Logarithm Approximation

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***Abstract*—In this paper, an approximation formula for logarithm is introduced. This formula gives close approximation to the actual logarithm value, provided the input is greater than one and the base is greater than or equal to two. Certain examples have been presented in this paper for various numbers and bases, while at the same time comparison with actual logarithm value is also made.**

***Keywords*—Logarithm Approximation, Logarithm, Representation of Logarithm Algebraically**

I. Introduction

The concept of Logarithm was introduced by John Napier. Logarithms are used to increase efficiency and decrease calculation load on a very large scale, logarithms are widely used in computer science, telecommunications, medicine etc. Various approximation methods of logarithm have been introduced in the past.The natural logarithm and logarithm approximations were given by Tylor, Padé. The approximations were applicable under certain definite conditions only. In this paper I will introduce an approximation formula which results in close approximation to the logarithm value for various bases.

II. Formula for Approximation

This formula was achieved by trial and error method. In this formula 'b' represents base,whereas 'c' is different for different numbers and is subject to change with base, however the value of 'c' for different bases can be approximated .where 's' is 1/(square of the base) and 'x' is input.

$$(\frac{x-1}{x})+(\frac{x}{x-1})-(\frac{x+1}{x})+[\frac{x-(b-2)}{x}]^2-(\frac{1}{b\sqrt{x}})+(\frac{1}{bx})+[\frac{1}{(x+b)(b+1)}]+(\frac{1}{x+b})+(\frac{x}{b^3})\mp(\frac{1}{pscx})$$

1/(pscx) is defined as 'S.R.S.S' which stands for 'section reserved for sum or subtraction'.'p' is either integral, fractional or mixed fractional number which is multiplied to 'scx' to produce final result .Final value is Formula value + S.R.S.S value.

A. Approximation for base 10

TABLE I
APPROXIMATION FOR BASE 10

$Log_{10}()$	**Formula value (approximate)**	**S.R.S.S approximate value**	**Final value(approximate till 4 decimal place)**	**Log value (approximate till 4 decimal place)**
89	1.907739134	+0.04165	1.9494	1.9494
89.1	1.908028667	+0.04185	1.9499	1.9499
89.2	1.908317798	+0.04205	1.9504	1.9504
89.3	1.908606526	+0.04224	1.9509	1.9509
89.4	1.908894853	+0.04243	1.9513	1.9513
89.5	1.90918278	+0.04265	1.9518	1.9518
89.6	1.909470309	+0.04284	1.9523	1.9523
89.7	1.909757441	+0.04303	1.9528	1.9528
89.8	1.910044177	+0.04323	1.9533	1.9533
89.9	1.910330518	+0.04343	1.9538	1.9538
69	1.839498537	-0.00065	1.8389	1.8389
52	1.754773516	-0.03877	1.7160	1.7160
44	1.698621936	-0.05517	1.6435	1.6435
31	1.562158282	-0.07080	1.4914	1.4914
20	1.351634536	-0.05061	1.3010	1.3010
12	1.076406132	+0.00278	1.0792	1.0792
7	0.949020934	-0.10392	0.8451	0.8451

For approximation till four-five decimal places

p= 1	s*c for 89 ≈ 0.2698	p=-1	s*cfor 69 ≈ 22.3
p= 1	s*c for 89.1≈ 0.2682	p=-1	s*cfor 52 ≈ 0.496
p= 1	s*c for 89.2 ≈ 0.2666	p=-1	s*cfor 44 ≈ 0.41196
p= 1	s*c for 89.3≈ 0.2651	p=-1	s*cfor 31 ≈0.45564
p= 1	s*c for 89.4 ≈ 0.2636	p=-1	s*cfor 20 ≈0.98805
p= 1	s*c for 89.5 ≈ 0.2620	p= 1	s*cfor 12 ≈30
p= 1	s*c for 89.6 ≈ 0.2605	p=-1	s*cfor 7 ≈1.37465
p= 1	s*c for 89.7 ≈ 0.2591		
p= 1	s*c for 89.8 ≈ 0.2576		
p= 1	s*c for 89.9 ≈ 0.2561		

B. Approximation for base 11

TABLE II
APPROXIMATION FOR BASE 11

Log_{11}()	**Formula value (approximate)**	**S.R.S.S approximate value**	**Final value(approximate till 4 decimal place)**	**Log value (approximate till 4 decimal place)**
89	1.865955974	+0.00595	1.8719	1.8719
89.1	1.866240916	+0.00614	1.8724	1.8724
89.2	1.866525414	+0.00631	1.8728	1.8728
89.3	1.866809469	+0.00650	1.8733	1.8733
89.4	1.867093083	+0.00668	1.8738	1.8738
89.5	1.867376257	+0.00686	1.8742	1.8742
89.6	1.867658993	+0.00705	1.8747	1.8747
89.7	1.867941292	+0.00723	1.8752	1.8752
89.8	1.868223155	+0.00742	1.8756	1.8756
89.9	1.868504583	+0.00760	1.8761	1.8761
69	1.797619785	-0.03186	1.7658	1.7658
52	1.710353646	-0.06256	1.6478	1.6478
44	1.651665078	-0.07354	1.5781	1.5781
31	1.508148450	-0.07607	1.4321	1.4321
20	1.289321675	-0.04000	1.2493	1.2493
12	1.024192215	+0.01209	1.0363	1.0363
7	1.006656029	-0.19515	0.8115	0.8115

Note- For any base below or above 10 we take value of s*c (of base 10) as ‘c’ and for base 10 ‘p’ is ±1 (Plus or minus one), for any other base it is either integral or fractional (including mixed fractions) multiple. The values of‘s*c’ is always positive, whereas ‘p’ can be either positive or negative.

For approximation till four-five decimal places

p≈847	s*c for 89 ≈ 0.00223	p ≈-2.4682	s*cfor 69 ≈ 0.1842975
p≈ 825	s*c for 89.1≈0.002217	p ≈-74.993	s*cfor 52 ≈ 0.0040992
p≈ 806	s*c for 89.2≈ 0.002203	p ≈-90.778	s*cfor 44 ≈ 0.0034046
p≈ 786	s*c for 89.3 ≈ 0.002191	p ≈ -112.62	s*cfor 31 ≈ 0.0037656
p≈ 768	s*c for 89.4 ≈ 0.002179	p ≈-153.063	s*cfor 20 ≈ 0.0081657
p≈ 752	s*c for 89.5 ≈ 0.0021653	p ≈ 27.791	s*c for 12 ≈ 0.2479339
p≈ 735	s*c for 89.6 ≈ 0.0021529	p ≈-64.436	s*c for 7 ≈ 0.01136074
p≈ 720	s*c for 89.7 ≈ 0.0021413		
p ≈ 705	s*c for 89.8 ≈ 0.0021289		
p≈691.7	s*c for 89.9 ≈ 0.0021165		

C. Approximation for base e

TABLE III
APPROXIMATION FOR BASE e

ln()	Formula value (approximate)	S.R.S.S approximate value	Final value(approximate till 4 decimal place)	Log value (approximate till 4 decimal place)
89	6.382838322	-1.89420	4.4886	4.4886
89.1	6.387849502	-1.89809	4.4898	4.4898
89.2	6.392860622	-1.90198	4.4909	4.4909
89.3	6.397871681	-1.90587	4.4920	4.4920
89.4	6.402882679	-1.90976	4.4931	4.4931
89.5	6.407893617	-1.91366	4.4942	4.4942
89.6	6.412904494	-1.91755	4.4954	4.4954
89.7	6.417915311	-1.92144	4.4965	4.4965
89.8	6.422926068	-1.92534	4.4976	4.4976
89.9	6.427936765	-1.92924	4.4987	4.4987
69	5.379054217	-1.14495	4.2341	4.2341
52	4.52188779	-0.57065	3.9512	3.9512
44	4.116112139	-0.33192	3.7842	3.7842
31	3.44984004	-0.01585	3.4340	3.4340
20	2.869823733	+0.12590	2.9957	2.9957
12	2.416230741	+0.06868	2.4849	2.4849
7	2.078849095	-0.13294	1.9459	1.9459

For approximation till four-five decimal places

p≈ -0.162454 s*cfor 89 ≈ 0.0365135 p≈ -0.004194 s*c for 69 ≈ 3.017977
p≈ -0.1629055 s*c for 89.1≈ 0.0362969 p≈ -0.50204 s*c for 52 ≈ 0.0671263
p≈ -0.163365 s*c for 89.2 ≈ 0.0360804p≈ -1.2281306 s*c for 44 ≈ 0.0557527
p≈ -0.16377 s*c for 89.3 ≈ 0.0358774p≈ -33.004728 s*c for 31 ≈ 0.061664
p≈ -0.164183 s*c for 89.4 ≈ 0.0356744 p≈ 2.97 s*c for 20 ≈ 0.13371803
p≈ -0.164665 s*c for 89.5 ≈ 0.0354578 p≈ 0.29885s*c for 12 ≈ 4.06006
p≈ -0.1650922 s*c for 89.6 ≈ 0.0352548 p≈-5.776258s*c for 7 ≈ 0.1860385
p≈ -0.1654635 s*c for 89.7 ≈ 0.0350654
p≈ -0.165905 s*c for 89.8 ≈ 0.0348624
p≈ -0.1663542 s*c for 89.9 ≈ 0.03465937

III. FOR GREATER ACCURACY

The approximation formula described belowcan produce accurate value up todecimal placesas desired. For example if it is desired to find the value of $\log_{10}$ (89) till 9 decimal places? The 'Formula value' of $\log_{10}(89) \approx 1.907739134$ + S.R.S.S value. The more accurate value of 'S*C' should be used in this case, for example S.R.S.S value of $\log_{10}(89) \approx 0.04165$, but in order to produce accurate result till 9 decimal places the accuracy of S.R.S.S value should be increased, thus S.R.S.S value of $\log_{10}(89) \approx 0.0416508732$ is to be used. The resulting value of $\log_{10}(89) \approx 1.949390007$ and actual logarithm value of $\log_{10}(89)$ till 9 decimal place is also 1.949390007.

$$(\frac{x-1}{x})+(\frac{x}{x-1})-(\frac{x+1}{x})+[\frac{x-(b-2)}{x}]^2-(\frac{1}{b\sqrt{x}})+(\frac{1}{bx})+[\frac{1}{(x+b)(b+1)}]+(\frac{1}{x+b})+(\frac{x}{b^3})\mp(\frac{1}{pscx})$$

IV. CONCLUSIONS

I have introduced a logarithm approximation formula which can produce approximate value of logarithm for various bases. Some of which were presented in this paper. The accuracy can be increased by using more accurate values of 'p' and's*c', which there by increases accuracy of S.R.S.S value, thus results in close approximation.

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