Ideal Theory in Triple Systems

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Abstract - The aim of this research article is to introduce the different notions of Ideal Theory in Triple Systems and investigate some results.

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Introduction :

Through this section T will denote an associative triple system. Let z denote the ring of all integers. The main results in this section are theorem 2 and theorem 3**.**

Theorem 1 Let $a \in T$. Then

(1) The right ideal of T generated by a is

$$(a)_r = z_a + < a, T, T >$$

(2) The middle ideal of T generated by a is

$$(a)_m = z_a + < T, a, T > + < T, T, < a, T, T >>$$

(3) The left ideal of T generated by a is

$$(a)_1 = z_a + < T, T, a >$$

(4) The ideal generated by a is

$$(a) = z_a + < T, T, a > + < T, a, T > + < a, T, T > + < T, T, < a, T, T >>$$

Where $z_a = \{na : n \in z\}$

Proof We prove only (4). The proof for (1), (2) and (3) are similar.

We first observe that $T^3 \subseteq T$,

$$< T, na, T > \subseteq < T, a, T >$$

$$< na, T, T > \subseteq < a, T, T >$$

$< T, T, na > \subseteq < T, T, a >$, n being integer.

Let J be the ideal of T generated by a. Set

$$K = z_a + < T, T, a > + < T, a, T > + < a, T, T > + < T, T, < a, T, T >>$$

Then

$$< K, T, T > = < z_a, T, T > + << T, T, a >, T, T > + << T, a, T >, T, T> + << a, T, T >, T, T >$$
$$+ < < T, T, < a, T, T > >, T, T >$$
$$\subseteq < a, T, T > + < T, T, < a, T, T > > + < T, A, T^3 > + < A, T^3 , T > + < < T, T, a > T^3, T >$$
$$\subseteq < a, T, T > + < T, T, < a, T, T > > + < T, a, T > + < a, T, T > + < < T, T, a >, T, T > >,$$
Since $T^3 \subseteq T$.
$$\subseteq < a, T, T > + < T, T, < a, T, T > > + < T, a, T > + < T, T, < a, T, T >>$$
$$\subseteq < a, T, T > + < T, T, < a, T, T > > + < T, a, T >$$
$$\subseteq z_a + < T, T, a > + < T, a, T > + < a, T, T > + < T, T, < a, T, T >>$$
$$= K$$

Thus $< K, T, T > \subseteq K$ and so K is a right ideal of T. In a similar manner it can be shown that k is middle as well as a left ideal of T. Thus K is an ideal of T containing a. But J is the ideal of T generate by a.

Therefore $J \subseteq K$. …………(1)

Again $a \in J$ implies that $Z_a \subseteq J$. Since J is an ideal of T, we have

$$< T, T, a > \subseteq < T, T, J > \subseteq J$$
$$< T, a, T > \subseteq < T, J, T > \subseteq J$$
$$< a, T, T > \subseteq < J, T, T > \subseteq J$$

and $< T, T, < a, T, T > > \subseteq < T, T, J > \subseteq J.$

Thus $K \subseteq J$……………….(2)

Therefore from (1) and (2) it follows that J = K. This completes the proof of (4).

Remark.1 Let ideal of T generated by a subspace S is given by,

$S + < T, T, S > + < T, S, T > + < S, T, T > + < T, T, < S, T, T >>$.

Theorem 2 Let B be a non – zero ideal of T. Then

$B_1 = \{x \in T: < x, T, B > = < B, T, x > = < B, x, t > = < 0 > \}$ is an ideal of T.

Proof Clearly B_1 is a subspace of T. Let $x \in B_1$ and $t_1, t_2 \in T$. Then

$$< x, T, B > = < T, x, B > = < B, T, x > = < B, x, T > = < 0 > \text{………..(1)}$$

We first show that B_1 is a right ideal of T.

Now $<< x, t_1, t_2 >, T, B > = < x, < t_1, t_2, T >, B >$
$$\subseteq < x, T, B >$$
$$= (0)$$

$< T, <x, t_1, t_2 >, B > = < T, x, < t_1, t_2, B >>$

$\subseteq < T, x, B >$

$= (0)$

$< B, T, < x, t_1, t_2 >> = < < B, T, x >, t_1, t_2 >$

$= << 0 >, t_1, t_2 >$

$= (0)$

And

$< B, < x, t_1, t_2 >. T > = < B, x, < t_1, t_2, T >>$

$\subseteq < B, x, T >$

$= (0)$

Thus $<x, t_1, t_2 > \in B_1$ and so $< B_1, T, T > \subseteq B_1$.

Hence B_1 is a right ideal of T.

We now show that B_1 is a left ideal of T.

$<< t_1, t_2, x >, T, B > = < t_1, t_2, < x, T, B >>$

$= < t_1, t_2, <0>>$

$= (0)$

$< T, < t_1, t_2, x >, B > = << T, t_1, t_2, >, x, B >$

$\subseteq < T, x, B >$

$= (0)$

$< B, T, < t_1, t_2, x >> = << B, T, t_1 >, t_2, x >$

$\subseteq < B, t_2, x >$ since B is an ideal.

$\subseteq < B, T, x >$

$= (0)$

And

$< B, < t_1, t_2, x >, T > = << B, t_1, t_2 >, x, T >$

$\subseteq < B, x, T >$ since B is an ideal.

$= (0)$

Hence $< t_1, t_2, x > \in B_1$ and so $< T, T, B_1 > \subseteq B_1$.

Thus B_1 is a left ideal of T.

Finally we show that B_1 is a middle ideal of T.

$<< t_1, x, t_2 >, T, B > = < t_1, x, < t_2, T, B >>$

$\subseteq < t_1, x, B >$ since B is an ideal.

$\subseteq < T, x, B >$

$= (0)$

$< T, <t_1, x, t_2 >, B > = < T, t_1, < x, t_2, B >>$

$= < T, t_1, < x, T, B >>$

$= < T, t_1, <0>>$

$= (0)$

$< B, T, < t_1, x, t_2 >>= << B, T, t_1>, x, t_2 >$

$= < B, x, t_2 >$ since B is an ideal.

$\subseteq < B, x, T >$

$= (0)$

And

$< B, < t_1, x, t_2 >, T >= << B, t_1, x >, t_2, T >$

$\subseteq \quad << B, T, x >, t_2, T >$

$= <<0>, t_2, T >$

$= (0)$

Thus $< t_1, x, t_2 > \in B_1$ and so $< T, B_1, T > \subseteq B_1$ implying that B_1 is a middle ideal of T. Hence B_1 is an ideal of T.

Theorem 3 Let A be an ideal of T and B , an ideal of A. If B^* is the ideal of T generated by B then $B^{*3} \subseteq$ B.

Proof We know, by remark.1,

$B^{*3} = B + < T, T, B > + < T, B, T > + < B, T, T > + < T, T, < B, T, T >>$.

Since A is an ideal of T and $B \subseteq A$ it follows that $B^* \subseteq A$.

Therefore,

$$B^{*3} = << B^*, B^*, B^* >, B^*, B^* >$$
$$\subseteq << A, A, B + < T, T, B > + < T, B, T > + < B, T, T >$$
$$+ < T, T, < B, T, T >>>, A, A >$$
$$= << A, A, B > + < A, A, < T, T, B >> + < A, A, < T, B, T >>$$
$$+ < A, A, < B, T, T >> + <A, A, < T, T, < B, T, T >>>, A, A >$$
$$= << A, A, B >, A, A > + << A, A, < T, T, B >>, A, A >$$
$$+ << A, A, < T, B, T >>, A, A > + << A, A, < B, T, T >>, A, A >$$

$+ \ll A, A, < T, T, < B, T, T >>>, A, A >$

$= \ll A, A, B >, A, A > + < A, <A, T, T >, < B, A, A >>$

$+ \ll A, A, T >, B, < T, A, A >> + \ll A, A, B >, < T, T, A>, A >$

$\subseteq < B A, A > + < B, A, B > + < A, B, A > + < B, A, A >$

$+ \ll A, A, B >A, A >$ (since B is an ideal A and A is an ideal of T)

$\subseteq B + B + B + B + B$

$\subseteq B$

Thus $B^{*3} \subseteq B$.

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