Variable Viscosity and Thermal Conductivity on Steady Free Convection Heat Transfer of Micro-polar Fluid Flow Over a Porous Hot Vertical Plate With Constant Heat Flux

Prof. G.C. Hazarika[#1], Probhat Hatimota[*2]

[#]Professor and Head, Department of Mathematics, Dibrugarh University,Dibrugarh, Assam, India

[*] Department of Mathematics, Dibrugarh University,Dibrugarh, Assam, India

Abstract— **This paper presents a numerical investigation of the effect of variable viscosity and thermal conductivity on natural heat transfer of steady incompressible micro-polar fluid flow over a porous hot vertical plate with constant heat flux .Here the flow governing equations are trans -formed into ordinary differential equations and the resulting boundary value problems are solved numerically by using Runge-Kutta shooting method . The results are represented graphically to illustrate influence of the various physical parameters on the velocity profiles, micro-rotation profile and heat transfer profile.**

Keywords— : **Variable viscosity and thermal conductivity , micro-polar fluid , porous medium , skin friction , shooting method.**

I. Introduction

Eringen formulated the micro-polar fluid theory in 1960 [8] as an extension of the Navier-Stokes model of classical hydro-dynamics to facilitate the description of the fluids with complex molecules. It is a suitable form of non-Newtonian fluid model that can be used to analyse the behaviour of exotic lubricants, colloidal suspensions, polymeric fluids, liquid crystals, human and animal bloods, and so forth. The micro-polar fluids deals with a class of fluids which exhibits certain microscopic effects arising from the local structure and micro-motions of the fluid elements. These fluids contain dilute suspensions of rigid macromolecules with individual motions that support stress and body moments and are influenced by spin inertia.

In recent years, the dynamics of micro-polar fluids has been popular area of research because of its wide and increasing application in heat power engineering, chemical engineering in various branches of industry and agriculture. Researcher have focussed mainly on the heat transfer of micro-polar fluid flow. Ahmadi studied the fluid flow characteristic of boundary layer flow of a micro-polar fluid over a semi-infinite plate, using a Runge-Kutta shooting method with Newtonian iteration [1].Hassanien and Gorla studied the mixed convection in stagnation flow of micro-polar fluid over a vertical surface with variable surface temperature and uniform surface heat flux [14].Gorla et al. analysed the heat transfer characteristic of a micro-polar fluid over a flat plate [12].Ezzat et al. studied some problems of micro-polar magneto -hydrodynamics boundary layer flow [9]. M. A. Ezzat studied about the free convection flow of conducting micro-polar fluid with thermal relaxation including heat sources applying the inversion of the Laplace transforms to carry out the numerical results [10].Patowary G and Sut D.K. studied about the micor-polar fluid flow near the stag-nation on a vertical plate with prescribed wall heat flux [17]. Rebhi A. Damsehet. al studied about unsteady natural convection heat transfer of micro-polar fluid over a vertical surface with constant heat flux [19] .They found that the temperature increases inside the boundary layer for the micro-polar flow, as compared to Newtonian flows also increasing the vortex viscosity parameter increases the rotation inside the boundary layer and it has a property to increase the coefficient of friction and to decrease the local Nusselt number .Borthakur P.J and Hazarika G. C. have studied about the effect of variable viscosity and thermal conductivity on the flow heat transfer of a stretching surface in a rotating micro-polar fluid with suction and blowing [4], boundary layer and heat transfer fluid near an axisymmetric stagnation point on a moving cylinder [5]. Borgohain B. and Hazarika G. C. studied about the effect of variable viscosity and thermal conductivity on the flow of a micro-polar fluid bounded by stretching sheet [3] , stretching surface with suction and blowing in presence of a magnetic field [6].

In many studies done earlier on the free convection of micro-polar fluid flow through porous medium, the viscosity and thermal conductivity were assumed to be constant. However these physical properties can change significantly with temperature and when the variable viscosity and thermal conduvicity are taken into account, the flow characteristic are substantially changed to the constant cases. Borah G and Hazarika G.C. has studied the effects of variable viscosity and thermal conductivity on steady laminar free convection flow of an electrically conducting Newtonian fluid along a porous hot vertical plate in presence of heat source/sink [2].

Literature survey reveals that no research regarding to the effect of variable viscosity and thermal conductivity heat on steady free convection heat transfer of incompressible micro-polar fluid flow over a porous hot vertical plate with constant flux . This paper aims to investigate to the effect of variable viscosity and thermal conductivity on steady free convection heat transfer of incompressible micro-polar fluid flow over a porous hot vertical plate with constant flux .

Here the flow governing equations are transformed into ordinary differential equations and the resulting boundary value problems are solved numerically by using Runge-Kutta shooting method [7,13,20]

II. MATHEMATICAL FORMULATION

Let us consider the steady one-dimensional vertical flow of incompressible electrically conducting micro-polar fluid past an infinite plane surface. The x^*-axis is taken in the vertical direction along the plate and the y^*-axis is normal to it. The velocity components of the fluids are $(u^*,v^*,0)$ and $(0,0,N_3^*)$ are local angular velocity acting in z-direction. A constant magnetic field with components $(0,H_0, 0)$ is assumed to be supplied transversely to the direction of the flow. The induced electric current due to the motion of the fluid that is caused by the buoyancy forces does not distort the applied magnetic field. Since the motion is two dimensional and the length of the plate is large, therefore, all the physical variables are independent of x^*. The system of the boundary-layer equations that govern the steady one dimensional free convectional flow through a conducting medium of micro-polar fluid in the presence of a constant magnetic field and if the body-force and body couple are absent the equations of continuity, motion, micro-rotation and energy are :-

The equation of continuity is :-

$$\frac{\partial v^*}{\partial y^*} = 0$$

$\Rightarrow$ independent of y^*, i.e. $v^*=-v_0$ (constant).

The momentum equation is :-

$$\rho v^* \frac{\partial u^*}{\partial y^*} = (\mu + k)\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial \mu}{\partial y^*}\frac{\partial u^*}{\partial y^*} + k\frac{\partial N_3^*}{\partial y^*} + \rho g \beta (T^* - T_\infty) - \sigma B_0^2 u^* \quad (1)$$

The angular momentum equation is

$$\rho j v^* \frac{\partial N_3^*}{\partial y^*} = \gamma \frac{\partial^2 N_3^*}{\partial y^{*2}} - k\left(\frac{\partial u^*}{\partial y^*} + 2N_3^*\right) \quad (2)$$

The generalized energy equation is

$$\rho C_p v^* \frac{\partial T^*}{\partial y^*} = \frac{\partial \lambda}{\partial y^*}\frac{\partial T^*}{\partial y^*} + \lambda \frac{\partial^2 T^*}{\partial y^{*2}} + S^*(T^* - T_\infty) + (\mu + k)\left(\frac{\partial u^*}{\partial y^*}\right)^2 \quad (3)$$

Where μ= the dynamic viscosity, k= the vortex viscosity, γ = the spin gradient viscosity, ρ= the density , j= the micro inertia density, λ=the thermal conductivity, β=the thermal expansion coefficient, g=acceleration due to gravity, B_0= external magnetic field coefficient, S^* =source/sink parameter, C_p=specific heat at constant pressure, σ=magnetic permeability, T_w^*=temperature of the plate, T_∞ =free stream temperature.

The boundary conditions are

At $y^*=0$, $u^*=0$, $N_3^* = -n\frac{\partial u^*}{\partial y^*}$ and $T^*=T_w$

When $y^* \to \infty$, $u^* \to 0$, $N_3^* \to 0$ and $T^* \to T_\infty$ (4)

Here n is a constant and 0<n<1 , [21]. The case n=0 is called strong concentration by Gram and Smith [11], corresponds to the case where the particle density is sufficiently large so that microelements close to the wall are unable to rotate [15]. The case n=0.5 represents a weak representation of the micro-elements and vanishing of anti-symmetric part of the stress tensor [21]. The case n=1 .0 corresponds to the turbulent flow inside the boundary layer [18].

Lai and Kulacki have assumed that viscosity and thermal conductivity of fluid to be an inverse linear relation of temperature [16] as

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}[1+\gamma_1\ (T-T_\infty)]\ \text{and}\ \frac{1}{\lambda}=\frac{1}{\lambda_\infty}[1+\gamma_2(T-T_\infty)]$$

Here μ_∞=the viscosity at infinity, λ_∞=the thermal conductivity at infinity, T_∞=the fluid temperature at infinity and γ_1 and γ_2 are constants.

Let us the following non-dimensional variables :-

$$\eta=\frac{v_0 y^*}{v_\infty}\ ,\ f=\frac{u^*}{v_0},\ g=\frac{v_\infty}{v_0^{\ 2}}N_3^{\ *}\ ,\ \theta=\frac{T^*-T_\infty}{T_w-T_\infty} \tag{5}$$

And let us assume $\mu=-\frac{\mu_\infty\theta_e}{\theta-\theta_e},\quad \lambda=-\frac{\lambda_\infty\theta_r}{\theta-\theta_r}$ (6)

Invoking the non-dimensional quantities above and using (6) in equations (1), (2), (3) and dropping the asterisks we get

$$\left[1-\mu_1\frac{\theta-\theta_e}{\theta_e}\right]f''=\frac{\theta-\theta_e}{\theta_e}\left[f'+\frac{\theta-\theta_e}{\theta_e}\theta' f'+\mu_1 g'+Gr\theta-M^2 f\right] \tag{7}$$

$$g''=R_2\left(f'+2g\right)-R_1 g' \tag{8}$$

$$\theta''=\frac{\theta'^2}{\theta-\theta_r}+\frac{\theta-\theta_r}{\theta_r}\left[\left(1+S\right)\theta'+E\left(\mu_1-\frac{\theta_e}{\theta-\theta_e}\right)f'^2\right]\Pr \tag{9}$$

where $\Pr=\frac{\mu_\infty c_p}{\lambda_\infty}$ (Prandtl number), $Gr=\frac{\rho\beta g(T_w-T_\infty)v_\infty^2}{v_0^3\mu_\infty}$ (Grashof number), $M^2=\frac{B_0^2 v_\infty^2\sigma}{v_0^2\mu_\infty}$ (Hartmann number), $E=\frac{v_0^2}{c_p(T_w-T_\infty)}$ (Eckert number)

and the following are dimensionless micro-polar parameters :-

$$\mu_1=\frac{k}{\mu_\infty}\quad ,R_1=\frac{\rho j v_\infty}{\gamma},\qquad R_2=\frac{k v_\infty^{\ 2}}{\gamma v_0^{\ 2}}$$

The dashes denote differentiation w. r. t. 'η' , the dimensionless distance parameter.

The boundary conditions on f, g and θ are

At η=0, f(0)=0, $g(0)=-nf'(0)$, θ(0)=1

When $\eta\to\infty$, f=0, g=0, $\theta\to 0, C\to 0$

The skin friction coefficient C_f, and the Nusselt number Nu of this problem are given by

$$C_f=\frac{2[(\mu+k)\frac{\partial f}{\partial\eta}+kg]_{\eta=0}}{\rho v_0^{\ 2}}$$

$$=2[(\mu_1-\frac{\theta_e}{\theta-\theta_e})f'(0)+\mu_1 g(0)]$$

$$N_u=\frac{y^*\lambda\frac{\partial T^*}{\partial y^*}]_{y^*=0}}{\lambda_\infty(T_w-T_\infty)}$$

$$=-\frac{\theta_r}{1-\theta_r}\theta'(0)$$

III. METHOD OF SOLUTION

To solve the boundary value problems (6)-(9) Runge-Kutta shooting method is applied. In this method the BVP are converted to initial value problems by estimating the missing initial values to a desired degree of accuracy by an iterative scheme. Hazarika[10] showed that through there is no guarantee of convergence of the iterative scheme, if the initial guesses for the missing initial values are on opposite sides of the true values. The convergence is rapid and agrees well with other methods.

IV. RESULT AND DISCUSSION

In this paper the system of differential equations governed by the respective boundary conditions are solved numerically by applying an efficient numerical technique based on the common Rung-Kutta shooting method. The whole numerical scheme can be programmed and applied easily. It is experienced that the convergence of the iteration process is quite rapid.

Using shooting method, the missing initial values viz. $f'(0)$, $g'(0)$ and $\Theta'(0)$ are estimated for various combination of parameters.

In Tables (1-2), missing values of $f'(0)$, $g'(0)$ and $\Theta'(0)$ were found for different values of M and Θ_e. It is observed that the missing values of $f'(0)$, $g'(0)$ and $\Theta'(0)$ decrease for M increases and increase for Θ_e increases.

In Tables (3-4) we observed that missing values of $f'(0)$, $g'(0)$ and $\Theta'(0)$ decrease with the increasing values of both the parameters M and Θ_r.

From Tables(5-7) it is seen that the missing values $f'(0)$, $g'(0)$ and $\Theta'(0)$ are increased for increasing values of R_1 and R_2 while that are decreased for increasing values of Θ_r.

In Table 8, it is observed that missing values $f'(0)$, $g'(0)$ and $\Theta'(0)$ are increased for R_2 increases and are decreased for Θ_e increases.

The variation of angular velocity profiles are shown in Fig.1 and Fig.2 for different values of R_1 and R_2 respectively. In both cases, angular velocity decrease with increasing R_1 and R_2.

Fig.3 and Fig.4 represent temperature profiles for thermal conductivity variation parameter Θ_r. In both the cases , temperature field decreases with increasing Θ_r but not significantly.

Fig. 5 represents temperature profiles for source/sink parameter S and it is seen that temperature profiles are decreased with increasing S.

In Fig. 6 we are observing that the effect of velocity profiles with the variation of micro-polar parameter R_1. It is observed that the fluid velocity increases as R_1 increases.

From the Fig.7 and Fig.8 it is seen that velocity profiles are decreased with both the parameters S and Θ_r increase.

In Fig.9 we study the effect of Θ_e, the variable viscosity parameter , on velocity profile. The study reveals that the velocity profile increases with the increase of the variable viscosity parameter Θ_e, but not significantly.

Figs.(10-12) illustrate the effects of the parameters Gr, μ_1 and M on the velocity profile and it is observed that the velocity profile increases with the increase of the parameters Gr, μ_1 and M.

Table 1:

Estimated missing values of $f'(0)$, $g'(0)$ and $\Theta'(0)$ for various Θ_e and Θ_r=3.00, μ_1=0.10, R_1=0.25, R_2=0.25, Gr=0.10, Pr=0.70, S=1, E=0.05, n=0.5

M→	0.25			0.25			0.75		
Θ_e ↓	f'	g'	Θ'	f'	g'	Θ'	f'	g'	Θ'
-10	0.102772	0.039004	-0.933561027	0.090928	0.034775	-0.93357265	0.078717	0.030368	-0.933582604
-9	0.103463	0.039281	-0.933560848	0.091524	0.035015	-0.933572531	0.079218	0.030571	-0.933582544
-8	0.104329	0.039626	-0.933560789	0.092269	0.035314	-0.933572292	0.079845	0.030824	-0.933582485
-7	0.105442	0.040071	-0.93356061	0.093228	0.035699	-0.933572114	0.080649	0.03115	-0.933582366
-6	0.106929	0.040665	-0.933560312	0.094506	0.036213	-0.933572114	0.081721	0.031583	-0.933582306
-5	0.109014	0.041498	-0.933559895	0.096297	0.036933	-0.933571875	0.083221	0.03219	-0.933582187
-4	0.11215	0.042753	-0.933559239	0.098984	0.038014	-0.933571518	0.085467	0.033099	-0.933582008
-3	0.117397	0.044852	-0.933558345	0.103466	0.039819	-0.933570802	0.0892	0.034612	-0.933581591
-2	0.127961	0.049085	-0.93355614	0.11244	0.043436	-0.933569431	0.096628	0.037625	-0.933580816

Table 2:

Estimated missing values of $f'(0)$, $g'(0)$ and $\Theta'(0)$ for various Θ_e and Θ_r=3.00, μ_1=0.10, R_1=0.25, R_2=0.25, Gr=0.10, Pr=0.70, S=1, E=0.05, n=0.5

M→	0.25			0.25			0.75		
θ_e ↓	f'	g'	θ'	f'	g'	θ'	f'	g'	θ'
2	0.066346	0.024514	-0.933564186	0.059064	0.02202	-0.933574319	0.051459	0.019384	-0.933583319
3	0.076263	0.028447	-0.933564425	0.067835	0.025522	-0.933574677	0.059058	0.022438	-0.933583617
4	0.081284	0.030442	-0.933564067	0.072248	0.027286	-0.933574438	0.062853	0.023966	-0.933583438
5	0.084316	0.031648	-0.93356365	0.074903	0.028349	-0.9335742	0.065127	0.024882	-0.933583438
6	0.086344	0.032455	-0.93356353	0.076676	0.029059	-0.933574021	0.066642	0.025493	-0.933583379
7	0.087797	0.033033	-0.933563292	0.077944	0.029567	-0.933574081	0.067723	0.025929	-0.933583319
8	0.088888	0.033468	-0.933563232	0.078896	0.029948	-0.933574021	0.068534	0.026256	-0.9335832
9	0.089738	0.033806	-0.933563054	0.079636	0.030245	-0.933573902	0.069164	0.02651	-0.933583319
10	0.090419	0.034078	-0.933562994	0.080229	0.030482	-0.933573723	0.069668	0.026713	-0.93358314

Table 3:

Estimated missing values of f'(0), g'(0) and θ'(0) for various Θ_r and Θ_e=2.00, μ_1=0.10, R_1=0.25, R_2=0.25, Gr=0.10, Pr=0.70, S=1, E=0.05, n=0.5

M→	0.25			0.25			0.75		
θ_r ↓	f'	g'	θ'	f'	g'	θ'	f'	g'	θ'
-10	0.058998	0.02224	-1.540232	0.052743	0.020044	-1.540242	0.046214	0.017724	-1.540251
-9	0.058848	0.022192	-1.555788	0.052614	0.020002	-1.555798	0.046107	0.017689	-1.555807
-8	0.058663	0.022133	-1.575233	0.052454	0.01995	-1.575243	0.045974	0.017645	-1.575252
-7	0.058428	0.022057	-1.600233	0.052251	0.019884	-1.600244	0.045805	0.01759	-1.600253
-6	0.058119	0.021958	-1.633568	0.051986	0.019798	-1.633578	0.045584	0.017517	-1.633587
-5	0.057697	0.021821	-1.680237	0.051622	0.019679	-1.680247	0.045281	0.017417	-1.680256
-4	0.057085	0.021622	-1.750241	0.051094	0.019506	-1.750251	0.044841	0.017271	-1.75026
-3	0.056114	0.021305	-1.866918	0.050257	0.019229	-1.866928	0.044143	0.017038	-1.866937
-2	0.054333	0.020716	-2.100291	0.048722	0.018716	-2.100301	0.042861	0.016605	-2.100309

Table 4:

Estimated missing values of f'(0), g'(0) and θ'(0) for various Θ_r and Θ_e=2.00, μ_1=0.10, R_1=0.25, R_2=0.25, Gr=0.10, Pr=0.70, S=1, E=0.05, n=0.5

M→	0.25			0.25			0.75		
θ_r ↓	f'	g'	θ'	f'	g'	θ'	f'	g'	θ'
2	0.07046	0.025715	-0.700233	0.062599	0.023065	-0.700243	0.05438	0.020257	-0.700251
3	0.066346	0.024514	-0.933564	0.059064	0.02202	-0.933574	0.051459	0.019384	-0.933583
4	0.06464	0.024	-1.05023	0.057597	0.021574	-1.05024	0.050245	0.01901	-1.050249
5	0.063699	0.023714	-1.12023	0.056788	0.021325	-1.12024	0.049574	0.018801	-1.120249
6	0.063102	0.02353	-1.166896	0.056274	0.021165	-1.166907	0.049148	0.018667	-1.166915

7	0.062689	0.023403	-1.200229	0.055919	0.021054	-1.20024	0.048854	0.018574	-1.200249
8	0.062386	0.023309	-1.225229	0.055659	0.020973	-1.22524	0.048638	0.018505	-1.225249
9	0.062155	0.023237	-1.244674	0.055459	0.02091	-1.244684	0.048472	0.018453	-1.244693
10	0.061972	0.02318	-1.260229	0.055302	0.020861	-1.26024	0.048342	0.018411	-1.260249

Table 5:
Estimated missing values of f'(0) ,g'(0) and ϴ'(0) for various Θ_r and Θ_e=2.00, μ_1=0.10, R_2=0.25, Gr=0.10, M=0.25, Pr=0.70, S=1, E=0.05, n=0.5

R_1→	0.25			0.75			1.25		
ϴ$_r$ ↓	**f'**	**g'**	**ϴ'**	**f'**	**g'**	**ϴ'**	**f'**	**g'**	**ϴ'**
-10	0.040589	0.015699	-1.540257931	0.040663	0.022699	-1.540257573	0.040698	0.030917	-1.540257692
-9	0.0405	0.015669	-1.555813432	0.040574	0.022654	-1.55581367	0.040609	0.030853	-1.555813551
-8	0.04039	0.015633	-1.575258493	0.040464	0.022598	-1.575258613	0.040499	0.030774	-1.575258493
-7	0.04025	0.015586	-1.600259543	0.040324	0.022527	-1.600259423	0.040359	0.030674	-1.600259423
-6	0.040066	0.015525	-1.633593798	0.040141	0.022433	-1.633593798	0.040176	0.030543	-1.633593798
-5	0.039814	0.015441	-1.680262566	0.03989	0.022305	-1.680262685	0.039925	0.030362	-1.680262685
-4	0.039449	0.015319	-1.750266433	0.039525	0.022117	-1.750266433	0.03956	0.030099	-1.750266433
-3	0.038868	0.015123	-1.86694324	0.038945	0.021819	-1.866943359	0.038981	0.029681	-1.86694324
-2	0.037799	0.014758	-2.100315809	0.037878	0.021266	-2.100315809	0.037915	0.028909	-2.100315809

Table 6:
Estimated missing values of f'(0), g'(0) and ϴ'(0) for various Θ_r and Θ_e=2.00, μ_1=0.10, R_2=0.25, Gr=0.10, M=0.25, Pr=0.70, S=1, E=0.05, n=0.5

R_1→	0.25			0.75			1.25		
ϴ$_r$ ↓	**f'**	**g'**	**ϴ'**	**f'**	**g'**	**ϴ'**	**f'**	**g'**	**ϴ'**
2	0.047316	0.017809	-0.700257659	0.047385	0.026008	-0.700257659	0.04742	0.035636	-0.700257659
3	0.044921	0.017085	-0.933589935	0.044991	0.024855	-0.933589816	0.045025	0.033978	-0.933589876
4	0.043922	0.016774	-1.050256014	0.043992	0.024366	-1.050256014	0.044026	0.033278	-1.050256014
5	0.043369	0.016599	-1.12025559	0.04344	0.024093	-1.12025559	0.043474	0.03289	-1.12025559
6	0.043018	0.016488	-1.166922212	0.043089	0.023919	-1.166922092	0.043123	0.032642	-1.166922092
7	0.042774	0.01641	-1.200255275	0.042846	0.023798	-1.200255394	0.04288	0.03247	-1.200255513
8	0.042596	0.016353	-1.22525537	0.042667	0.023709	-1.225255251	0.042701	0.032344	-1.22525537
9	0.042459	0.016309	-1.244699836	0.042531	0.023641	-1.244699597	0.042565	0.032247	-1.244699717
10	0.042351	0.016274	-1.260255337	0.042423	0.023587	-1.260255337	0.042457	0.032171	-1.260255218

Table 7:

Estimated missing values of f'(0), g'(0) and ϴ'(0) for various Θ_e and Θ_r=3.00, μ_1=0.10, R_1=0.25, Gr=0.10, M=0.25, Pr=0.70, S=1, E=0.05, n=0.5

R_2→	0.25			0.75			1.25		
Θ_e ↓	f'	g'	ϴ'	f'	g'	ϴ'	f'	g'	ϴ'
-10	0.068323	0.026571	-0.933589518	0.068711	0.038587	-0.933589399	0.06886	0.045283	-0.933589459
-9	0.068746	0.026743	-0.933589459	0.069141	0.038853	-0.93358928	0.069293	0.045605	-0.93358928
-8	0.069275	0.026958	-0.933589518	0.069677	0.039184	-0.933589339	0.069833	0.046009	-0.93358928
-7	0.069953	0.027234	-0.933589518	0.070366	0.03961	-0.93358916	0.070526	0.046527	-0.93358916
-6	0.070857	0.027601	-0.933589399	0.071283	0.040178	-0.93358928	0.071449	0.047217	-0.93358922
-5	0.072118	0.028115	-0.933589339	0.072565	0.040971	-0.93358922	0.07274	0.048182	-0.93358922
-4	0.074004	0.028882	-0.93358916	0.074481	0.042158	-0.933589101	0.074669	0.049628	-0.933589101
-3	0.077129	0.030156	-0.933589101	0.077659	0.04413	-0.933588862	0.07787	0.052032	-0.933588803
-2	0.083311	0.032677	-0.933588624	0.083953	0.048045	-0.933588386	0.084215	0.056814	-0.933588326

Table 8:

Estimated missing values of f'(0), g'(0) and ϴ'(0) for various Θ_r and Θ_e=2.00, μ_1=0.10, R_1=0.25, Gr=0.10, M=0.25, Pr=0.70, S=1, E=0.05, n=0.5

R_2→	0.25			0.75			1.25		
Θ_r ↓	f'	g'	ϴ'	f'	g'	ϴ'	f'	g'	ϴ'
-10	0.040589	0.015699	-1.540257931	0.040715	0.022579	-1.540257692	0.040752	0.026338	-1.540257573
-9	0.0405	0.015669	-1.555813432	0.040626	0.022546	-1.555813551	0.040663	0.026306	-1.555813551
-8	0.04039	0.015633	-1.575258493	0.040516	0.022504	-1.575258493	0.040553	0.026266	-1.575258374
-7	0.04025	0.015586	-1.600259543	0.040376	0.022451	-1.600259304	0.040413	0.026214	-1.600259185
-6	0.040066	0.015525	-1.633593798	0.040192	0.022381	-1.633594036	0.04023	0.026146	-1.633594036
-5	0.039814	0.015441	-1.680262566	0.039941	0.022284	-1.680262327	0.039979	0.026052	-1.680262208
-4	0.039449	0.015319	-1.750266433	0.039576	0.022143	-1.75026679	0.039614	0.025913	-1.750266194
-3	0.038868	0.015123	-1.86694324	0.038996	0.021914	-1.86694324	0.039034	0.025686	-1.866943359
-2	0.037799	0.014758	-2.100315809	0.03793	0.021481	-2.100315571	0.037968	0.025252	-2.100315571

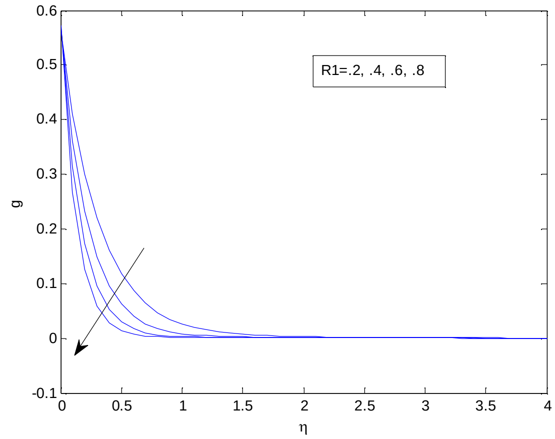

Fig. 1: Angular velocity profile for different R_1

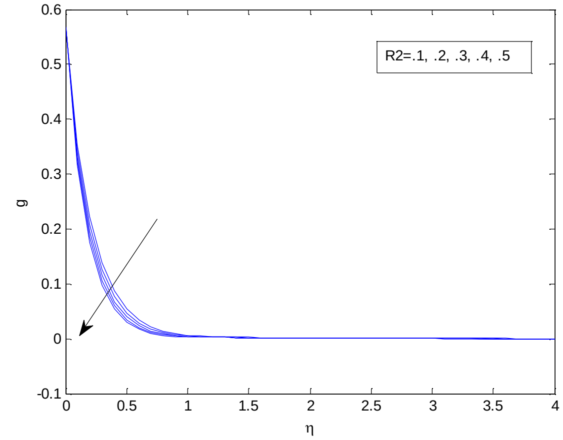

Fig. 2: Angular velocity profile for different R_2

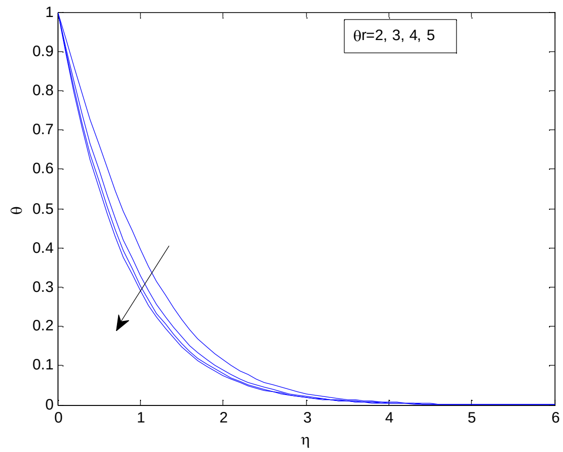

Fig. 3: Temperature profile for different Θ_r

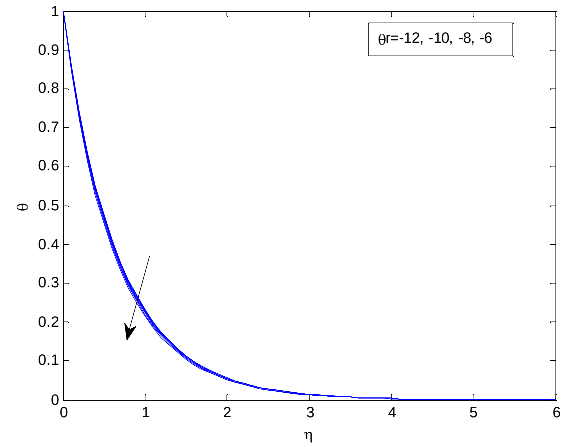

Fig. 4: Temperature profile for different Θ_r

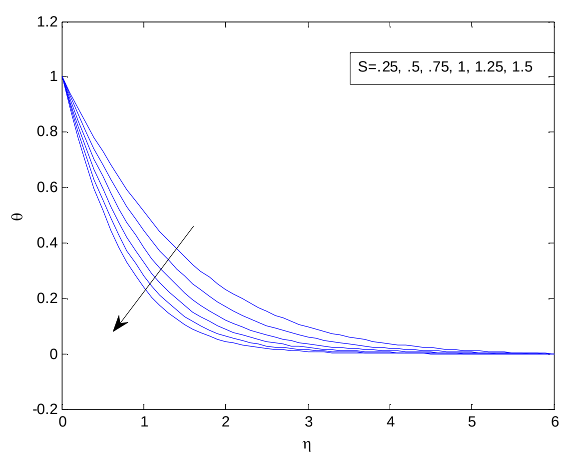

Fig. 5: Temperature profile for different S

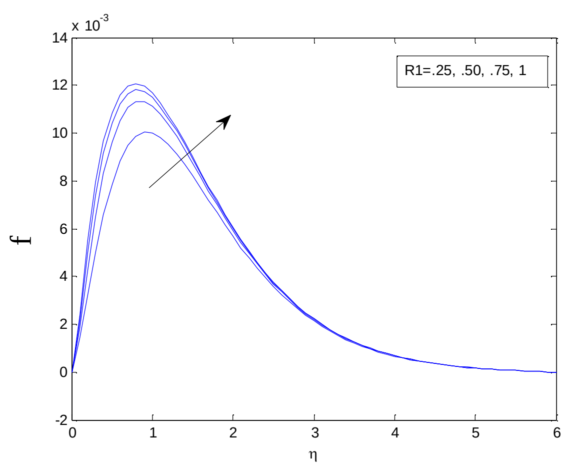

Fig. 6: Velocity profile for different R_1

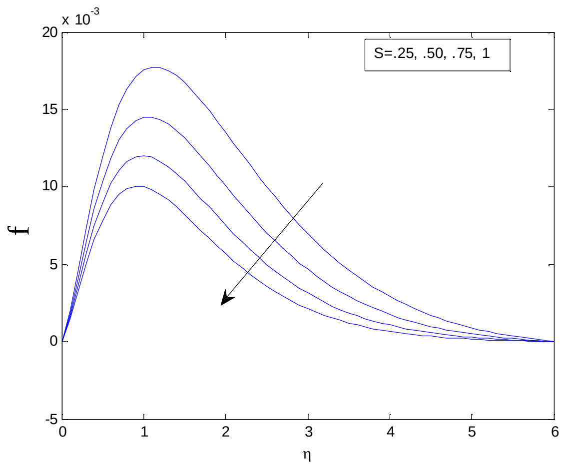

Fig. 7: Velocity profile for different S

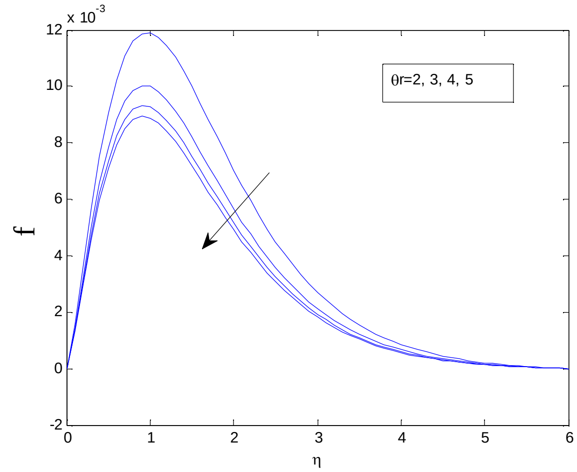

Fig. 8: Velocity profile for different Θ_r

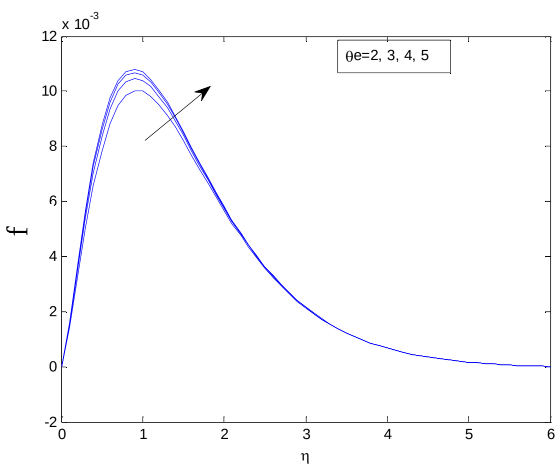

Fig. 9: Velocity profile for different Θ_e

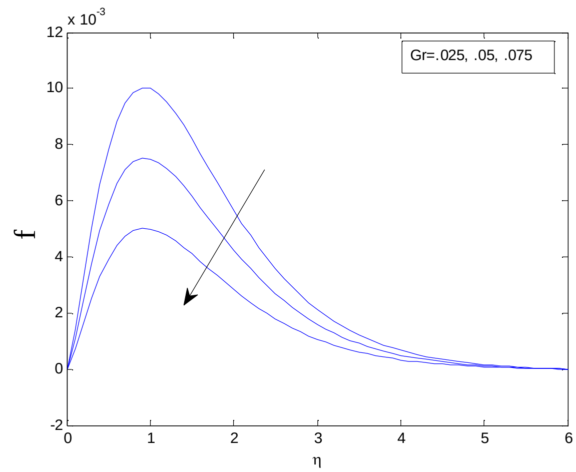

Fig. 10: Velocity profile for different G_r

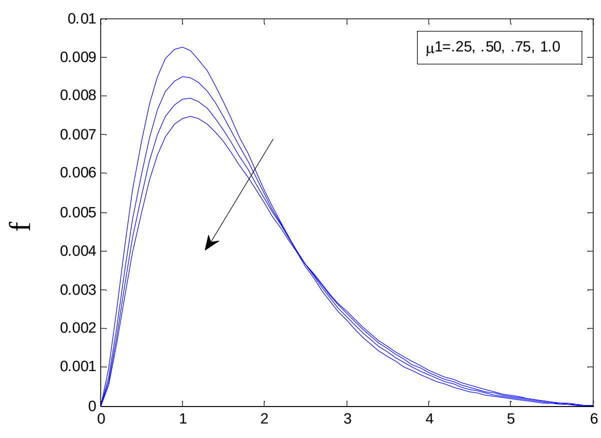

Fig. 11: Velocity profile for different μ_1

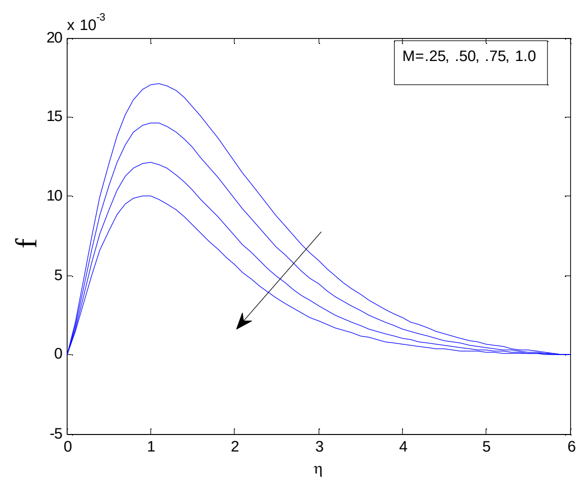

Fig. 12: Velocity profile for different M

V. CONCLUSIONS

The analysis presented above has shown that the micro-polar fluid flow is influenced by the variation of the thermal conductivity parameter and the viscosity parameters. Therefore we can conclude that to predict more accurate results ,the variable viscosity and thermal conductivity effects have to be taken into consideration on heat and mass transfer due to non-Newtonian viscous fluid flow on free convection through porous medium with constant heat flux to avoid the spoilt of energy. The results discussed above can be applied to engineering and industrial problems to save energy and economical benefit.

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