

FEW RESULTS on GEOMETRIC MEAN GRAPHS

Si.P.Viji, S2. Somasundaram, S3.S.Sandhya

1. Department of Mathematics
K.N.S.K. College of Engineering, Therekalputhooor, Tamilnadu, India - 629004
2. Department of Mathematics
M.S. University, Tirunelveli, Tamilnadu, India-627012
3. Department of Mathematics
SreeAyyappa College for Women, Chunkankadai, Tamilnadu, India-629003

ABSTRACT

A Graph $G = (V,E)$ with p vertices and q edges is said to be a Geometric mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1, 2, \dots, q+1$ in such a way that when each edge $e=uv$ is labeled with $f(e=uv) = \lfloor \sqrt{f(u)f(v)} \rfloor$ (or) $\lceil \sqrt{f(u)f(v)} \rceil$, then the resulting edge labels are all distinct. In this case, f is called Geometric mean labeling of G . In this paper we prove that, Triple Triangular snake, Alternate Triple Triangular snake and Subdivision on Triple Triangular and Alternate Triple Triangular snakes are Geometric mean graphs.

Keywords: Graph, Geometric mean graph, Triple Triangular snake, Alternate Triple Triangular snake, Subdivision on graphs.

I. INTRODUCTION

Throughout this paper we consider finite, undirected and simple graphs. Let $G=(V,E)$ be a graph with p vertices and q edges. For all terminologies and notations we follow Harary [1]. There are several types of labeling and detailed survey can be found in [2]. The concept of mean labeling has been introduced in [3] and the basic results was proved in [4] and [5]. The concept of Double Triangular snake and Double Quadrilateral snake has been proved in [6]. In this paper we prove that the Geometric mean labeling behaviour of Triple Triangular snake, Alternative Triple Triangular snake graphs. Also we wish to investigate Subdivision of Triple Triangular snake and Alternate Triple Triangular snake graphs are Geometric mean graphs. The following definitions are necessary for our present investigation.

Definition 1.1: A Graph $G=(V,E)$ with p vertices and q edges is said to be a Geometric mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1, 2, \dots, q+1$ in such way that when each edge $e=uv$ is labeled with $f(e=uv) = \lfloor \sqrt{f(u)f(v)} \rfloor$ (or) $\lceil \sqrt{f(u)f(v)} \rceil$, then the resulting edge labels are all distinct. In this case, f is called Geometric mean labeling of G .

Definition 1.2: A Triangular snake T_n is obtained from a path $u_1u_2 \dots u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n-1$. That is, every edge of a path is replaced by a triangle C_3 .

Definition 1.3: An Alternate Triangular snake $A(T_n)$ is obtained from a path $u_1u_2 \dots u_n$ by joining u_i and u_{i+1} (Alternatively) to new vertex v_i . That is, every alternate edge of a path is replaced by C_3 .

Definition 1.4: A Double Triangular snake $D(T_n)$ consists of two Triangular snakes that have a common path.

Definition 1.5: An Alternate Double Triangular snake $A[D(T_n)]$ consists of two Alternate Triangular snakes that have a common path.

Definition 1.6: A Triple Triangular snake $T(T_n)$ consists of three Triangular snakes that have a common path.

Definition 1.7: An Alternate Triple Triangular snake $A(T(T_n))$ consists of three Alternate Triangular snakes that have a common path.

II. MAIN RESULTS

Theorem 2.1: Triple Triangular snakes are Geometric mean graphs.

Proof: Let G be the graph obtained from a path $u_1u_2\dots u_n$ add new vertices v_i, w_i and t_i , where $1 \leq i \leq n-1$, such that v_i is joined with u_i and u_{i+1} , w_i is joined with u_i and u_{i+1} and t_i is joined with u_i and u_{i+1} . The resultant graph is T_n . The labeling pattern is shown in the following figure.

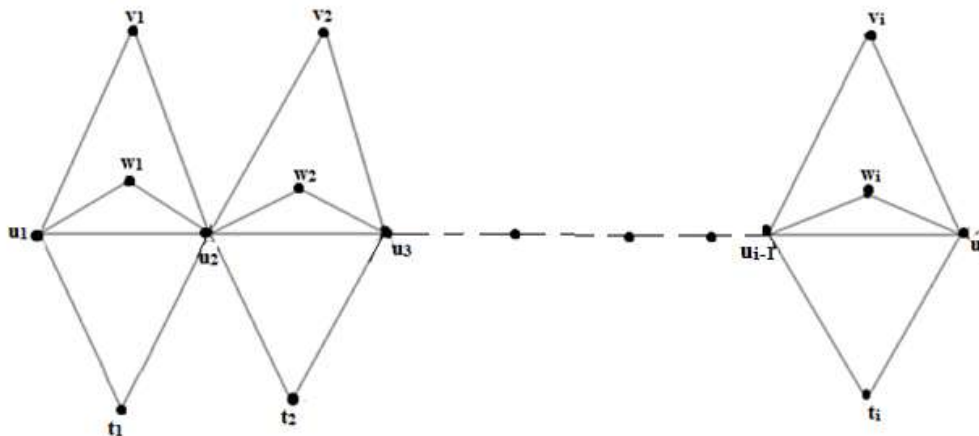


Figure: 1

Define a function $f: V(G) \rightarrow \{1, 2, \dots, q+1\}$ by

$$f(u_1) = 3$$

$$f(u_i) = 7i - 6, 1 \leq i \leq n$$

$$f(v_1) = 1$$

$$f(v_i) = 7i - 5, 1 \leq i \leq n-1$$

$$f(w_i) = 7i - 3, 1 \leq i \leq n-1$$

$$f(t_i) = 7i, 1 \leq i \leq n-1.$$

From the above labeling pattern, we get distinct edge labels.

Thus f provides a Geometric mean labeling for G .

Example 2.2: The labeling pattern of Triple Triangular snake obtained from six vertices is given below

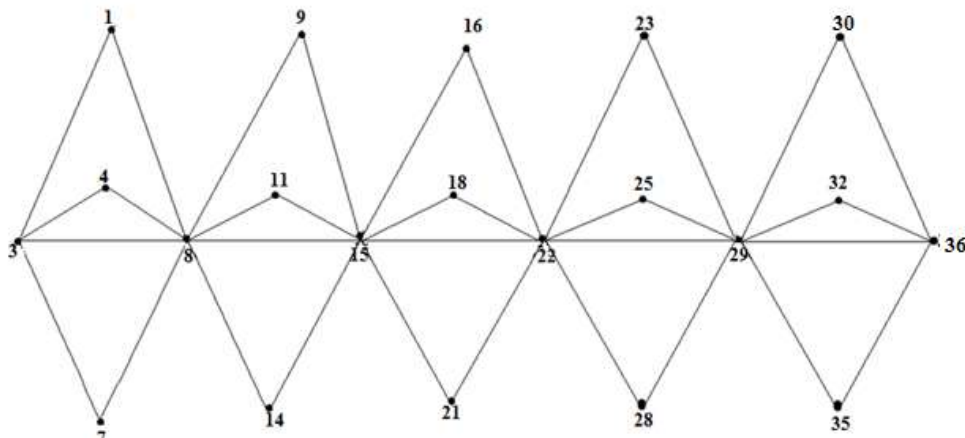


Figure: 2

Theorem 2.3: Alternate Triple Triangular snakes are Geometric mean graphs.

Proof: Let $G = A[T(T_n)]$ be the Alternate Triple Triangular snake graph and its vertices be v_i, w_i and t_i , where $1 \leq i \leq n-1$, such that v_i is joined with u_i and u_{i+1} , w_i is joined with u_i and u_{i+1} and t_i is joined with u_i and u_{i+1} (Alternatively). The resultant graph is $A(T_n)$ Here we consider two cases.

Case (i): If the triangle starts from u_1 then we need to consider two subcases.

Subcase (i) (a): If n is odd then.

Define a function $f: V(G) \rightarrow \{1, 2, \dots, q+1\}$ by

$$f(u_1) = 3$$

$$f(u_i) = 4i-3, \quad \forall i = 3, 5, 7, \dots, n$$

$$4i, \quad \forall i = 2, 4, 6, \dots, n-1.$$

$$f(v_1) = 1$$

$$f(v_i) = 8i-6, \quad \forall i = 2, 3, 4, \dots, \frac{n-1}{2}$$

$$f(w_i) = 8i-4, \quad \forall i = 1, 2, 3, \dots, \frac{n-1}{2}$$

$$f(t_i) = 8i-2, \quad \forall i = 1, 2, 3, \dots, \frac{n-1}{2}$$

From the above labeling pattern, we get distinct edge labels.

Thus f provide a Geometric mean labeling for G . Geometric mean labeling of G is obtained from seven vertices, which is given below.

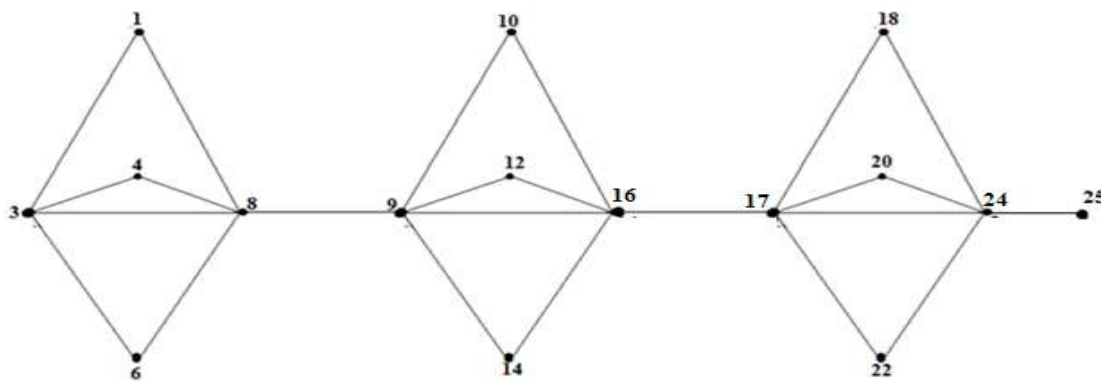


Figure: 3

Subcase(i)(b): If n is even then

Define a function $f: V(G) \rightarrow \{1, 2, \dots, q+1\}$ by

$$f(u_1) = 3$$

$$f(u_i) = 4i, \quad \forall i = 2, 4, 6, \dots, n$$

$$f(u_i) = 4i-3 \quad \forall i = 3, 5, 7, \dots, n-1$$

$$f(v_1) = 1$$

$$f(v_i) = 8i-6, \quad \forall i = 2, 3, 4, \dots, \frac{n}{2}$$

$$f(w_i) = 8i-4, \quad \forall i = 1, 2, 3, \dots, \frac{n}{2}$$

$$f(t_i) = 8i-2, \quad \forall i = 1, 2, 3, \dots, \frac{n}{2}$$

From the above labeling pattern, we get distinct edges labels.

Thus f provide a Geometric mean labeling for G and its labeling pattern is shown below.

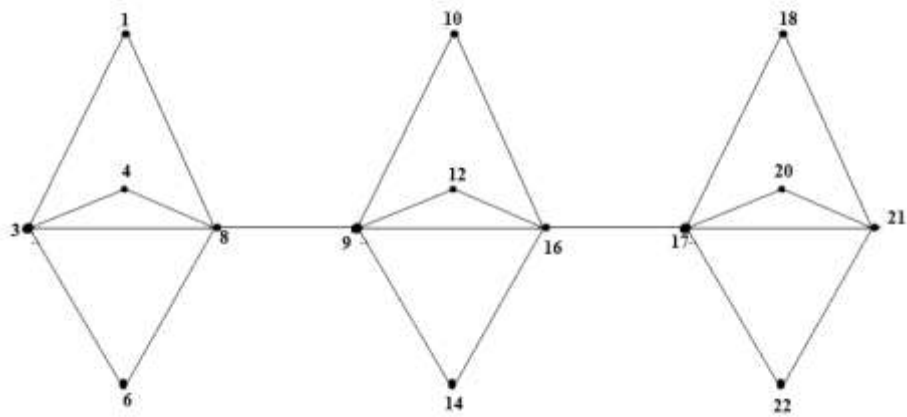


Figure : 4

Case (ii): If the triangular starts from u_2 then we need to consider two subcases

Subcase (ii) (a): If n is odd, then

Define a function $f: V(G) \rightarrow \{1, 2, \dots, q+1\}$ by

$$f(u_i) = 4i-3, \quad \forall i=1, 3, 5, \dots, n.$$

$$f(u_i) = 4i-6, \quad \forall i=2, 4, 6, \dots, n-1.$$

$$f(v_i) = 8i-4, \quad \forall i=1, 2, 3, \dots, \frac{n-1}{2}$$

$$f(w_i) = 8i-3, \quad \forall i=1, 2, 3, \dots, \frac{n-1}{2}.$$

$$f(t_i) = 8i-1, \quad \forall i=1, 2, 3, \dots, \frac{n-1}{2}.$$

From the above labeling pattern, we get distinct edge labels. Thus f is Geometric mean labeling of G and its labeling pattern is shown below.

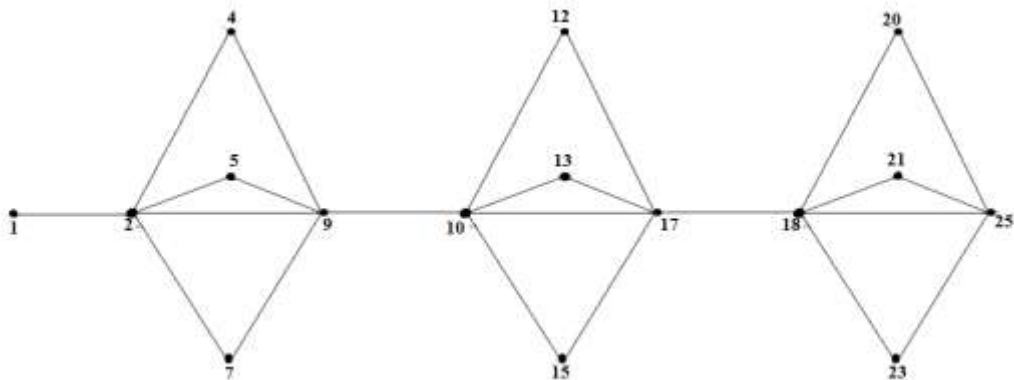


Figure: 5

Subcase(ii) (b): If n is even

Define a function $f: V(G) \rightarrow \{1, 2, \dots, q+1\}$ by

$$f(u_i) = 4i-3, \quad \forall i=1, 3, 5, \dots, n-1.$$

$$f(u_i) = 4i-6, \quad \forall i=2, 4, 6, \dots, n.$$

$$f(v_i) = 8i-4, \quad \forall i=1, 2, \dots, \frac{n-2}{2}$$

$$f(w_i) = 8i-3, \quad \forall i=1, 2, \dots, \frac{n-2}{2}$$

$$f(t_i) = 8i-1, \quad \forall i=1, 2, 3, \dots, \frac{n-2}{2}$$

Then we get distinct edge labels. Thus f provides a Geometric mean labeling of G and the labeling pattern of G is obtained from eight vertices is shown below.

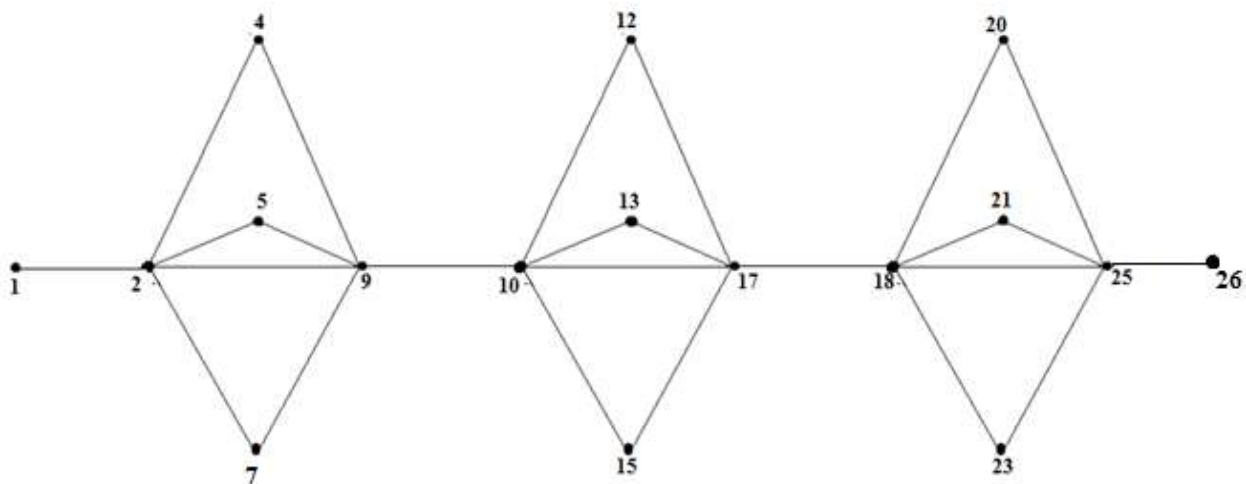


Figure: 6

From above all the cases, we conclude that Alternate Triple Triangular snakes are Geometric mean graphs.

Theorem 2.4: Subdivision of any Triple Quadrilateral snake is a Geometric mean graph.

Proof: Consider a path u_1, u_2, \dots, u_n and join u_i and u_{i+1} and its vertices be v_i, w_i , and z_i , where $1 \leq i \leq n-1$. Let, $S[T(T_n)] = T(T_n)$ be the graph is obtained by subdividing all the edges of $T(T_n)$. Let $x_i, y_i, r_i, s_i, n_i, m_i$ and t_i $1 \leq i \leq n-1$ be the new vertices of the edges $u_i v_i, v_i u_{i+1}, u_i w_i, w_i u_{i+1}, u_i z_i, u_{i+1} z_i$ and $u_i u_{i+1}$. The resultant graph is $S[T(T_n)]$ and its labeling pattern is shown below.

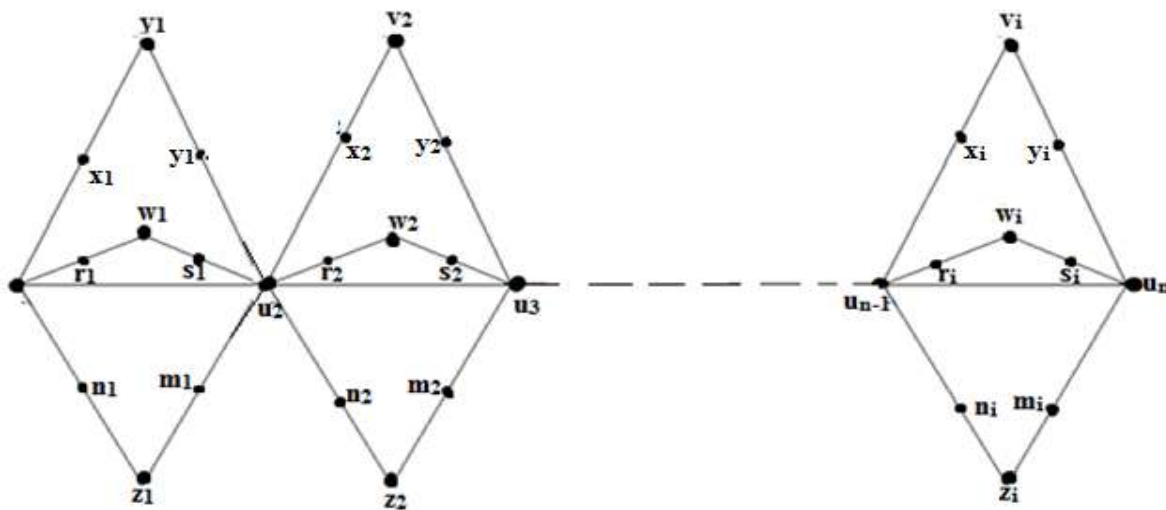


Figure: 7

Define a function $f: V[T(T_n)] \rightarrow \{1, 2, \dots, q+1\}$ by

$$\begin{aligned}
 f(u_1) &= 3, & f(u_i) &= 14i-13, 2 \leq i \leq n \\
 f(v_1) &= 2, & f(v_i) &= 14i-11, 2 \leq i \leq n-1. \\
 f(w_1) &= 10, & f(w_i) &= 14i-5, \quad 2 \leq i \leq n-1. \\
 f(z_1) &= 11, & f(z_i) &= 14i-6, \quad 2 \leq i \leq n-1. \\
 f(x_1) &= 1, & f(x_i) &= 14i-12, 2 \leq i \leq n-1. \\
 f(y_1) &= 5, & f(y_i) &= 14i-8, \quad 2 \leq i \leq n-1. \\
 f(r_1) &= 4, & f(r_i) &= 14i-9, \quad 2 \leq i \leq n-1.
 \end{aligned}$$

$$f(s_1)=12, \quad f(s_i) = 14i-4, \quad 2 \leq i \leq n-1.$$

$$f(n_1)= 8, \quad f(n_i) = 14i-7, \quad 2 \leq i \leq n-1.$$

$$f(m_i) = 14i, \quad 1 \leq i \leq n-1.$$

$$f(t_1) = 9, \quad f(t_i) = 14i-3, \quad 2 \leq i \leq n-1.$$

From the above labeling pattern, we get distinct edge labels

Hence $T(T_N)$ is a Geometric mean graph.

Example 2.5: The Geometric mean labeling of $T(T_N)$ of four vertices is given below.

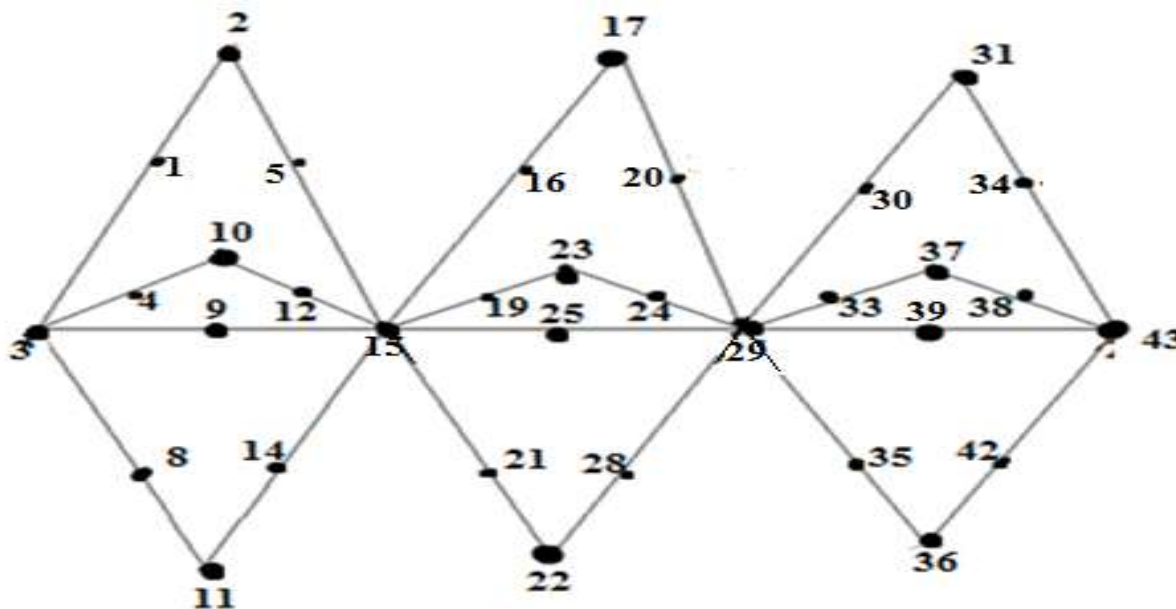


Figure: 8

Theorem 2.6: Subdivision of any Alternate Triple Triangular snake is a Geometric mean graph.

Proof: Let G be an Triple Alternative Triangular snake and its vertices be v_i, w_i and z_i where $1 \leq i \leq n-1$.

Let $S [A(T(T_n))] = A[T(T_N)]$ be a graph obtained by subdividing all the edges of $A[T(T_n)]$ and its new vertices be $x_i, y_i, r_i, s_i, n_i, m_i$ and t_i where $1 \leq i \leq n-1$. The resultant graph is $A[T(T_N)]$. Here we consider two cases.

Case(i) : If the Triangular starts from u_1 then we need to consider two subcases.

Subcase (i) (a): If n is odd, then

Define a function $f: V(T(T_N)) \rightarrow \{1, 2, \dots, q+1\}$ by

$$f(u_1) = 3$$

$$f(u_i) = 8i-1, \quad \forall i = 2, 4, 6, \dots, n-1.$$

$$f(u_i) = 8i-7, \quad \forall i = 3, 5, 7, \dots, n.$$

$$f(t_1) = 5$$

$$f(t_i) = 8i, \quad \forall i = 2, 4, 6, \dots, n-1.$$

$$f(t_i) = 8i-2, \quad \forall i = 3, 5, 7, \dots, n-1.$$

$$f(v_i) = 16i-9, \quad \forall i = 3, 5, 7, \dots, \frac{n-1}{2}$$

$$f(w_i) = 11, \quad f(w_i) = 16i-6 \quad \forall i = 2, 3, \dots, \frac{n-1}{2}$$

$$f(z_1) = 9, \quad f(z_i) = 16i-8, \quad \forall i = 2, 3, \dots, \frac{n-1}{2}$$

$$f(r_1) = 4, \quad f(r_i) = 16i-13, \quad \forall i = 2, 3, 4, \dots, \frac{n-1}{2}$$

$$f(s_1) = 12, \quad f(s_i) = 16i-2, \quad \forall i = 2, 3, 4, \dots, \frac{n-1}{2}$$

$$f(x_i) = 1, \quad f(x_i) = 16i-14, \quad \forall i=2,3,4,\dots,\frac{n-1}{2}$$

$$f(y_i) = 14, \quad f(y_i) = 16i-7, \quad \forall i=2,3,\dots,\frac{n-1}{2}$$

$$f(n_i) = 6, \quad f(n_i) = 16i-11, \quad \forall i=2,3,\dots,\frac{n-1}{2}$$

$$f(m_i) = 10, \quad f(m_i) = 16i-4, \quad \forall i=2,3,\dots,\frac{n-1}{2}$$

From the above labeling pattern we get distinct edge labels.

Thus f provides a Geometric mean labeling of G . Its labeling pattern is shown in the following figure.

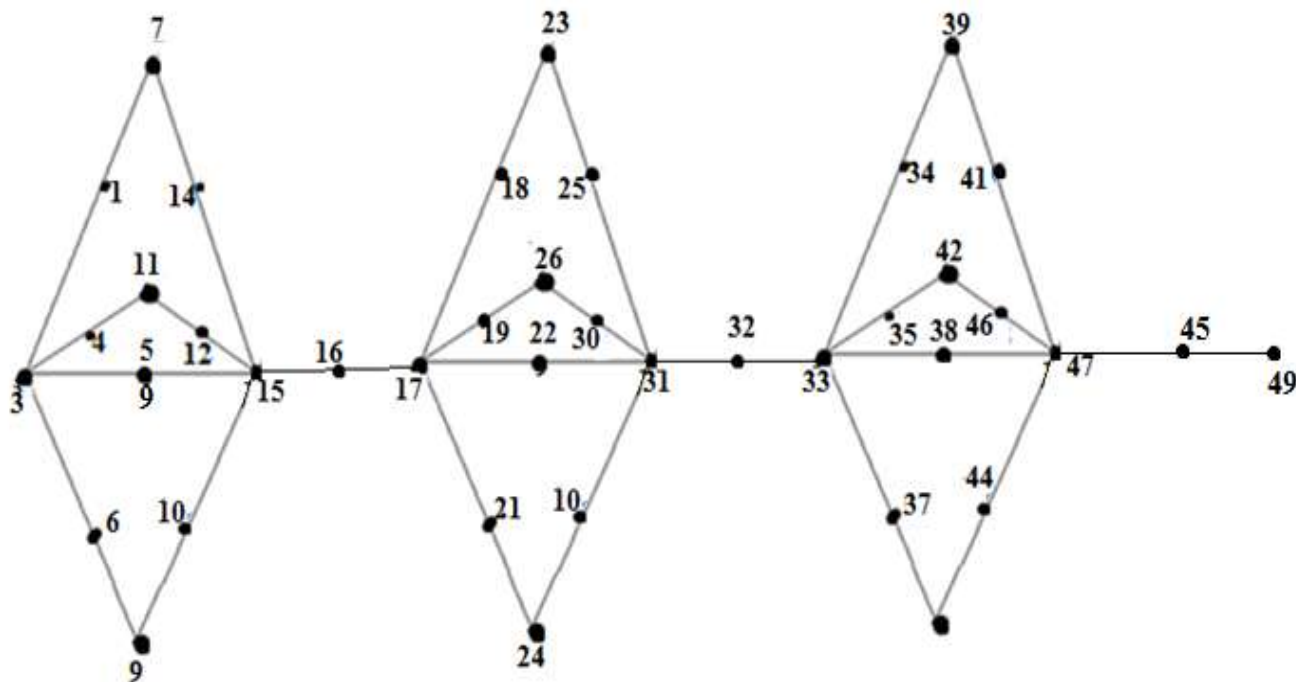


Figure : 9

Subcase (i) (b) : If n is even, then

Define a function $f: V(T(T_N)) \rightarrow \{1,2,\dots,q+1\}$ by

$$f(u_i) = 3$$

$$f(u_i) = 8i-1, \quad \forall i=2,4,6,\dots,n.$$

$$f(u_i) = 8i-7, \quad \forall i=3,5,7,\dots,n-1.$$

$$f(t_i) = 5$$

$$f(t_i) = 8i, \quad \forall i=2,4,6,\dots,n-1.$$

$$f(t_i) = 8i-2, \quad \forall i=3,5,7,\dots,n-1.$$

$$f(v_i) = 16i-9, \quad \forall i=3,5,7,\dots,\frac{n}{2}$$

$$f(w_i) = 11, f(w_i) = 16i-6, \quad \forall i=2,3,\dots,\frac{n}{2}$$

$$f(z_i) = 9, f(z_i) = 16i-8, \quad \forall i=2,3,\dots,\frac{n}{2}$$

$$f(r_i) = 4, f(r_i) = 16i-13, \quad \forall i=2,3,4,\dots,\frac{n}{2}$$

$$f(s_i) = 12, f(s_i) = 16i-2, \quad \forall i=2,3,4,\dots,\frac{n}{2}$$

$$f(x_i) = 1, f(x_i) = 16i-14, \quad \forall i=2,3,4,\dots,\frac{n}{2}$$

$$f(y_i) = 14, f(y_i) = 16i-7, \quad \forall i= 2,3,\dots,\frac{n}{2}$$

$$f(n_i) = 6, f(n_i) = 16i-11, \quad \forall i= 2,3,\dots,\frac{n}{2}$$

$$f(m_i) = 10, f(m_i) = 16i-4, \quad \forall i= 2,3,\dots,\frac{n}{2}$$

From the above labeling pattern we get distinct edge labels. Thus f provides a Geometric mean, labeling of G and its labeling pattern is shown below.

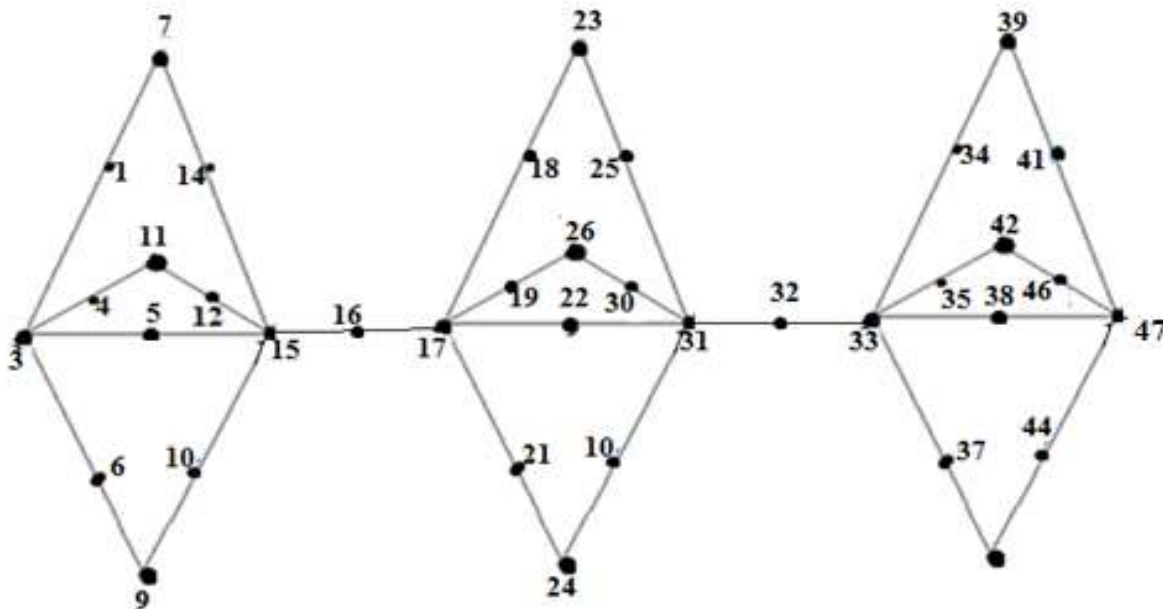


Figure: 10

Case (ii): If the triangular starts from u_2 , here also we consider two subcases.

Subcase (ii) (a): If n is odd, then

Define a function $f: V[T(T_N)] \rightarrow \{1,2,\dots,q+1\}$ by

$$f(u_1) = 1$$

$$f(u_i) = 8i-13, \quad \forall i= 2,4,6,\dots,n-1$$

$$f(u_i) = 8i-7, \quad \forall i= 3,5,7,\dots,n.$$

$$f(t_1) = 2$$

$$f(t_i) = 8i-9, \quad \forall i= 2,4,6,\dots,n-1.$$

$$f(t_i) = 8i-6, \quad \forall i= 3,5,7,\dots,n-1.$$

$$f(v_i) = 16i-4, \quad \forall i= 1,2,3,\dots,\frac{n-1}{2}.$$

$$f(w_i) = 16i-2, \quad \forall i= 1,2,3,\dots,\frac{n-1}{2}$$

$$f(z_i) = 16i-2, \quad \forall i= 1,2,3,\dots,\frac{n-1}{2}$$

$$f(x_i) = 16i-11, \quad \forall i= 1,2,3,\dots,\frac{n-1}{2}$$

$$f(y_i) = 16i-3, \quad \forall i= 1,2,3,\dots,\frac{n-1}{2}$$

$$f(r_i) = 16i-12, \quad \forall i= 1,2,3,\dots,\frac{n-1}{2}$$

$$f(s_i) = 16i, \quad \forall i= 1,2,3,\dots,\frac{n-1}{2}$$

$$f(n_i) = 16i-7, \quad \forall i= 1,2,3,\dots,\frac{n-1}{2}$$

$$f(m_i) = 16i-5, \quad \forall i= 1,2,3,\dots,\frac{n-1}{2}$$

From the above labeling pattern we get distinct edge labels.

Thus f provides a Geometric mean labeling for G and its labeling pattern is shown below.

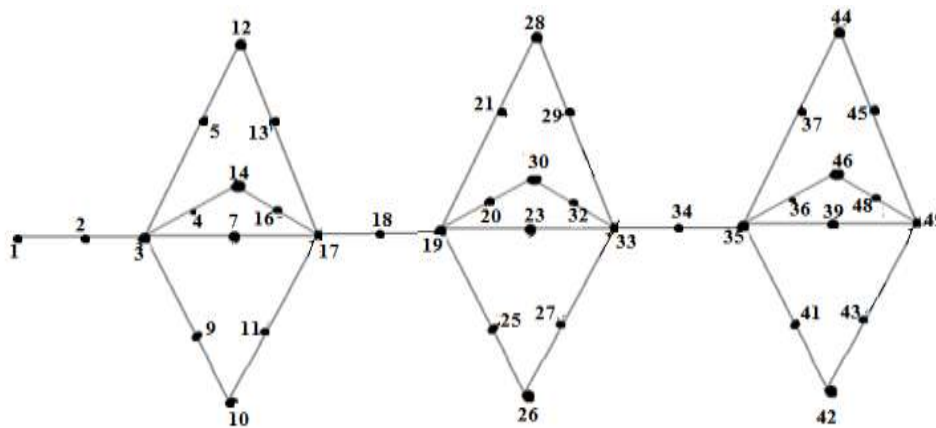


Figure: 11

Subcase (ii) (b): If n is even, then

Define a function $f: V[T(T_N)] \rightarrow \{1, 2, \dots, q+1\}$ by

$$f(u_1) = 1$$

$$f(u_i) = 8i - 13, \quad \forall i = 2, 4, 6, \dots, n-1$$

$$f(u_i) = 8i - 7, \quad \forall i = 3, 5, 7, \dots, n.$$

$$f(t_1) = 2$$

$$f(t_i) = 8i - 9, \quad \forall i = 2, 4, 6, \dots, n-1.$$

$$f(t_i) = 8i - 6, \quad \forall i = 3, 5, 7, \dots, n-1.$$

$$f(v_i) = 16i - 4, \quad \forall i = 1, 2, 3, \dots, \frac{n-2}{2}.$$

$$f(w_i) = 16i - 2, \quad \forall i = 1, 2, 3, \dots, \frac{n-2}{2}$$

$$f(z_i) = 16i - 2, \quad \forall i = 1, 2, 3, \dots, \frac{n-2}{2}$$

$$f(x_i) = 16i - 11, \quad \forall i = 1, 2, 3, \dots, \frac{n-2}{2}$$

$$f(y_i) = 16i - 3, \quad \forall i = 1, 2, 3, \dots, \frac{n-2}{2}$$

$$f(r_i) = 16i - 12, \quad \forall i = 1, 2, 3, \dots, \frac{n-2}{2}$$

$$f(s_i) = 16i, \quad \forall i = 1, 2, 3, \dots, \frac{n-2}{2}$$

$$f(n_i) = 16i - 7, \quad \forall i = 1, 2, 3, \dots, \frac{n-2}{2}$$

$$f(m_i) = 16i - 5, \quad \forall i = 1, 2, 3, \dots, \frac{n-2}{2}$$

$$f(m_i) = 16i - 5, \quad \forall i = 1, 2, 3, \dots, \frac{n-2}{2}$$

From the above labeling pattern we get distinct edge labels.

Thus f provides a Geometric mean labeling for G and its labeling pattern is shown below.

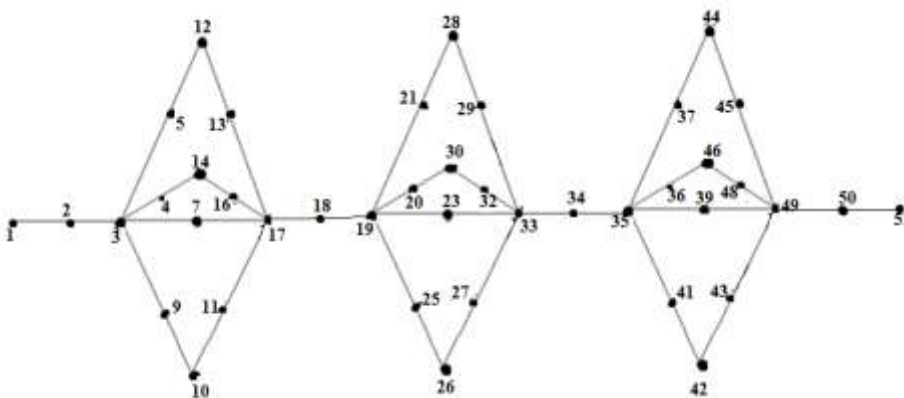


Figure: 11

From all the above cases, it is clear that subdivision on Alternate Triple Triangular snake graphs are Geometric mean graphs.

Conclusion: All graphs are not Geometric mean graphs. It is very interesting to investigate graphs which admit Geometric mean labeling. In this paper we proved that Triple Triangular snakes are Geometric mean graphs. It is possible to investigate similar results for several other graphs in the context of different labeling techniques.

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