Figure 1 variation of axial velocity is studied for Hartmann number M=0, 1, 2, 3. The values of other parameters are assumed as $Re = .7$, $R_m = 2$. It is observed that the fluid velocity decreases for increasing value of Hartmann number and it is due to the fact that as the magnetic field applied, the Lorentz force opposes the flow of blood and hence reduces the velocity. Again the influence of Reynolds number on axial velocity with radial distance is shown in the Figure 2 and the velocity profiles increases for increasing values of Reynolds number Re. Figure 3 shows that the axial velocity decreases for increasing values of theta. These results indicate that the velocity profile can be controlled by changing the inclination of external magnetic field. The magnetic field distribution increases for increasing values of Magnetic Reynolds number and the angle of inclination of the external magnetic field which are described in Figure 4 and Figure 5.

The variation of blood flow rate with axial distance is shown in Figure 6 and Figure 7 for values of Hartmann number M=0,1,2,3 and Reynolds number Re=1, 2, 3, 4 respectively. The graphical results shows that the flow rate decreases for increasing values of Hartmann number and it increases for increasing values of Reynolds number . Also from Figure 8 we have seen that the wall shear rate decreases with increasing value of Hartmann number.

III. CONCLUSIONS

From the above analysis we can conclude that a significant variation takes place in flow characteristics due to application of external induced magnetic field. It is seen that the flow rate and wall shear rate decreases with the increase in strength of Magnetic field. So the present study would be helpful in blood pressure control.

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