Antiflexible Rings with Weak Novikov Identity

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Abstract **- If R is an antiflexible ring of characteristic ≠ 2, 3 with Weak Novikov identity (w, x, y z) = y (w, x, z) then Strong Novikov identity x (y z) = y(x z). Using this results we prove that, if R is a prime not associative antiflexible ring of characteristic ≠ 2, 3 satisying the Weak Novikov identity (w, x, yz) = y (w, x, z) then R is either an alternative ring (or) strongly (-1,1) ring.**

Key words **- Antiflexible rings, Weak Novikov identity, Strong Novikov identity, alternative ring, strongly (-1, 1) ring.**

I. INTRODUCTION

E. Kleinfeld in1994 [1] proved that a prime non-associative Weakly Novikov ring (x, y, z) = (x, z, y) must be Strong Novikov. Again Kleinfeld in 1996 [2] proved that a semi prime ring of characteristic ≠ 2 satisfying the variations of the Novikov identities (x y) z = (x z) y and (x, y, z) = - (x, z, y) is associative. In the another paper of Kleinfeld [3], it is proved that a prime right alternative ring with minimum condition on right ideals which satisfies the identity (w, x, yz) = y (w, x, z) must be associative. Lastly, K. Subhashini in [4] has proved that, if R is a prime (-1,1) ring of characteristic ≠ 2, 3 then R must be commutative and associative.In this paper, first we prove that, a Weak Novikov identity is a Strong Novikov identity. Using this condition of Weak Novikov identity, we prove that an antiflexible ring of characteristic ≠ 2, 3 is either an alternative ring (or) strongly (-1, 1) ring.

II. PRELIMINARIES

A ring is said to be antiflexible ring if it satisfy the identity

A(x, y, z) = (x, y, z) - (z, y, x) -------(1)

The identity (w, x, y z) = y (w, x, z) ------(2)

is known as Weak Novikov identity.

Where as the identity x(yz) = y(xz) ------(3)

is refered as Strong Novikov identity.

A ring is Strong Novikov then it is Weakly Novikov. Moreover, Weakly Novikov rings are a subclass of associative rings where as Strong Novikov rings are not.

The Teichmuller identity which holds in any ring.

B(w, x, y, z) = (wx, y, z) - (w, xy, z) + (w, x, yz) – w(x, y, z) – (w, x, y)z = 0 ------(4)

An antiflexible ring R is a non-associative ring in which the following identities hold.

(w, (x, y, z)) = 0 by [5] ------(5)

The Semi-Jacobi identity is

C(x, y, z) = (x y, z) – x (y, z) – (x, z) y – (x, y, z) – (z, x, y) + (x, z, y) = 0 ------(6)

The nucleus N of any ring is defined as

$N = \{ n \in R \,/\, (n, R, R) = (R, R, n) = (R, n, R) = 0\}$.

An alternative ring R is a ring in which

$(x\, x)\, y = x\, (x\, y),\ y\, (x\, x) = (y\, x)\, x$, for all x, y in R. ------(7)

These equations are known as the left and right alternative laws respectively.

A right alternative ring R satisfying the identity ((R, R), R) = 0 is called a strongly (-1,1) ring.

Lemma 2.1 : Let $n \in N$ then $(R,N) \subseteq N$.

Proof: Let $w, x, y, z \in R$ and $n \in N$.

We now take a turn letting one of four elements in Teichmuller identity (4) be in the nucleus N. Thus

$(n\, x, y, z) = n\, (x, y, z)$

$(w\, n, y, z) = (w, n\, y, z)$ -------(8)

$(w, x\, n, z) = (w, x, n\, z)$ -------(9)

$(w, x, y\, n) = (w, x, y)\, n$ -----(10)

By using equations (5), (1), (10), (1) and (9), we have

W = n in (5)

$n\, (x, y, z) = (x, y, z)\, n$ (by (5))

$= (z, y, x)\, n$ (by (1))

$= (z, y, x\, n)$ (by (10))

$= (x\, n, y, z)$ (by (1))

$= (x, n\, y, z)$ (by (8))

$= (x, y, n\, z)$ (by (9))

$= (x, y, z)\, n$ (by (10))

$= (x, y, z\, n)$ (by (10))

$= (x, y, n\, z)$

Hence $(x, y, zn) – (x, y, nz) = 0$

implies $(x, y, (z, n)) = 0$ ------(11)

Hence $(R, N) \subseteq N$. ♦

Lemma 2.2 : The nucleus N of R is an ideal such that NA = 0. If R is prime and non-associative ring then N = 0.

Proof: For arbitrary elements $x, y, z \in R$ and $n \in N$.

From (2), we have

$(x, y, z\, n) = z\, (x, y, n) = 0$

also from (10) $(x, y, n\, z) = (x, y, z\, n) = 0$.

Therefore N is both left and right ideal have an ideal of R.

Again using (2) and (5), we have

$(x, y, n\, z) = 0 = n\, (x, y, z) = (x, y, z)\, n$

i.e., N A = A N = 0

Since R is prime and not associative

and hence N = 0. ♦

Lemma 2.3 : If R is prime and not associative then R is Strongly Novikov.

Proof : Through the repeated use of (2) and (1),

For any a, b $\in$ R, we obtain,

$(a, b, x.yz) = x (a, b, y z)$ (by (2))

$= x (y z, b, a)$ (by (1))

$= (y z, b, x a)$ (by (2))

$= (x a, b, y z)$ (by (1))

$= y (x a, b, z)$ (by (2))

$= y (z, b, x a)$ (by (1))

$= y.x (z, b, a)$ (by (2))

$= y.x (a, b, z)$ (by (1))

$= y (a, b, x z)$ (by (1))

$= (a, b, y.xz)$ (by (2))

Therefore $(a, b, x.yz) = (a, b, y.xz)$

$\Rightarrow (a, b, x.yz) - (a, b, y.xz) = 0$

$\Rightarrow (a, b, x.yz - y.xz) = 0$

Therefore $x.yz - y.xz \in N$.

From lemma 2.2, N = 0,

Hence we have Strong Novikov identity $x.yz = y.xz$ holds in R. ♦

Lemma 2.4: If R is a prime and not associative ring then U is an ideal.

Proof : Note that

$(xy, y) = xy.y - y.xy$

$= xy.y - x.yy$ (by (3))

$= xy^2 - xy^2$

$= 0.$

Linearization results in $(xy, z) = -(xz, y)$

If $u \in U$ and $y = u$ then $(xu, z) = 0$

Thus U is a left ideal.

Since xu = ux, it follows that

U is an ideal of R. ♦

Consider the equation $(y, (x, x, y)) = 0$

Replacing y by $y + (a, b)$ in the equation then we obtain ((a, b),(x, x, y)) = - (y, (x, x, (a, b))) ------(12)

In D(x, y, z) = (x, (yz)) + (y, (zx)) + (z, (x, y)) = 0

Put y = (R, R, R) an arbitrary associator and apply (5), then we have

((R, R, R), (z, x)) = 0 ------(13)

Let I be the linear span of the alternators in R.

Obviously I is an ideal of R. ♦

Lemma 2. 5: Let I be an ideal of an antiflexible ring with characteristic $\neq 2, 3$ then

(a) ann(I) = {x ∈ R/ xI = Ix = 0} is an ideal.

(b) ANN(I) = { x ∈ ann(I) / (I, R, x) = 0 } is the largest ideal of R containing in ann(I).

Proof :

By virtue of B(x, y, z) = (x, y, z) + (y, z, x) + (z, x, y) = 0

Then we claim that ann(I) is an ideal.

Let t ∈ I, h ∈ ann(I), k ∈ ANN(I) and x, y ∈ R.

Since ANN(I) ⊆ ann(I)

We know that all six associators

(k, t, x) = (k, x, t) = (x, k, t) = (t, x, k) = (t, k, x) = 0

Thus kx.t = k.xt = 0

And t.kx = tk.x = 0

i.e., kx ∈ ann(I).

Also from D(x, w, y, z) ≡ (xw, y, z) - (x, w, yz) + (x, y, wz) – (x, w, z)y – (x, y, z)w = 0

We have 0≡ (t, y, kx) + (t, k, yx) – (t, y, x)k – (t, k, x)y = (t, y, kx)

Since (t, y, x) ∈ I

Therefore ANN(I) is a right ideal.

Now (xk)t = x(kt) = 0 and

t(xk) = (tx)k = 0

so xk ∈ ann(I)

⇒ ann(I) is an ideal.

To show (t, y, xk) = 0

We consider B(t, x, k, y) = (tx, k, y) – (t, xk, y) + (t, x, ky) – t(x, k, y) – (t, x, k)y = 0

Since I is an ideal, ANN(I) is a right ideal contained in ann(I) and any associator with elements from R, I and ANN(I) is zero, then these two identities reduce to

-(t, xk, y) – t(x, k, y) = 0 and (x, k, y)t = 0

Adding these two identities and applying (5) we have

ANN(I) ⊆ ann(I)

Thus (t, y, xk) = 0.

Which establishes ANN(I) is an ideal of R. ♦

Theorem 2.1 : Let R be a prime not associative antiflexible ring of characteristic $\neq 2,3$ satisfying the weak Novikov identity (w, x, yz) = y(w, x, z), then R is either an alternative ring or a strongly (-1,1) ring.

Proof : By semi-Jacobi identity , we have

C(x, y, z) = (x y, z) – x (y, z) – (x, z) y – (x, y, z) – (z, x, y) + (x, z, y) = 0

Interchanging x and y in this equation, we have

C(y, x, z) = (y x, z) – y (x, z) – (y, z) x – (y, x, z) – (z, y, x) + (y, z, x) = 0

Subtracting these two equations, we have

(x y, z) – x (y, z) – (x, z) y – (x, y, z) – (z, x, y) + (x, z, y) – (y x, z) + y (x, z) + (y, z) x + (y, x, z) + (z, y, x) – (y, z, x) = 0.

$\Rightarrow$(xy - yx, z) – (x(y, z) - (y, z)x) + (y(x, z) - (x, z)y) = 0 ------(14)

Since I is an ideal and also from (14), (I, Z) = 0, we obtain

(x(y ,z) - (y, z)x)= 0

$\Rightarrow$ x(y, z) = (y, z)x

Let x = (x, x, z) be an alternator and z = (R, R), we have

(x, x, z) (y, (R, R)) = 0 = (y, (R, R)) (x, x, z)

Thus we have established (y, (R, R)) $\in$ ann(I)

Next using linearized (14) and the fact that I is an ideal, we have

(I, R, (y, (R, R))) = - (R, I, (y, (R, R))) = 0

Thus (y, (R, R)) $\in$ ANN(I)

But ANN(I) is an ideal of R from Lemma 6

Since I. (ANN(I)) = 0 and R is prime.

Then either Z = 0 or (R, (R, R)) = 0.

If I = 0 then R is alternative ring.

If (R, (R, R)) = 0 then R is strongly (-1, 1) ring. ♦

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