# Soft C-Sets and a Decomposition of Soft Continuity in Soft Topological Spaces

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*Abstract*— **In the present paper, we define some new concepts in soft topological spaces such as soft $\alpha^*$-sets, soft C-sets. We introduce these concepts in soft topological spaces which are defined over an initial universe with a fixed set of parameters. We discuss their relationships with different types of subsets of soft topological spaces. We also investigate the concept of soft C-continuous functions and discuss their relationships with soft continuous and other forms of soft continuous functions. With the help of counterexamples, we show the noncoincidence of these various types of functions. Finally we give a decomposition of soft continuity.**

*Keywords*— **Soft set, Soft topological space, Soft $\alpha^*$-set, Soft C-set, Soft C-continuous function**

## I. INTRODUCTION

Molodtsov initiated a novel concept, which is called soft set, as a new mathematical tool for dealing with uncertainties [9]. In fact, a soft set is a parameterized family of subsets of a given universe set. The way of parameterization in problem solving makes soft set theory convenient and simple for application. Maji et al. [8] carried out Molodtsov's idea by introducing several operations in soft set theory. M. Shabir and M. Naz [10] presented soft topological spaces and they investigated some properties of soft topological spaces. Later, many researchers [13, 4, 3, 2, 1, 5, 12, 11, 7] studied some of basic concepts and properties of soft topological spaces.

In this paper, we continue the study in a soft topological space. We introduce some new concepts in soft topological spaces such as soft $\alpha^*$-sets, soft C-sets. We discuss their relationships with different types of subsets of soft topological spaces with the help of counterexamples. We also investigate the concept of soft C-continuous functions and discuss their relationships with soft continuity and other forms of soft continuous functions. Finally, we give a decomposition of soft continuity.

## II. PRELIMINARIES

In this section, we present the basic definitions and results of soft set theory which may be found in earlier studies.

Let $X$ be an initial universe set and $E$ be the set of all possible parameters with respect to $X$. Parameters are often attributes, characteristics or properties of the objects in $X$. Let $P(X)$ denote the power set of $X$. Then a soft set over $X$ is defined as follows.

*Definition 2.1* [9]*:*

A pair $(F, A)$ is called a soft set over $X$ where $A \subseteq E$ and $F : A \to P(X)$ is a set valued mapping. In other words, a soft set over $X$ is a parameterized family of subsets of the universe $X$. For $\forall \varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of $\varepsilon -$ approximate elements of the soft set $(F, A)$. It is worth noting that $F(\varepsilon)$ may be arbitrary. Some of them may be empty, and some may have nonempty intersection.

The set of all soft sets over $X$ is denoted by $SS(X)_E$.

*Definition 2.2* [8]*:*

A soft set $(F, A)$ over $X$ is said to be null soft set denoted by $\Phi$ if for all $e \in A$, $F(e) = \varnothing$. A soft set $(F, A)$ over $X$ is said to be an absolute soft set denoted by $\tilde{A}$ if for all $e \in A$, $F(e) = X$.

*Definition 2.3* [10]*:*

Let $Y$ be a nonempty subset of $X$, then $\tilde{Y}$ denotes the soft set $(Y, E)$ over $X$ for which $Y(e) = Y$, for all $e \in E$. In particular, $(X, E)$ will be denoted by $\tilde{X}$.

*Definition 2.4* [8]*:*

For two soft sets $(F, A)$ and $(G, B)$ over $X$, we say that $(F, A)$ is a soft subset of $(G, B)$ if $A \subseteq B$ and for all $e \in A$, $F(e)$ and $G(e)$ are identical approximations. We write $(F, A) \tilde{\subset} (G, B)$. $(F, A)$ is said to be a soft super set of $(G, B)$, if $(G, B)$ is a soft subset of $(F, A)$. We denote it by $(G, B) \tilde{\subset} (F, A)$. Then $(F, A)$ and $(G, B)$ are said to be soft equal if $(F, A)$ is a soft subset of $(G, B)$ and $(G, B)$ is a soft subset of $(F, A)$.

*Definition 2.5* [8]*:*

The union of two soft sets $(F, A)$ and $(G, B)$ over $X$ is the soft set $(H, C)$, where $C = A \cup B$ and for all $e \in C$, $H(e) = F(e)$ if $e \in A - B$, $G(e)$ if $e \in B - A$, $F(e) \cup G(e)$ if $e \in A \cap B$. We write $(F, A) \tilde{\cup} (G, B) = (H, C)$.

*Definition 2.6* [8]*:*

The intersection $(H, C)$ of $(F, A)$ and $(G, B)$ over $X$, denoted $(F, A) \tilde{\cap} (G, B)$, is defined as $C = A \cap B$ and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

*Definition 2.7* [10]*:*

The difference $(H, E)$ of two soft sets $(F, E)$ and $(G, E)$ over $X$, denoted by $(F, E) \setminus (G, E)$, is defined as $H(e) = F(e) \setminus G(e)$ for all $e \in E$.

*Definition 2.8* [10]*:*

The relative complement of a soft set $(F, E)$ is denoted by $(F, E)^c$ and is defined by $(F, E)^c = (F^c, E)$ where $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X - F(e)$ for all $e \in E$.

*Definition 2.9* [10]*:*

Let $\tau$ be the collection of soft sets over $X$, then $\tau$ is said to be a soft topology on $X$ if

     (1) $\Phi, \tilde{X} \in \tau$

     (2) If $(F, E), (G, E) \in \tau$, then $(F, E) \tilde{\cap} (G, E) \in \tau$

     (3) If $\{(F_i, E)\}_{i \in I} \in \tau$, $\forall i \in I$, then $\tilde{\cup}_{i \in I} (F_i, E) \in \tau$.

The triplet $(X, \tau, E)$ is called a soft topological space over $X$. Every member of $\tau$ is called a soft open set in $X$. A soft set $(F, E)$ over $X$ is called soft closed set in $X$ if its relative complement $(F, E)^c$ belongs to $\tau$. We will denote the family of all soft open sets (resp., soft closed sets) of a soft topological space $(X, \tau, E)$ by SOS

$(X,\tau,E)$ (resp., SCS $(X,\tau,E)$).

*Definition 2.10:*

Let $(X,\tau,E)$ be a soft topological space over $X$ and $(F,E)$ be a soft set over $X$.

    (1) [10] The soft closure of $(F,E)$ is the soft set $cl(F,E) = \cap\{(G,E):(G,E)$ is soft closed and $(F,E) \tilde{\subseteq} (G,E)\}$.

    (2) [13] The soft interior of $(F,E)$ is the soft set $int(F,E) = \cup\{(H,E):(H,E)$ is soft open and $(H,E) \tilde{\subseteq} (F,E)\}$.

Clearly, $cl(F,E)$ is the smallest soft closed set over $X$ which contains $(F,E)$ and $int(F,E)$ is the largest soft open set over $X$ which is contained in $(F,E)$.

*Theorem 2.1* [10]*:*

Let $(X,\tau,E)$ be a soft topological space and $(F,E)$, $(G,E)$ soft sets over $X$. Then

    (1) $cl(\Phi) = \Phi$ and $cl(\tilde{X}) = \tilde{X}$.

    (2) $(F,E) \tilde{\subseteq} cl(F,E)$.

    (3) $(F,E)$ is a soft closed set if and only if $(F,E) = cl(F,E)$.

    (4) $cl(cl(F,E)) = cl(F,E)$.

    (5) $(F,E) \tilde{\subseteq} (G,E)$ implies $cl(F,E) \tilde{\subseteq} cl(G,E)$.

    (6) $cl((F,E) \tilde{\cup} (G,E)) = cl(F,E) \tilde{\cup} cl(G,E)$.

    (7) $cl((F,E) \tilde{\cap} (G,E)) \tilde{\subseteq} cl(F,E) \tilde{\cap} cl(G,E)$.

*Theorem 2.2* [4]*:*

Let $(X,\tau,E)$ be a soft topological space and $(F,E)$, $(G,E)$ soft sets over $X$. Then

    (1) $int(\Phi) = \Phi$ and $int(\tilde{X}) = \tilde{X}$.

    (2) $int(F,E) \tilde{\subseteq} (F,E)$.

    (3) $(F,E)$ is a soft open set if and only if $(F,E) = int(F,E)$.

    (4) $int(int(F,E)) = int(F,E)$.

    (5) $(F,E) \tilde{\subseteq} (G,E)$ implies $int(F,E) \tilde{\subseteq} int(G,E)$.

    (6) $int((F,E) \tilde{\cap} (G,E)) = int(F,E) \tilde{\cap} int(G,E)$.

    (7) $int((F,E) \tilde{\cup} (G,E)) \tilde{\supseteq} int(F,E) \tilde{\cup} int(G,E)$.

Throughout the paper, the spaces $X$ and $Y$ (or $(X,\tau,E)$ and $(Y,\nu,K)$) stand for soft topological spaces assumed unless stated otherwise.

*Definition 2.11:*

Let $(X,\tau,E)$ be a soft topological space. A soft set $(F,E)$ is called

    (1) a soft semiopen set in $X$ [3] if $(F,E) \tilde{\subseteq} cl(int(F,E))$,

    (2) a soft preopen set in $X$ [2] if $(F,E) \tilde{\subseteq} int(cl(F,E))$,

    (3) a soft $\alpha$-open set in $X$ [1] if $(F,E) \tilde{\subseteq} int(cl(int(F,E)))$,

    (4) a soft $\beta$-open set in $X$ [5] if $(F,E) \tilde{\subseteq} cl(int(cl(F,E)))$,

    (5) a soft regular open set in $X$ [12] if $(F,E) = int(cl(F,E))$,

    (6) a soft A-set in $X$ [11] if $(F,E) = (G,E) \backslash (H,E)$, where $(G,E)$ is a soft open set and $(H,E)$

is a soft regular open set in $X$ ,

(7) a soft $t$-set in $X$  [11] if $int(cl(F,E)) = int(F,E)$ ,

(8) a soft B-set in $X$  [11] if $(F,E) = (G,E) \tilde{\cap} (H,E)$ , where $(G,E)$ is a soft open set and $(H,E)$ is a soft $t$-set in $X$ .

The relative complement of a soft semiopen (soft preopen, soft $\alpha$-open, soft $\beta$-open, soft regular open) set is called a soft semiclosed (soft preclosed, soft $\alpha$-closed, soft $\beta$-closed, soft regular closed) set.

We will denote the family of all soft semiopen sets (resp., soft preopen sets, soft $\alpha$-open sets, soft $\beta$-open sets, soft regular open sets, soft A-sets and soft B-sets) of a soft topological space $(X, \tau, E)$ by $SSOS(X)$ (resp., $SPOS(X)$, $S\alpha OS(X)$, $S\beta OS(X)$, $SROS(X)$, $SAS(X)$ and $SBS(X)$ ).

*Remark 2.1:*

In a soft topological space $(X, \tau, E)$ ;

(1) every soft open set is a soft $\alpha$-open set [1],
(2) every soft $\alpha$-open set is soft preopen and soft semiopen [1],
(3) every soft regular open set is soft open [12],
(4) every soft open set is a soft A-set [11],
(5) every soft A-set is soft semiopen [11],
(6) every soft open set is a soft B-set [11],
(7) every soft A-set is a soft B-set [11].

*Lemma 2.1* [5]*:*

Let $(X, \tau, E)$ be a soft topological space and $(F,E)$ and $(G,E)$ be two soft sets over $X$ . If either $(F,E)$ is soft semiopen or $(G,E)$ is soft semiopen, then

$$int(cl((F,E) \tilde{\cap} (G,E))) = int(cl(F,E)) \tilde{\cap} int(cl(G,E)).$$

*Definition 2.12* [6]*:*

Let $SS(X)_E$ and $SS(Y)_K$ be families of soft sets, $u : X \to Y$ and $p : E \to K$ be mappings. Then the mapping $f_{pu} : SS(X)_E \to SS(Y)_K$ is defined as:

(1)   Let $(F,E) \in SS(X)_E$ .   The   image   of   $(F,E)$   under   $f_{pu}$ ,   written   as $f_{pu}(F,E) = (f_{pu}(F), p(E))$ , is a soft set in $SS(Y)_K$ such that

$$f_{pu}(F)(y) = \begin{cases} \cup_{x \in p^{-1}(y) \cap A} u(F(x)), & p^{-1}(y) \cap A \neq \varnothing \\ \varnothing, & otherwise \end{cases}$$

for all $y \in K$ .

(2)   Let $(G,K) \in SS(Y)_K$ .   The   inverse   image   of   $(G,K)$   under   $f_{pu}$ ,   written   as $f_{pu}^{-1}(G,K) = (f_{pu}^{-1}(G), p^{-1}(K))$ , is a soft set in $SS(X)_E$ such that

$$f_{pu}^{-1}(G)(x) = \begin{cases} u^{-1}(G(p(x))), & p(x) \in K \\ \varnothing, & otherwise \end{cases}$$

for all $x \in E$ .

Definition 2.13:

Let $(X, \tau, E)$ and $(Y, \upsilon, K)$ be soft topological spaces and $f_{pu} : SS(X)_E \to SS(Y)_K$ be a function. Then $f_{pu}$ is called

(1) soft continuous [13] if for each $(G, K) \in \upsilon$ we have $f_{pu}^{-1}(G, K) \in \tau$ ,

(2) soft semicontinuous [7] if for each $(G, K) \in SOS(Y)$ we have $f_{pu}^{-1}(G, K) \in SSOS(X)$ ,

(3) soft $\alpha$ -continuous [1] if for each $(G, K) \in SOS(Y)$ we have $f_{pu}^{-1}(G, K) \in S\alpha OS(X)$ ,

(4) soft precontinuous [1] if for each $(G, K) \in SOS(Y)$ we have $f_{pu}^{-1}(G, K) \in SPOS(X)$ ,

(5) soft $\beta$ -continuous [5] if for each $(G, K) \in SOS(Y)$ we have $f_{pu}^{-1}(G, K) \in S\beta OS(X)$ ,

(6) soft A-continuous [11] if for each $(G, K) \in SOS(Y)$ , $f_{pu}^{-1}(G, K)$ is a soft A-set in $X$ ,

(7) soft B-continuous [11] if for each $(G, K) \in SOS(Y)$ , $f_{pu}^{-1}(G, K)$ is a soft B-set in $X$ .

*Remark 2.2:*

Let $(X, \tau, E)$ and $(Y, \upsilon, K)$ be soft topological spaces and $f_{pu} : SS(X)_E \to SS(Y)_K$ be a function. Then,

(1) every soft continuous function is soft $\alpha$ -continuous [1],
(2) every soft $\alpha$ -continuous function is soft semicontinuous and soft precontinuous [1],
(3) every soft continuous function is soft A-continuous [11],
(4) every soft A-continuous function is soft semicontinuous [11],
(5) every soft continuous function is soft B-continuous [11],
(6) every soft A-continuous function is soft B-continuous [11].

III. SOFT C-SETS

*Definition 3.1:*

Let $(X, \tau, E)$ be a soft topological space. A soft set $(F, E)$ is called a soft $\alpha^*$ -set in $X$ if $int(cl(int(F, E))) = int(F, E)$ .

*Proposition 3.1:*

Let $(X, \tau, E)$ be a soft topological space. Then the following are equivalent for a soft set $(F, E)$ over $X$ :

$(i)$ $(F, E)$ is a soft $\alpha^*$ -set in $X$ ,

$(ii)$ $(F, E)$ is soft $\beta$ -closed,

$(iii)$ $int(F, E)$ is soft regular open.

*Proof:* The proof is obvious.

*Proposition 3.2:*

Let $(X, \tau, E)$ be a soft topological space and $(F, E)$ be a soft set over $X$ . If $(F, E)$ is a soft t-set, then $(F, E)$ is a soft $\alpha^*$ -set.

*Proof:* Let $(F, E)$ be a soft t-set and

$$int(cl(int(F,E))) = int(cl(F,E)) = int(F,E).$$

Hence we obtain $(F,E)$ is a soft $\alpha^*$-set in $X$.

*Remark 3.1:*

The converse of Proposition 3.2 is not true in general as shown in the following example.

*Example 3.1:*

Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1,E), (F_2,E),...,(F_{11},E)\}$ where $(F_1,E), (F_2,E),...,(F_{11},E)$ are soft sets over $X$, defined as follows:

$$(F_1,E) = \{(e_1, \{x_1\}), (e_2, \{x_1\})\},$$
$$(F_2,E) = \{(e_1, \{x_2\}), (e_2, \{x_2\})\},$$
$$(F_3,E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2\})\},$$
$$(F_4,E) = \{(e_1, \{x_1, x_2, x_3\}), (e_2, \{x_1, x_3\})\},$$
$$(F_5,E) = \{(e_1, \{x_1, x_2, x_4\}), (e_2, \{x_1, x_2, x_3\})\},$$
$$(F_6,E) = \{(e_1, \{x_2\}), (e_2, \varnothing)\},$$
$$(F_7,E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1\})\},$$
$$(F_8,E) = \{(e_1, \{x_1, x_2, x_3\}), (e_2, \{x_1, x_2, x_3\})\},$$
$$(F_9,E) = \{(e_1, X), (e_2, \{x_1, x_2, x_3\})\},$$
$$(F_{10},E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2, x_3\})\},$$
$$(F_{11},E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_3\})\}.$$

Then $\tau$ defines a soft topology on $X$ and thus $(X, \tau, E)$ is a soft topological space over $X$.

Let $(G,E)$ be a soft set over $X$ such that $(G,E) = \{(e_1, \{x_1, x_4\}), (e_2, \{x_2, x_3\})\}$. Since $int(G,E) = int(cl(int(G,E)))$, $(G,E)$ is a soft $\alpha^*$-set but not a soft t-set.

*Proposition 3.3:*

Let $(X, \tau, E)$ be a soft topological space and $(F,E)$ be a soft set over $X$. Then a soft semiopen set $(F,E)$ is a soft t-set if and only if $(F,E)$ is a soft $\alpha^*$-set.

*Proof:* Let $(F,E)$ be a soft semiopen set and a soft $\alpha^*$-set. Since $(F,E)$ is soft semiopen $cl(int(F,E)) = cl(F,E)$ and hence $int(cl(F,E)) = int(cl(int(F,E))) = int(F,E)$. Therefore $(F,E)$ is a soft t-set.

Conversely, let $(F,E)$ be a soft semiopen set and a soft t-set. Since $(F,E)$ is soft semiopen $int(F,E) \tilde{\subseteq} int(cl(int(F,E)))$ and since $(F,E)$ is a soft t-set $int(F,E) = int(cl(F,E)) \tilde{\supseteq} int(cl(int(F,E)))$. Hence we obtain $(F,E)$ is a soft $\alpha^*$-set.

*Proposition 3.4:*

Let $(X, \tau, E)$ be a soft topological space and $(F,E)$ be a soft set over $X$. Then $(F,E)$ is soft $\alpha$-open and a

soft $\alpha^*$-set if and only if $(F,E)$ is soft regular open.

*Proof:* Let $(F,E)$ be soft $\alpha$-open and a soft $\alpha^*$-set. By Proposition 3.1 and the definition of a soft $\alpha$-open set, we have $int(cl(int(F,E))) = (F,E)$ and hence $int(cl(F,E)) = int(cl(int(F,E))) = (F,E)$. The converse is obvious.

*Remark 3.2:*

Let $(X,\tau,E)$ be a soft topological space. The following example shows that

      $i)$   a soft open set need not be a soft $\alpha^*$-set,

      $ii)$   the notion of soft $\alpha$-open sets is different from that of soft $\alpha^*$-sets.

*Example 3.2:*

Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1,E), (F_2,E), (F_3,E)\}$ where $(F_1,E), (F_2,E), (F_3,E)$ are soft sets over $X$, defined as follows:

$$(F_1,E) = \{(e_1, \{x_1\}), (e_2, \{x_1\})\},$$
$$(F_2,E) = \{(e_1, \{x_2\}), (e_2, \{x_2\})\},$$
$$(F_3,E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2\})\}.$$

Then $\tau$ defines a soft topology on $X$ and thus $(X,\tau,E)$ is a soft topological space over $X$. Since $int(cl(int(F_3,E))) = \tilde{X} \neq int(F_3,E)$, $(F_3,E)$ is not a soft $\alpha^*$-set but a soft open set.

*Example 3.3:*

Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Let us take the soft topology $\tau$ on $X$ and the soft set $(F_3,E)$ in Example 3.2. $(F_3,E)$ is a soft $\alpha$-open set since every soft open set is soft $\alpha$-open. But $(F_3,E)$ is not a soft $\alpha^*$-set by Example 3.2.

*Example 3.4:*

Let $X = \{x_1, x_2, x_3, x_4\}$ and $E = \{e_1, e_2\}$. Let us take the soft topology $\tau$ on $X$ in Example 3.1. Let $(G,E)$ be a soft set over $X$ such that $(G,E) = \{(e_1, \{x_3, x_4\}), (e_2, \{x_2\})\}$. Since $int(cl(int(G,E))) = \Phi = int(G,E)$, $(G,E)$ is a soft $\alpha^*$-set. But $(G,E)$ is not a soft $\alpha$-open set.

*Remark 3.3:*

The union of two soft $\alpha^*$-sets need not to be a soft $\alpha^*$-set, as shown in the following example.

*Example 3.5:*

Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Let us take the soft topology $\tau$ on $X$ and the soft sets $(F_1,E)$ and $(F_2,E)$ in Example 3.2. Clearly, $(F_1,E)$ and $(F_2,E)$ are soft $\alpha^*$-sets in $(X,\tau,E)$ but $(F_1,E) \tilde{\cup} (F_2,E)$ is not a soft $\alpha^*$-set.

*Proposition 3.5:*

Arbitrary intersection of soft $\alpha^*$-sets is a soft $\alpha^*$-set.

*Proof:* Let $\{(F_i, E) : i \in I\}$ be a collection of soft $\alpha^*$-sets. Then for each $i \in I$,

$$int(cl(int(F_i, E))) = int(F_i, E).$$

Now

$$\underset{i \in I}{\tilde{\cap}} int(F_i, E) = \underset{i \in I}{\tilde{\cap}} int(cl(int(F_i, E)))$$

$$int(\underset{i \in I}{\tilde{\cap}}(F_i, E)) = int(\underset{i \in I}{\tilde{\cap}} cl(int(F_i, E)))$$

$$\tilde{\supseteq} int(cl(\underset{i \in I}{\tilde{\cap}} int(F_i, E)))$$

$$= int(cl(int(\underset{i \in I}{\tilde{\cap}}(F_i, E)))).$$

Also

$$int(\underset{i \in I}{\tilde{\cap}}(F_i, E)) \tilde{\subseteq} cl(int(\underset{i \in I}{\tilde{\cap}}(F_i, E)))$$

$$int(int(\underset{i \in I}{\tilde{\cap}}(F_i, E))) \tilde{\subseteq} int(cl(int(\underset{i \in I}{\tilde{\cap}}(F_i, E))))$$

$$int(\underset{i \in I}{\tilde{\cap}}(F_i, E)) \tilde{\subseteq} int(cl(int(\underset{i \in I}{\tilde{\cap}}(F_i, E)))).$$

Hence we obtain $int(\underset{i \in I}{\tilde{\cap}}(F_i, E)) = int(cl(int(\underset{i \in I}{\tilde{\cap}}(F_i, E))))$ and so $\underset{i \in I}{\tilde{\cap}}(F_i, E)$ is a soft $\alpha^*$-set.

*Definition 3.2:*

Let $(X, \tau, E)$ be a soft topological space. A soft set $(F, E)$ is called soft C-set in $X$ if there exist a soft open set $(G, E)$ and a soft $\alpha^*$-set $(H, E)$ such that $(F, E) = (G, E) \tilde{\cap} (H, E)$.

*Proposition 3.6:*

Let $(X, \tau, E)$ be a soft topological space. Then every soft $\alpha^*$-set is a soft C-set and every soft open set is a soft C-set.

*Proof:* Since $\tilde{X}$ is both a soft open set and a soft $\alpha^*$-set, the proof is obvious.

*Remark 3.4:*

The converse of Proposition 3.6 is not true in general as shown in the following example.

*Example 3.6:*

Let $X = \{x_1, x_2, x_3, x_4\}$ and $E = \{e_1, e_2\}$. Let us take the soft topology $\tau$ on $X$ and the soft set $(F_5, E)$ in Example 3.1. Since $(F_5, E) = (F_5, E) \tilde{\cap} \tilde{X}$ such that $(F_5, E)$ is soft open and $\tilde{X}$ is a soft $\alpha^*$-set, $(F_5, E)$ is a soft C-set but not a soft $\alpha^*$-set.

*Example 3.7:*

Let $X = \{x_1, x_2, x_3, x_4\}$ and $E = \{e_1, e_2\}$. Let us take the soft topology $\tau$ on $X$ and the soft set $(G, E) = \{(e_1, \{x_1, x_4\}), (e_2, \{x_2, x_3\})\}$ in Example 3.1. Since $(G, E)$ is a soft $\alpha^*$-set, $(G, E)$ is a soft C-set but not soft open.

*Proposition 3.7:*

In a soft topological space $(X, \tau, E)$, every soft B-set is a soft C-set.

*Proof:* By Proposition 3.2, every soft t-set is a soft $\alpha^*$-set. Hence we obtain every soft B-set is a soft C-set.

*Remark 3.5:*

The converse of Proposition 3.7 is not true in general as shown in the following example.

*Example 3.8:*

Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Let us take the soft topology $\tau$ on $X$ in Example 3.2. Let $(G, E)$ be a soft set over $X$ such that $(G, E) = \{(e_1, \{x_3\}), (e_2, \{x_1, x_3\})\}$. Since $int(cl(int(G, E))) = int(G, E)$, $(G, E)$ is a soft $\alpha^*$-set and so it is a soft C-set. But $(G, E)$ is not a soft t-set, since $int(cl(G, E)) \neq int(G, E)$ and so $(G, E)$ is not a soft B-set.

*Remark 3.6:*

Let $(X, \tau, E)$ be a soft topological space. The notion of soft $\alpha$-open sets is different from that of soft C-sets.

The following examples show that a soft C-set need not be a soft $\alpha$-open set and a soft $\alpha$-open set need not be a soft C-set.

*Example 3.9:*

*Let $X = \{x_1, x_2, x_3, x_4\}$ and $E = \{e_1, e_2\}$. Let us take the soft topology $\tau$ on $X$ in Example 3.1 and the soft set $(G, E) = \{(e_1, \{x_3, x_4\}), (e_2, \{x_2\})\}$ in Example 3.4. $(G, E)$ is a soft $\alpha^*$-set by Example 3.4. Also, $(G, E)$ is a soft C-set since every soft $\alpha^*$-set is a soft C-set. But $(G, E)$ is not a soft $\alpha$-open set.*

*Example 3.10:*

Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F, E)\}$ where $(F, E)$ is a soft set over $X$, defined as follows:

$$(F, E) = \{(e_1, \{x_1\}), (e_2, \{x_2\})\}.$$

Then $\tau$ defines a soft topology on $X$ and thus $(X, \tau, E)$ is a soft topological space over $X$.

Let $(G, E)$ be a soft set over $X$ such that $(G, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_2\})\}$. Since $(G, E) \tilde{\subseteq} int(cl(int(G, E))) = \tilde{X}$, $(G, E)$ is a soft $\alpha$-open set but not a soft C-set.

*Proposition 3.8:*

Let $(X, \tau, E)$ be a soft topological space. A soft set $(F, E)$ over $X$ is soft open if and only if it is a soft $\alpha$-open set and a soft C-set.

*Proof:* By Proposition 3.6 and Remark 2.1, every soft open set is a soft C-set and a soft $\alpha$-open set. Let $(F, E)$ be a soft C-set and a soft $\alpha$-open set. Since $(F, E)$ is a soft C-set, $(F, E) = (G, E) \tilde{\cap} (H, E)$ where

$(G, E)$ is soft open and $(H, E)$ is a soft $\alpha^*$-set. Since $(F, E)$ is a soft $\alpha$-open set, by using Lemma 2.1, we have

$$(F, E) \tilde{\subseteq} int(cl(int(F, E)))$$
$$= int(cl(int((G, E) \tilde{\cap} (H, E))))$$
$$= int(cl(G, E)) \tilde{\cap} int(cl(int(H, E)))$$
$$= int(cl(G, E)) \tilde{\cap} int(H, E)$$

and hence

$$(F, E) = (G, E) \tilde{\cap} (F, E)$$
$$\tilde{\subseteq} (G, E) \tilde{\cap} (int(cl(G, E)) \tilde{\cap} int(H, E))$$
$$= (G, E) \tilde{\cap} int(H, E)$$
$$\tilde{\subseteq} (F, E).$$

Consequently, we obtain $(F, E) = (G, E) \tilde{\cap} int(H, E)$ and thus $(F, E)$ is soft open.

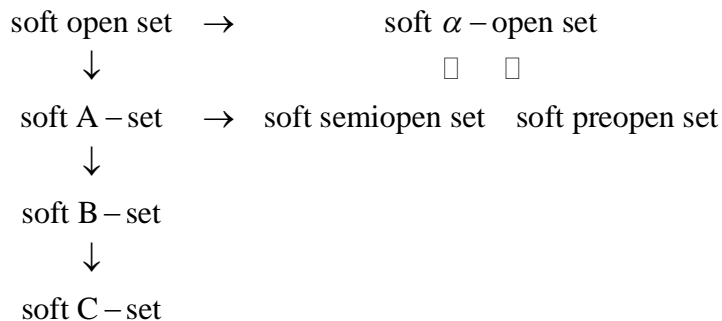We have implications as shown in FIGURE 1 for a soft topological space $(X, \tau, E)$.

soft open set $\rightarrow$        soft $\alpha$ − open set
      $\downarrow$                  □     □

soft A − set $\rightarrow$ soft semiopen set    soft preopen set
      $\downarrow$

soft B − set
      $\downarrow$

soft C − set

FIGURE 1

## IV. DECOMPOSITION OF SOFT CONTINUITY

*Definition 4.1:*
Let $(X, \tau, E)$ and $(Y, \upsilon, K)$ be soft topological spaces. Let $u : X \rightarrow Y$ and $p : E \rightarrow K$ be mappings. Let $f_{pu} : SS(X)_E \rightarrow SS(Y)_K$ be a function. Then the function $f_{pu}$ is called soft C-continuous if for each $(G, K) \in SOS(Y)$, $f_{pu}^{-1}(G, K)$ is a soft C-set in $X$.

*Proposition 4.1:*
Every soft B-continuous function is soft C-continuous.

*Proof:* The proof easily follows from Proposition 3.7.

*Remark 4.1:*
The converse of implication in Proposition 4.1 is not true, which is clear from the following example.

*Example 4.1:*
Let $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2, y_3\}$, $E = \{e_1, e_2\}$, $K = \{k_1, k_2\}$,

$\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ in Example 3.2, $\upsilon = \{\Phi, \tilde{Y}, (H, K)\}$ such that $(H, K) = \{(k_1, \{y_1\}), (k_2, \{y_1, y_3\})\}$ and $(X, \tau, E)$ and $(Y, \upsilon, K)$ be soft topological spaces. Define $u : X \to Y$ and $p : E \to K$ as

$$u(x_1) = \{y_3\}, u(x_2) = \{y_2\}, u(x_3) = \{y_1\} \text{ and } p(e_1) = \{k_1\}, p(e_2) = \{k_2\}.$$

Let $f_{pu} : SS(X)_E \to SS(Y)_K$ be a function. Then $(H, K)$ is soft open in $Y$ and $f_{pu}^{-1}(H, K) = (G, E) = \{(e_1, \{x_3\}), (e_2, \{x_1, x_3\})\}$ is a soft C-set but not a soft B-set in $X$ by Example 3.8. Therefore, $f_{pu}$ is a soft C-continuous function but not soft B-continuous.

*Theorem 4.1:*

Let $(X, \tau, E)$ and $(Y, \upsilon, K)$ be soft topological spaces and $f_{pu} : SS(X)_E \to SS(Y)_K$ be a function. Then $f_{pu}$ is a soft continuous function if and only if it is soft $\alpha$-continuous and soft C-continuous.

*Proof:* This is an immediate consequence of Proposition 3.6 and 3.8.
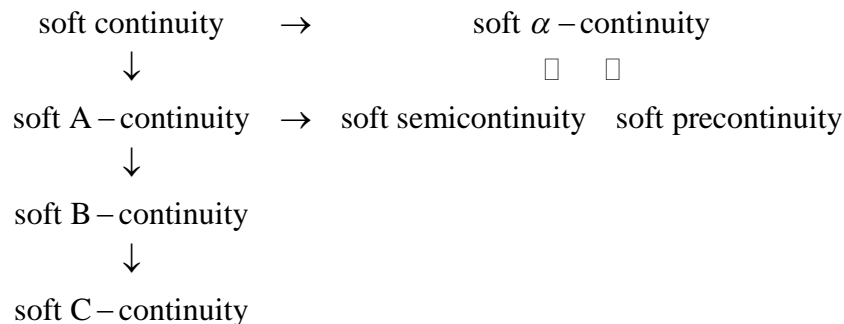
We have implications as shown in FIGURE 2.

$$
\begin{array}{ccc}
\text{soft continuity} & \rightarrow & \text{soft } \alpha - \text{continuity} \\
\downarrow & & \square \quad \square \\
\text{soft A} - \text{continuity} & \rightarrow & \text{soft semicontinuity} \quad \text{soft precontinuity} \\
\downarrow & & \\
\text{soft B} - \text{continuity} & & \\
\downarrow & & \\
\text{soft C} - \text{continuity} & &
\end{array}
$$

FIGURE 2

## V. CONCLUSION

In the present study, we have introduced soft $\alpha^*$-sets and soft C-sets in soft topological spaces which are defined over an initial universe with a fixed set of parameters. We have presented their basic properties with the help of some counterexamples. Also, we have introduced soft C-continuous functions and we have obtained a new decomposition of soft continuity. We expect that results in this paper will be helpfull for further applications in soft topological spaces.

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