Logarithm Approximation for Small Values of ‘x’

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***Abstract*— In this paper, an approximation formula for logarithm($\log_{10}(1+x)$, $\log_{10}(1-x)$ and $\log_{10}(x)$) is introduced. These formulas give close approximation to the actual logarithm value. The equation for $\log_{10}(1+x)$ and $\log_{10}(1-x)$ will give close approximation only when the input i.e $x \in [-1,1]$ and for $\log_{10}(x)$ will give close approximation only when $x \in [0,2]$. Certain examples have been presented in this paper, while at the same time comparison with actual logarithm value is also made. Graphical representation of logarithm and my equation is also provided.**

***Keywords* — Logarithm Approximation for small values, Logarithm, Representation of Logarithm Algebraically, Logarithm Approximation For $\log_{10}(1+x)$, $\log_{10}(1-x)$ and $\log_{10}(x)$.**

I. Introduction

The concept of Logarithm was introduced by John Napier. Few instances where Logarithms used are in Ritcher and Decibel scale . Various approximation methods of logarithm have been introduced in the past. The logarithm approximations for $\log_{10}(1+x)$, $\log_{10}(1-x)$ and for $\log_{10}(x)$ were given by likes of Tylor and Padé. In this paper I will introduce approximation formulas which results in close approximation to the logarithm value for $\log_{10}(1+x)$, $\log_{10}(1-x)$ and $\log_{10}(x)$.

II. Formula for approximation

This formula was achieved by trial and error method. ‘c’ is different for different numbers, whereas ‘x’ is input.

$$log(x) \approx (\frac{1}{\sqrt{x}}) + 1 \mp [\frac{1}{(c*x)}]$$

$$log(1+x) \approx -(\frac{1}{\sqrt{1+x}}) + 1 \mp [\frac{1}{(c*x)}]$$

$$log(1-x) \approx -(\frac{1}{\sqrt{1-x}}) + 1 \mp [\frac{1}{(c*x)}]$$

1/(cx) is defined as ‘D Difference’.

A. Approximation for $\log_{10}(1+x)$

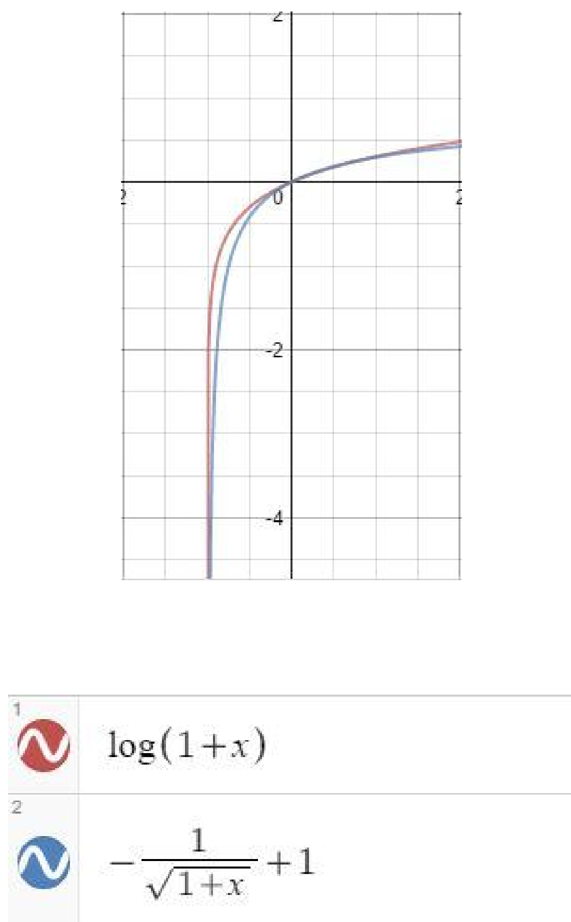
TABLE I
APPROXIMATION FOR $\log_{10}(1+x)$

$\log_{10}(1+x)$	**Formula value (approximate value)**	**D Difference (approximate value)**	**Final value(approximate till 4 decimal place)**	**Log value (approximate till 4 decimal place)**
1.1	0.04653741075	-0.005145	0.0414	0.0414
1.2	0.08712907082	-0.007948	0.0792	0.0792
1.3	0.1229419807	-0.008999	0.1139	0.1139
1.4	0.158457453	-0.012329	0.1461	0.1461
1.5	0.1835034191	-0.007412	0.1761	0.1761
1.6	0.209430585	-0.005311	0.2041	0.2041
1.7	0.2330350112	-0.002586	0.2304	0.2304
1.8	0.2546440075	+0.000628	0.2553	0.2553
1.9	0.2745237499	+0.00423	0.2788	0.2788

For approximation till four-five decimal places

1/(C) for 1.1≈ -0.005659
1/(C) for 1.2 ≈ -0.009537
1/(C) for 1.3 ≈ -0.011698
1/(C) for 1.4 ≈ -0.017261
1/(C) for 1.5 ≈ -0.011118
1/(C) for 1.6 ≈ -0.008497
1/(C) for 1.7 ≈ -0.004396
1/(C) for 1.8 ≈ +0.001131
1/(C) for 1.9 ≈ +0.008037

2. Graph Of The equation $\log_{10}(1+x)$ Without ‘d difference’.



B. Approximation for log(1-x)

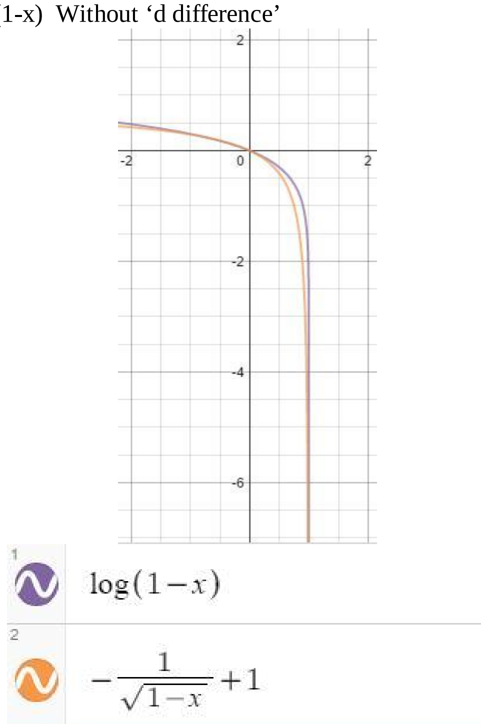
TABLE II
APPROXIMATION FOR $log_{10}(1-x)$

$\log_{10}(1-x)$	Formula value (approximate value)	D Difference (approximate value)	Final value(approximate till 4 decimal place)	Log value (approximate till 4 decimal place)
0.9	-0.0540925539	+0.008336	-0.0458	-0.0458
0.8	-0.1180339887	+0.021124	-0.0969	-0.0969
0.7	-0.1952286093	+0.040327	-0.1549	-0.1549
0.6	-0.2909944487	+0.069145	-0.2218	-0.2218
0.5	-0.4142135624	+0.113184	-0.3010	-0.3010
0.4	-0.5811388301	+0.01832	-0.3979	-0.3979
0.3	-0.8257418584	+0.302863	-0.5229	-0.5229
0.2	-1.236067977	+0.53706	-0.6990	-0.6990
0.1	-2.16227766	+1.162278	-1	-1

1. For approximation till four-five decimal places

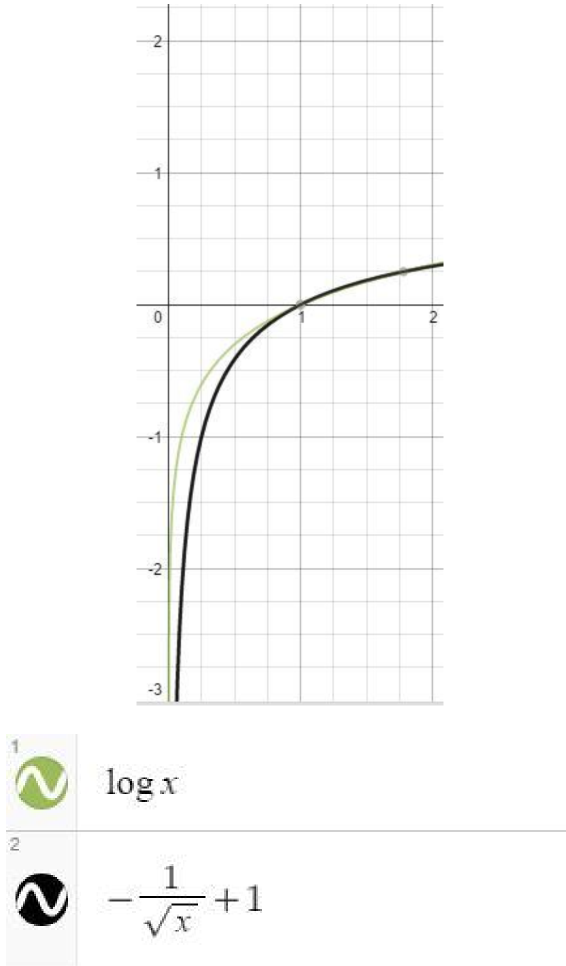
   1/(C) for 0.9 ≈ +0.007502
   1/(C) for 0.8≈ +0.016899
   1/(C) for 0.7≈ +0.028229
   1/(C) for 0.6 ≈ +0.041487
   1/(C) for 0.5 ≈ +0.056592
   1/(C) for 0.4 ≈ +0.073280
   1/(C) for 0.3 ≈ +0.090859
   1/(C) for 0.2 ≈ +0.107412
   1/(C) for 0.1 ≈ +0.116228

2. Graph Of The equation $\log_{10}(1-x)$ Without 'd difference'



Note: - These Formulas will result in close approximation only for small values of 'x'.

3. Graph Of The equation $\log_{10}(x)$ Without 'd difference'



III. Conclusions

I have introduced logarithm approximation formulas which can produce approximate value of logarithm for $\log_{10}(1+x)$, $\log_{10}(1-x)$ and $\log_{10}(x)$. Some of which were presented in this paper. The accuracy can be increased by using more accurate value of 'C', which thereby increases accuracy of 'D Difference' value, thus results in close approximation.

References

[1] Logarithm Wikipedia website [Online]. Available: http://en.wikipedia.org/wiki/Logarithm
[2] Padé Approximation website [Online]. Available: http://www.nezumi.demon.co.uk/consult/logx.htm
[3] SWAPNIL PALIWAL "Logarithm Approximation" in International Journal Of Mathematics, vol. 16 number 1, p. 12-14, Dec. 2014
[4] Tylor series Wikipedia website [Online].Available: http://en.wikipedia.org/wiki/Taylor_series
[5] Approximation for log function website [Online].Available: http://planetmath.org/approximationofthelogfunction