Nano Generalized A-Continuous and Nano A-Generalized Continuous Functions in Nano Topological Spaces

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***Abstract*— The purpose of this paper is to introduce and investigate the notions of Nano generalized α-continuous and Nano α-generalized continuous functions in Nano Topological spaces. Also, examine some of the properties of such functions.**

***Keywords*— Nα-continuous, Ngα-continuous, Nαg-continuous, Ngα-open and closed functions and Nαg- open and closed functions.**

1. INTRODUCTION

One of the main concepts of topology is continuous functions. In 1991, Balachandran[1] et al., was introduced and studied the notions of generalized continuous and generalized α-continuous functions. Different types of generalizations of continuous functions were introduced and studied by various authors in the recent development of topology. The concepts of Nano topology was introduced by Lellis Thivagar [6, 7], which was defined in terms approximations and boundary region of a subset of a universe using an equivalence relation on it. He has also defined a Nano continuous functions, Nano open mappings, Nano closed mappings and Nano homeomorphisms and their representations in terms of Nano closure and Nano interior. Bhuvaneswari[2] et al., was introduced & studied the Nano generalized closed sets, Nano α-generalized closed set and Nano generalized α-closed set in Nano topological spaces.

2. PRELIMINARIES

Definition: 2.1 A subset A of (X, τ) is called

(i) generalized closed (briefly g-closed) [4] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

(ii) generalized α-closed (briefly gα-closed) [8] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ).

(iii) α – generalized closed (briefly αg-closed) [8] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

Definition: 2.2 A map f: (X, τ)→(Y, σ) is called

(i) g-continuous [5] if $f^{1}(V)$ is g-open in (X, τ) for every open set V in (Y, σ).

(ii) gα-continuous [3] if $f^{1}(V)$ is gα-open in (X, τ) for every open set V in (Y, σ).

(iii) αg-continuous [3] if $f^{1}(V)$ is αg-open in (X, τ) for every open set V in (Y, σ).

(iv) α-continuous [3] if $f^{1}(V)$ is α-open in (X, τ) for every open set V in (Y, σ).

Definition: 2.3 [6] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with in another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = U\{R(x): R(x) \subseteq X\}$, Where R(x) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = U\{R(x): R(x) \cap X \neq \phi\}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Property: 2.4[6] If (U, R) is an approximation space and X, $Y \subseteq U$, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$.
(ii) $L_R(\phi) = U_R(\phi) = \phi$ & $L_R(U) = U_R(U) = U$.
(iii) $U_R(XUY) = U_R(X)\ U\ U_R(Y)$.
(iv) $L_R(XUY) \supseteq L_R(X)\ U\ L_R(Y)$.
(v) $U_R(X\cap Y) \subseteq U_R(X) \cap U_R(Y)$.
(vi) $L_R(X\cap Y) = L_R(X) \cap L_R(Y)$.
(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$, whenever $X \subseteq Y$.
(viii) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = (U_R(X))^C$.
(ix) $U_R\ U_R\ (X) = L_R\ U_R(X) = U_R(X)$.
(x) $L_R\ L_R(X) = U_R\ L_R(X) = L_R(X)$.

Definition: 2.5[6] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 2.4, $\tau_R(X)$ satisfies the following axioms:

(i) U and $\phi \in \tau_R(X)$
(ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets.

Remark: 2.6[6] If $\tau_R(X)$ is the Nano topology on U with respect to X, the the set $B = \{U, L_R(X), U_R(X)\}$ is the basis for $\tau_R(X)$.

Definition: 2.7[6] If $(U, \tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$ then the Nano interior of A is defined as the union of all Nano open subsets of A and it is denoted by NInt(A). That is, NInt(A) is the largest Nano open subset of A. The Nano closure of A is defined as the intersection of all Nano closed sets containing A and it is denoted by NCl(A). That is, NCl(A) is the smallest Nano closed set containing A.

Definition: 2.8[6] A Nano topological space $(U, \tau_R(X))$ is said to be extremely disconnected, if the Nano closure of each Nano open set is Nano open.

Definition: 2.9[6] Let $(U, \tau_R(X))$ be a Nano topological space and $A \subseteq U$. Then A is said to be

(i) Nano Semi open if $A \subseteq NCl(NInt(A))$
(ii) Nano Pre-open if $A \subseteq NInt(NCl(A))$
(iii) Nano α–open if $A \subseteq NInt(NCl(NInt(A)))$

NSO(U, X), NPO(U, X) and $\tau_R^{\alpha}(X)$ respectively, denote the families of all Nano semi open, Nano pre open and Nano α – open subsets of U.

Definition: 2.10[7] Let $(U, \tau_R(X))$ and $(V, \sigma_R(Y))$ be Nano topological spaces. Then a mapping $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$ is Nano continuous on U if the inverse image of every Nano open set in V is Nano open in U.

3. NANO GENERALIZED α-CONTINUOUS IN NANO TOPOLOGICAL SPACES.

Definition: 3.1 A function $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$ is called Nano generalized α-continuous (briefly Ngα-continuous) if $f^{-1}(S)$ is Ngα-open (resp. Ngα-closed) in $(U, \tau_R(X))$ for every Nano open set (resp. Nano closed set) S in $(V, \sigma_R(Y))$. That is, if the inverse image of every Nano open (resp. Nano closed) set in $(V, \sigma_R(Y))$ is Ngα-open (resp. Ngα-closed) in $(U, \tau_R(X))$.

Theorem: 3.2 Let U and V are any two Nano Topological spaces. Let $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$ be Nano continuous function. If f is Nano continuous function, then f is Ngα-conitnuous but not conversely.

Proof: Let S be any Nano closed set in $(V, \sigma_R(Y))$. Since, [2] every Nano closed set is Ngα-closed. Then, $f^{-1}(S)$ is Ngα-closed in $(U, \tau_R(X))$. Therefore, f is $f^{-1}(S)$ is Ngα-continuous. The converse of the theorem need not be true as seen from the following example.

Example: 3.3 Let U = {a, b, c, d} with U/R = {{a}, {c}, {b,d}} and X = {a,b}. Then, $\tau_R(X)$ = {U, ϕ, {a}, {a,b,d}, {b,d}}. Let V = {a, b, c, d} with V/R = {{b}, {d}, {a,c}} and X = {a,b}. Then $\sigma_R(Y)$ = {V, ϕ, {b}, {a, b, c}, {a,c}}. Define a mapping f: U→V as f(a) = c; f(b) = a; f(c) = d; f(d) = b. Then f is Ngα-conitnuous but not Nano continuous as the inverse image of an Nano closed set {a, c, d} in V is {a, b, c} is which is not Nano closed in U.

Theorem: 3.4 Let f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) be Nano continuous function and U&V are any two Nano Topological spaces. If f is Nano α-continuous function, then f is Ngα-conitnuous but not conversely.

Proof: Let S be any Nano closed set in (V, $\sigma_R(Y)$). Then $f^{-1}(S)$ is Nα-closed in (U, $\tau_R(X)$) as f is α-continuous. Since, [2] every Nano α-closed set Ngα-closed. Hence, $f^{-1}(S)$ is Ngα-closed in (U, $\tau_R(X)$). Therefore f is Ngα-conitnuous. The converse of the above theorem neet not be true as seen from the following example.

Example: Let U = {a, b, c} with U/R = {{c}, {a,b}} and X = {b,c}. Then $\tau_R(X)$ = {U, ϕ, {c}, {a,b}}. Let V = {x, y, z} with V/R = {{y}, {x,z}} and X = {y,z}. Then $\sigma_R(Y)$ = {V, ϕ, {y}, {x,z}}. Define f: U→V as f(a) = y; f(b) = z; f(c) = x. Then f is Ngα-conitnuous but not Nano α-conitnuous.

Theorem: 3.5 If a function f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) is α-irresolute then it is Ngα-continuous.

Proof: Assume that f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) is α-irresolute. Let S be any Nano closed set in (V, $\sigma_R(Y)$). Since [2] every Nano closed set is Nano α-closed. Then S is Nano α-closed in (V, $\sigma_R(Y)$). Since f is α-irresolute, $f^{-1}(S)$ is Nano α-closed in (U, $\tau_R(X)$). We know that, [2] every Nano α-closed set is Ngα-closed. Hence, $f^{-1}(S)$ is Ngα-closed in (U, $\tau_R(X)$).

Theorem: 3.6 A function f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) Ngα-continuous if and only if the inverse image of every Nano closed in V is Ngα-closed in U.

Proof: Let F be a closed set in V and f be Ngα-conitnuous. That is, V-F is Nano closed in V. Since, f is Ngα-continuous f^{-1}(V-F) is Ngα-closed in U. But f^{-1}(V-F) = U - f^{-1}(F). Hence, f^{-1}(F) is Ngα-closed in (U, $\tau_R(X)$). Conversely, let the inverse image of every Nano closed set in V is Ngα-closed in U. Assume S is an Nano closed set in V, then S^C is Nano closed in V. By the assumption, $f^{-1}(S^C)$ = U - f^{-1}(S) Ngα-closed in (U, $\tau_R(X)$). Hence, f^{-1}(S) is Ngα-open in U. Hence, f is Ngα-continuous.

4. NANO α-GENERALIZED CONTINUOUS IN NANO TOPOLOGICAL SPACES

Definition: 4.1 A function f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) is called Nano α-generalized continuous (briefly Nαg-continuous) if $f^{-1}(S)$ is Nαg-open (resp. Nαg-closed) in (U, $\tau_R(X)$) for every Nano open set (resp. Nano closed set) S in (V, $\sigma_R(Y)$). That is, if the inverse image of every Nano open (resp. Nano closed) set in (V, $\sigma_R(Y)$) is Nαg-open (resp. Nαg-closed) in (U, $\tau_R(X)$).

Theorem: 4.2 Let U and V are any Nano Topological spaces. Let f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) be Nano continuous function. If f is Nano continuous function, then f is Nαg-conitnuous but not conversely.

Proof: Let S be any Nano closed set in (V, $\sigma_R(Y)$). Then $f^{-1}(S)$ is Nαg-closed in (U, $\tau_R(X)$). Since, [2] every Nano closed set is Nαg-closed. Hence, $f^{-1}(S)$ is Nαg-closed in (U, $\tau_R(X)$). Therefore, f is $f^{-1}(S)$ is Nαg-continuous. The converse of the theorem need not be true as seen from the following example.

Example: 4.3 Let U = {a, b, c, d} with U/R = {{a}, {c}, {b,d}} and X = {a,b}. Then, $\tau_R(X)$ = {U, ϕ, {a}, {a,b,d}, {b,d}}. Let V = {x, y, z, w} with V/R = {{y}, {w}, {x,z}} and X = {x,y}. Then $\sigma_R(Y)$ = {V, ϕ, {y}, {x, y, z}, {x,z}}. Define a mapping f: U→V as f(a) = z; f(b) = x; f(c) = w; f(d) = y. Then f is Nαg-conitnuous but not Nano continuous as the inverse image of a Nano closed set {a, c, d} in V is {x, y, z} is which is not Nano closed in U.

Theorem: 4.4 Let f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) be Nano continuous function and U&V are any two Nano Topological spaces. If f is Nano α-continuous function, then f is Nαg-conitnuous but not conversely.

Proof: Let S be any Nano closed set in (V, $\sigma_R(Y)$). Then $f^{-1}(S)$ is Nα-closed in (U, $\tau_R(X)$) as f is Nano α-continuous. Since, [2] every Nano α-closed set Nαg-closed. Hence, $f^{-1}(S)$ is Nαg-closed in (U, $\tau_R(X)$). Therefore f is Nαg-conitnuous. The converse of the above theorem neet not be true as seen from the following example.

Example: 4.5 Let U = {a, b, c} with U/R = {{c}, {a,b}} and X = {b,c}. Then $\tau_R(X)$ = {U, ϕ, {c}, {a,b}}. Let V = {x, y, z} with V/R = {{y}, {x,z}} and X = {y,z}. Then $\sigma_R(Y)$ = {V, ϕ, {y}, {x,z}}. Define f: U→V as f(a) = y; f(b) = z; f(c) = x. Then f is Nαg-conitnuous but not Nano α-conitnuous.

Theorem: 4.6 Let f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) be Nano continuous function and U&V are any two Nano Topological spaces. If f is Nano g-continuous function, then f is Nαg-conitnuous but not conversely.

Proof: Let f be Ng-continuous function and S be an Nano closed set in (V, $\sigma_R(Y)$). Then, $f^{-1}(S)$ is Ng-closed in (U, $\tau_R(X)$). Since, every Ng closed set is Nαg-closed. Thus, $f^{-1}(S)$ is Nα-closed. Hence, f is Nαg-continuous.

Theorem: 4.7 Let f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) be Nano continuous function and U&V are any two Nano Topological spaces. If f is Nano Ngα -continuous function, then f is Nαg-conitnuous but not conversely.

Proof: Assume that f is Ngα-continuous. Let S be an Nano closed set in (V, $\sigma_R(Y)$). Then, $f^{-1}(S)$ is Ngα-closed. Since, [2] every Ngα-closed set is Nαg-closed set. Thus, $f^{-1}(S)$ is Nαg-closed. Hence f is Nαg-continuous.

5. NANO GENERALIZED α-OPEN AND CLOSED FUNCTIONS

Definition: 5.1 A function f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) is said to be Ngα-open (resp. Ngα-closed) function if the image of every Nano open (resp. Nano closed) set in (U, $\tau_R(X)$) is Ngα-open (resp. Ngα-closed) in (V, $\sigma_R(Y)$).

Theorem: 5.2 Let f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) be a function. If f is an Nano open function, then f is Ngα-open function.

Proof: Let f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) be a Nano open map and S be a Nano open set in U. Then f(S) is Nano open and hence f(S) is Ngα-open in V. Thus f is Ngα-open.

Theorem: 5.3 Let f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) be a function. If f is an Nano closed function, then f is Ngα-closed function.

Proof: Let f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) be a Nano closed map and S be a Nano closed set in U. Then f(S) is Nano closed and hence f(S) is Ngα-closed in V. Thus f is Ngα-closed.

Theorem: 5.4 A function f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) is Ngα-closed if and only if for each subset B of V and for each Nano open set G containing $f^{-1}(B)$ there exists a Ngα-open set F of V such that $B \subseteq F$ and $f^{-1}(B) \subseteq G$.

Proof: (Necessity) Let G be a Nano open subset of (U, $\tau_R(X)$) and B be subset of V such that $f^{-1}(B) \subseteq G$.Define, F = V-f(U-G). Since, f is Ngα-closed. Then F is Ngα-open set containing B such that $f^{-1}(F) \subseteq G$.

(Sufficiency) Let E be Nano closed subset of (U, $\tau_R(X)$). Then $(f^{-1}(V – f(E)) \subseteq (U – E)$ and (U – E) is Nano open. By hypothesis, there is a Ngα-open set F of (V, $\sigma_R(Y)$) such that $V – f(E) \subseteq F$ and $f^{-1}(B) \subseteq U – E$. Therefore, $E \subseteq U\text{-}f^{-1}(F)$. Hence, $V – F \subseteq f(E) \subseteq f(U\text{-}f^{-1}(F)) \subseteq V – F$. Which implies that f(E) = V – F and hence f(E) is Ngα-closed in (V, $\sigma_R(Y)$). Therefore, f is Ngα-closed function.

Theorem: 5.5 If a function f: U→V is Nano closed and a map g: V→W is Ngα-closed then their composition gof: U→W is Ngα-closed.

Proof: Let H be a Nano closed set in U. Then f(H) is Nano closed in V and (gof)(H) = g(f(H)) is Ngα-closed, as g is Ngα-closed. Hence, gof is Ngα-closed.

6. NANO α-GENERALIZED OPEN AND CLOSED FUNCTIONS

Definition: 6.1 A function f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) is said to be Nαg-open (resp. Nαg-closed) function if the image of every Nano open (resp. Nano closed) set in (U, $\tau_R(X)$) is Nαg-open (resp. Nαg-closed) in (V, $\sigma_R(Y)$).

Theorem: 6.2 Let f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) be a function. If f is an Nano open function, then f is Nαg-open function.

Proof: Let f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) be a Nano open map and S be a Nano open set in U. Then f(S) is Nano open and hence f(S) is Nαg-open in V. Thus f is Nαg-open.

Theorem: 6.3 Let f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) be a function. If f is an Nano closed function, then f is Nαg-closed function.

Proof: Let f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) be a Nano closed map and S be a Nano closed set in U. Then f(S) is Nano closed and hence f(S) is Nαg-closed in V. Thus f is Nαg-closed.

Theorem: 6.4 A function f: (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) is Nαg-closed if and only if for each subset B of V and for each Nano open set G containing $f^{-1}(B)$ there exists a Nαg-open set F of V such that $B \subseteq F$ and $f^{-1}(B) \subseteq G$.

Proof: (Necessity) Let G be a Nano open subset of (U, $\tau_R(X)$) and B be subset of V such that $f^{-1}(B) \subseteq G$. Define, F = V- f(U-G). Since, f is Nαg-closed. Then F is Nαg-open set containing B such that $f^{-1}(F) \subseteq G$.

(Sufficiency) Let E be Nano closed subset of (U, $\tau_R(X)$). Then $(f^{-1}(V - f(E)) \subseteq (U - E)$ and (U – E) is Nano open. By hypothesis, there is a Nαg-open set F of (V, $\sigma_R(Y)$) such that $V - f(E) \subseteq F$ and $f^{-1}(B) \subseteq U - E$. Therefore, $E \subseteq U - f^{-1}(F)$. Hence, $V - F \subseteq f(E) \subseteq f(U-f^{-1}(F)) \subseteq V - F$. Which implies that f(E) = V – F and hence f(E) is Nαg-closed in (V, $\sigma_R(Y)$). Therefore, f is Nαg-closed function.

Theorem: 6.5 If a function f: U→V is Nano closed and a map g: V→W is Nαg-closed then their composition gof: U→W is Nαg-closed.

Proof: Let H be a Nano closed set in U. Then f(H) is Nano closed in V and (gof)(H) = g(f(H)) is Nαg-closed, as g is Nαg-closed. Hence, gof is Nαg-closed.

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