International Journal of Mathematics Trends and Technology- Volume 15 Number 2 Nov 2014

Root Square Mean Labeling of Some New Disconnected Graphs

S.S.Sandhya[1] , S.Somasundaram[2] , S.Anusa[3*]

1.Department of Mathematics,Sree Ayyappa College for women,Chunkankadai:629003,TamilNadu, India.

2.Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli:627012,TamilNadu, India

3.Department of Mathematics, Arunachala College of Engineering for Women,Vellichanthai-629203,TamilNadu, India

Abstract

A graph $G = (V, E)$ with p vertices and q edges is called a Root Square Mean graph if it is possible to label the vertices $x \epsilon V$ with distinct labels $f(x)$ from $1,2,\dots,q+1$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then the edge labels are distinct. In this case f is called Root Square mean labeling of G. In this paper we prove that some disconnected graphs are Root Square Mean graphs.

Key Words: Graph, Root Square Mean labeling, Path, Cycle, Comb, Triangular Snake, Quadrilateral Snake.

1. INTRODUCTION

The graph considered here are simple, finite and undirected graphs. Let $V(G)$ denote the vertex set and $E(G)$ denote the edge set of G. For detailed survey of graph labeling we refer to Gallian[1]. For all other standard terminology and notations we follow Harary [2] . S.S.Sandhya , S.Somasundaram and S.Anusa introduced the concept of Root Square Mean labeling of graphs in [4] and studied their behavior in [5] , [6], [7]. Also we proved that some disconnected graphs are also Root Square Mean graphs in [8] and [9]. In this paper we prove that some new disconnected graphs are Root Square Mean graphs. The following definitions and theorems are necessary for our future study.

Definition1.1:A graph $G = (V, E)$ with p vertices and q edges is called a Root Square Mean graph if it is possible to label the vertices $x \epsilon V$ with distinct labels $f(x)$ from $1,2,\dots,q+1$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then the edge labels are distinct. In this case f is called Root Square mean labeling of G.

Definition1.2: The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1 \cup G_2$ with vertex set $V = V_1 \cup V_2$ and the edge set $E = E_1 \cup E_2$.

Definition1.3: The Corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the ith vertex of G_1 is adjacent to every vertex in the ith copy of G_2.

Definition1.4: A Triangular Snake T_n , is obtained from a path $u_1 u_2 \cdots u_n$ by joining u_i and u_{i+1} to a new vertex v_i , $1 \leq i \leq n-1$.

Definition1.5: A Double Triangular Snake $D(T_n)$ consists of two Triangular Snakes that have a common path.

Definition1.6: A Quadrilateral Snake Q_n is obtained from a path $u_1 u_2 \cdots u_n$ by joining u_i and u_{i+1} to two new vertices v_i and w_i respectively and then joining v_i and w_i , $1 \leq i \leq n-1$.

Definition1.7: A Double Quadrilateral Snake $D(Q_n)$ consists of two Quadrilateral Snakes that have a common path.

Theorem1.8: Triangular Snake T_n is a Root Square Mean graph.

Theorem1.9: Double Triangular Snake $D(T_n)$ is a Root Square Mean graph.

Theorem1.10: Quadrilateral Snake Q_n is a Root Square Mean graph.

Theorem1.11: Double Quadrilateral Snake $D(Q_n)$ is a Root Square Mean graph.

2. MAIN RESULTS

Theorem2.1: $C_m \cup T_n$ is a Root Square Mean graph.

Proof: Let $u_1 u_2 \cdots u_m u_1$ be the cycle C_m .Let $v_1 v_2 \cdots v_n$ be the path P_n. Let T_n be the triangular snake obtained from the path P_n by joining v_i and v_{i+1} to new vertex w_i ,$1 \leq i \leq n-1$. Let $G = C_m \cup T_n$.

Define a function $f: V(G) \to \{1,2,\ldots, q+1\}$ by

$f(u_i) = i$,$1 \leq i \leq m$

$f(v_i) = m + 3i - 2$,$1 \leq i \leq n$

$f(w_i) = m + 3i - 1$,$1 \leq i \leq n-1$

Then the edge labels are distinct. Hence f is a Root Square Mean labeling of G.

Example2.2: Root Square Mean labeling of $C_7 \cup T_6$ is given below.

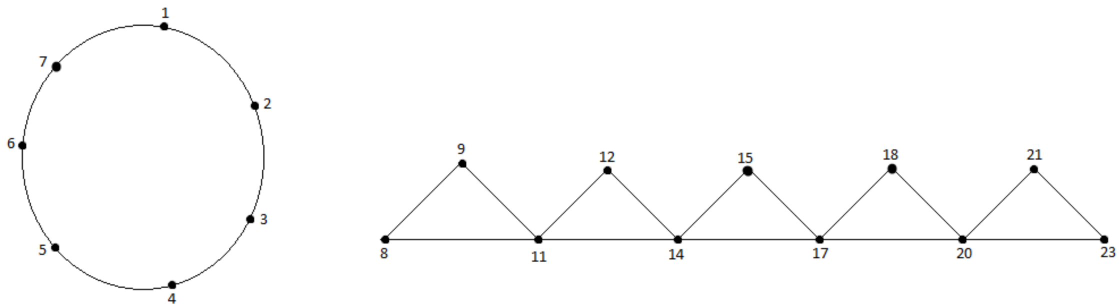

Figure1

Theorem2.3: $(C_m \odot K_1) \cup T_n$ is a Root Square Mean graph.

Proof: Let $u_1 u_2 \cdots u_m u_1$ be the cycle C_m . Let v_i be the vertex of K_1 which is attached to the vertex u_i , $1 \le i \le m$ of the cycle C_m . Let $w_1 w_2 \cdots w_n$ be the path P_n . Let T_n be the triangular snake obtained from P_n by joining w_i and w_{i+1} to a new vertex x_i , $1 \le i \le n-1$ .Let $G = (C_m \odot K_1) \cup T_n$.

Define a function $f: V(G) \to \{1,2, \dots, q+1\}$ by

$f(u_i) = 2i - 1$,$1 \le i \le m$

$f(v_i) = 2i$,$1 \le i \le m$

$f(w_i) = 2m + 3i - 2$,$1 \le i \le n$

$f(x_i) = 2m + 3i - 1$,$1 \le i \le n - 1$

Then the edge labels are distinct. Hence f is a Root Square Mean labeling of G.

Example2.4: Root Square Mean labeling of $(C_7 \odot K_1) \cup T_5$ is given below.

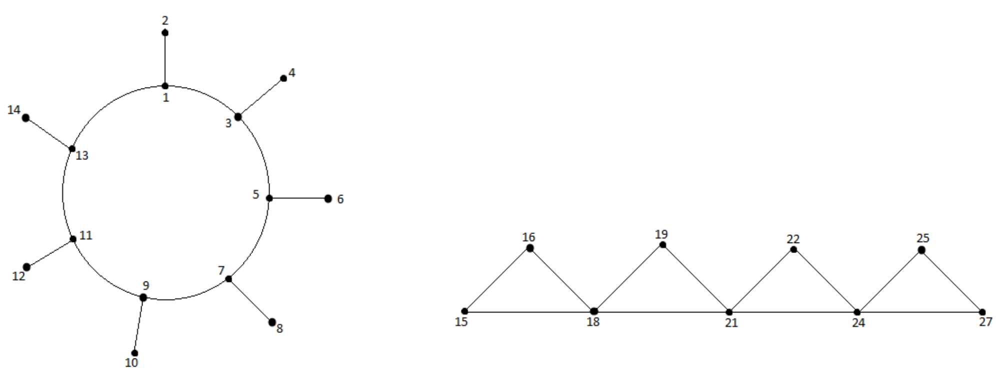

Figure2

Theorem2.5: $C_m \odot D(T_n)$ is a Root Square Mean graph.

Proof: Let $u_1 u_2 \cdots u_m u_1$ be the cycle C_m .Let $v_1 v_2 \cdots v_n$ be the path P_n. The double triangular snake $D(T_n)$ is obtained from the path P_n by joining v_i and v_{i+1} to two new vertices x_i and y_i , $1 \le i \le n-1$.

Let $G = C_m \odot D(T_n)$. Define a function $f: V(G) \to \{1,2,\dots,q+1\}$ by

$f(u_i) = i$,$1 \le i \le m$

$f(v_i) = m + 5i - 4$,$1 \le i \le n$

$f(x_i) = m + 5i - 3$,$1 \le i \le n-1$

$f(y_i) = m + 5i - 2$,$1 \le i \le n-1$

Then the edge labels are distinct. Hence f is a Root Square Mean labeling of G.

Example2.6: Root Square Mean labeling of $C_7 \odot D(T_5)$ is given below.

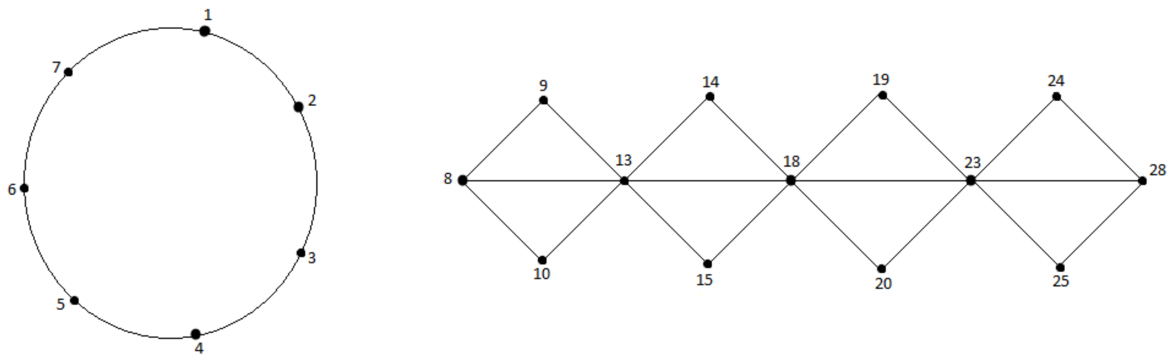

Figure3

Theorem2.7: $(C_m \odot K_1) \cup (D(T_n))$ is a Root Square Mean graph.

Proof: Let the cycle C_m be $u_1 u_2 \cdots u_m u_1$. Let v_i be the vertex of K_1 which is attached to the vertex u_i , $1 \le i \le m$ of the cycle C_m . Let $w_1 w_2 \cdots w_n$ be the path P_n . The double triangular snake $D(T_n)$ is obtained by joining w_i and w_{i+1} to two new vertices x_i and y_i , $1 \le i \le n-1$.

Let $G = (C_m \odot K_1) \cup (D(T_n))$. Define a function $f: V(G) \to \{1,2,\dots,q+1\}$ by

$f(u_i) = 2i - 1$,$1 \le i \le m$

$f(v_i) = 2i$,$1 \le i \le m$

$f(w_i) = 2m + 5i - 4$,$1 \le i \le n$

$f(x_i) = 2m + 5i - 3$,$1 \le i \le n-1$

$f(y_i) = 2m + 5i - 2$,$1 \le i \le n - 1$

Then the edge labels are distinct. Hence f is a Root Square Mean labeling of G.

Example2.8: Root Square Mean labeling of $(C_6 \odot K_1) \cup (D(T_5))$ is given below.

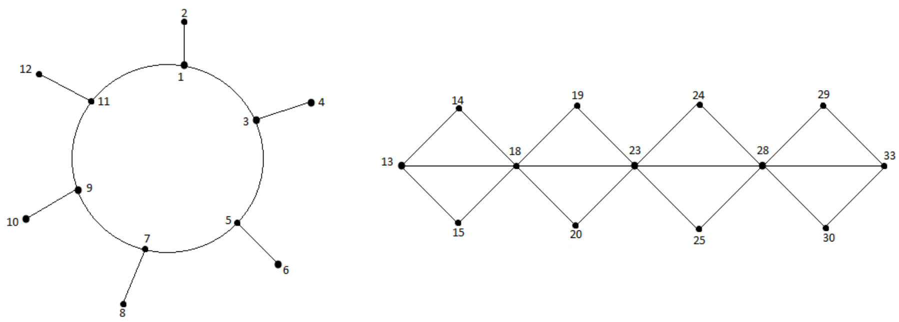

Figure4

Theorem2.9: $C_m \cup Q_n$ is a Root Square Mean graph.

Proof: Let $u_1 u_2 \cdots u_m u_1$ be the cycle C_m .Let $v_1 v_2 \cdots v_n$ be the path P_n. Let Q_n be the Quadrilateral snake obtained by joining v_i and v_{i+1} to two new vertices x_i and y_i ,$1 \le i \le n - 1$ respectively and then joining x_i and y_i . Let $G = C_m \cup Q_n$. Define a function $f: V(G) \to \{1,2, \dots, q + 1\}$ by

$f(u_i) = i$,$1 \le i \le m$

$f(v_i) = m + 4i - 3$,$1 \le i \le n$

$f(x_i) = m + 4i - 2$,$1 \le i \le n - 1$

$f(y_i) = m + 4i - 1$,$1 \le i \le n - 1$

Then the edge labels are distinct. Hence f is a Root Square Mean labeling of G.

Example2.10: The labeling pattern of $C_7 \cup Q_5$ is given below.

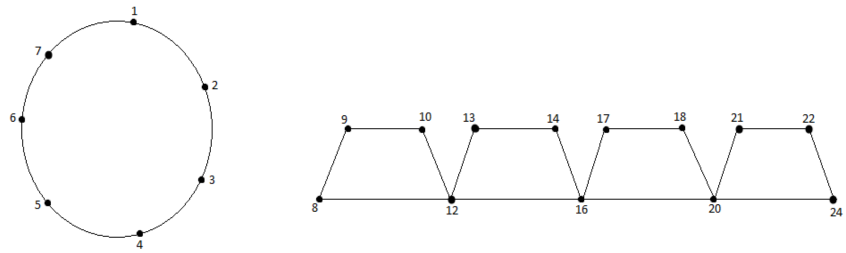

Figure5

Theorem2.11: $(C_m \odot K_1) \cup Q_n$ is a Root Square Mean graph.

Proof: Let $u_1u_2 \cdots u_m u_1$ be the cycle C_m . Let v_i be the vertex of K_1 which is attached to the vertex u_i , $1 \le i \le m$ of the cycle C_m . Let $w_1w_2 \cdots w_n$ be the path P_n . Let x_i and y_i , $1 \le i \le n-1$ be the vertices which are joined to w_i and w_{i+1} respectively. Join x_i and y_i .Let $G = (C_m \odot K_1) \cup Q_n$.

Define a function $f: V(G) \rightarrow \{1,2, \dots, q+1\}$ by

$f(u_i) = 2i-1$,$1 \le i \le m$

$f(v_i) = 2i$,$1 \le i \le m$

$f(w_i) = 2m + 4i - 3$,$1 \le i \le n$

$f(x_i) = 2m + 4i - 2$,$1 \le i \le n-1$

$f(y_i) = 2m + 4i - 1$,$1 \le i \le n-1$

Then the edge labels are distinct. Hence f is a Root Square Mean labeling of G.

Example2.12: The labeling pattern of $(C_7 \odot K_1) \cup Q_5$ is given below.

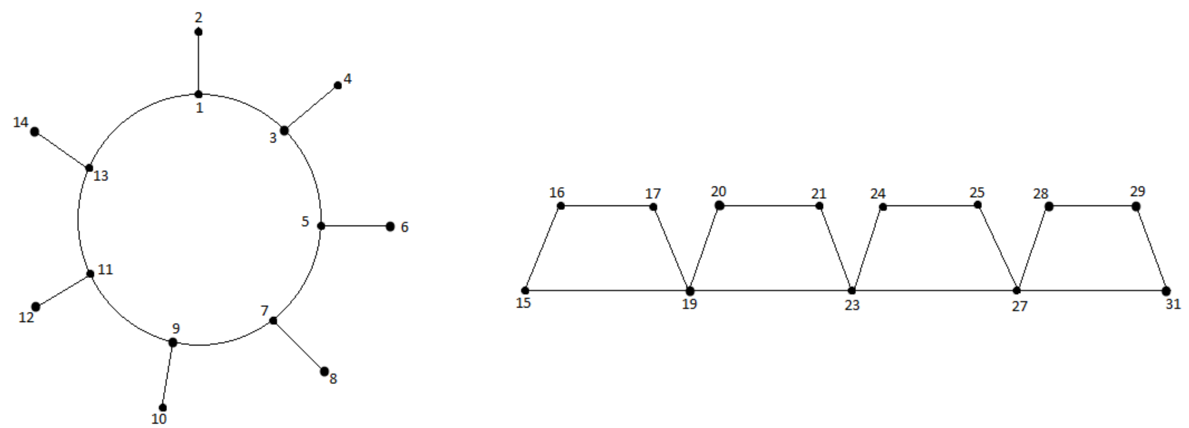

Figure6

Theorem2.13: $C_m \odot D(Q_n)$ is a Root Square Mean graph.

Proof: Let $u_1u_2 \cdots u_m u_1$ be the cycle C_m .Let v_i , x_i, y_i, x_i' , y_i' be the vertices of $D(Q_n)$.

Let $G = C_m \odot D(Q_n)$. Define a function $f: V(G) \rightarrow \{1,2, \dots, q+1\}$ by

$f(u_i) = i$,$1 \le i \le m$

$f(v_i) = m + 7i - 6$,$1 \le i \le n$

$f(x_i) = m + 7i - 5, 1 \leq i \leq n - 1$

$f(y_i) = m + 7i - 2, 1 \leq i \leq n - 1$

$f(x_i') = m + 7i - 4, 1 \leq i \leq n - 1$

$f(y_i') = m + 7i - 1, 1 \leq i \leq n - 1$

Then the edge labels are distinct. Hence f is a Root Square Mean labeling of G.

Example2.14: The labeling pattern of $C_7 \odot D(Q_5)$ is given below.

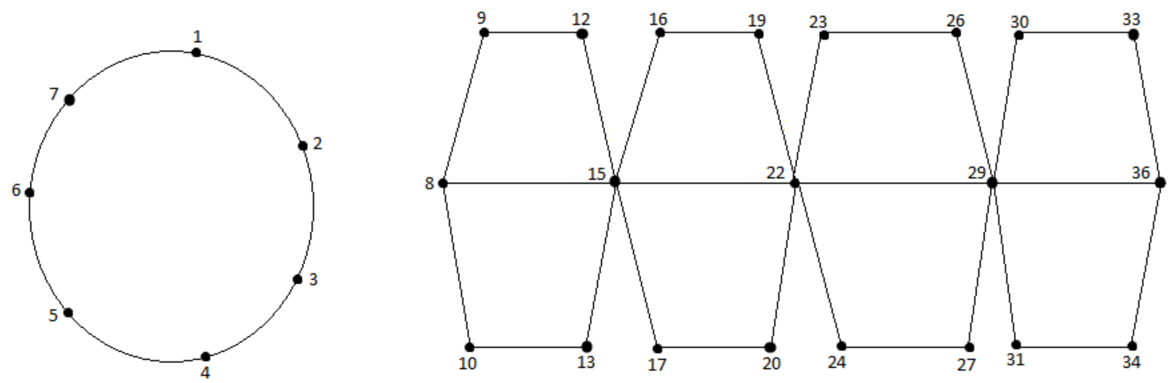

Figure7

Theorem2.15: $(C_m \odot K_1) \cup D(Q_n)$ is a Root Square Mean graph.

Proof: Let $u_1 u_2 \cdots u_m u_1$ be the cycle C_m . Let v_i be the vertex of K_1 which is attached to the vertex u_i , $1 \leq i \leq m$ of the cycle C_m . Let w_i , x_i, y_i, x_i' , y_i' be the vertices of $D(Q_n)$.

Let $G = (C_m \odot K_1) \cup D(Q_n)$ .Define a function $f: V(G) \rightarrow \{1, 2, \dots, q + 1\}$ by

$f(u_i) = 2i - 1, 1 \leq i \leq m$

$f(v_i) = 2i, 1 \leq i \leq m$

$f(w_i) = 2m + 7i - 6, 1 \leq i \leq n$

$f(x_i) = 2m + 7i - 5, 1 \leq i \leq n - 1$

$f(y_i) = 2m + 7i - 2, 1 \leq i \leq n - 1$

$f(x_i') = 2m + 7i - 4, 1 \leq i \leq n - 1$

$f(y_i') = 2m + 7i - 1, 1 \leq i \leq n - 1$

Then the edge labels are distinct. Hence f is a Root Square Mean labeling of G.

Example2.16: The labeling pattern of $(C_6 \odot K_1) \cup D(Q_5)$ is given below.

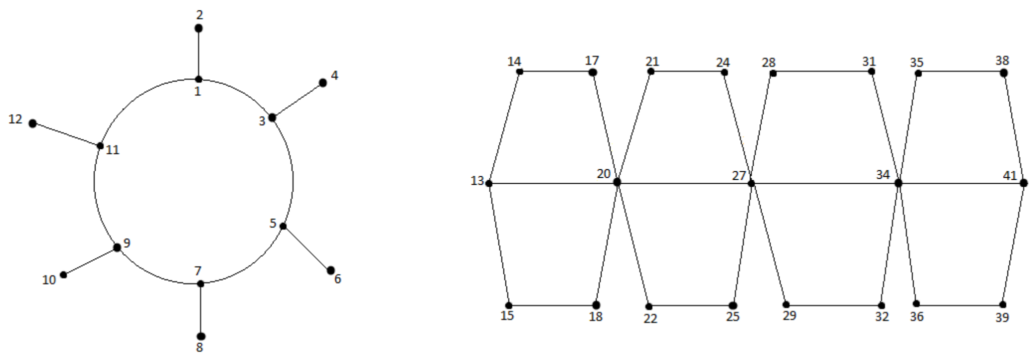

Figure8

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