Super Geometric Mean Labeling On Double Triangular Snakes

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ABSTRACT

Let f: V(G) $\rightarrow$ {1,2,…,p+q} be an injective function. For a vertex labeling "f" the induced edge labeling f* (e=uv) is defined by,

f* (e) $= \left\lceil \sqrt{f(u)f(v)} \right\rceil$ or $\left\lfloor \sqrt{f(u)f(v)} \right\rfloor$.Then f is called a **Super Geometric mean labeling** if $\{f(V(G))\} \cup \{f(e): e \in E(G)\} = \{1,2,\ldots,p+q\}$, A graph which admits Super Geometric mean labeling is called **Super Geometric mean graph**.

In this paper, we prove that Double Triangular Snakes and Alternate Double Triangular Snake graphs and Super Geometric mean graphs.

Key words: Graph, Geometric mean graph, Super Geometric mean graph, Double Triangular snake, Alternate Double Triangular snake.

1. Introduction

All graphs in this paper are finite, simple and undirected graph G = (V,E) with p vertices and q edges. For all detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notations we follow Harary [2]. The concept of Geometric mean labeling has been introduced by S. Somasundaram, R. Ponraj and P. Vidhyarani in [5]. We will provide a brief summary of definitions and other informations which are necessary for our present investigation.

Definition 1.1

A graph G = (V,E) with p vertices and q edges is called a **Geometric mean graph** if it is possible to label vertices $x \in V$ with distinct label f(x) from 1,2,…,q+1 in such a way that when each edge e=uv is labeled with, f(e=uv)= $\left\lceil \sqrt{f(u)f(v)} \right\rceil$ or $\left\lfloor \sqrt{f(u)f(v)} \right\rfloor$then the edge labels are distinct. In this case "f" is called **Geometric mean labeling** of G.

Definition 1.2:

Let f: V(G) →{1,2,…,p+q} be an injective function. For a vertex labeling "f", the induced edge labeling f* (e=uv) is defined by,

f* (e) = $\left\lceil\sqrt{f(u)f(v)}\right\rceil$ or $\left\lfloor\sqrt{f(u)f(v)}\right\rfloor$ then "f" is called a **Super Geometric mean labeling** if $\{f(V(G)\} \cup \{f(e): e \in E(G)\} = \{1,2,\ldots,p+q\}$. A graph which admits Super Geometric mean labeling is called **Super Geometric mean graph**.

Definition 1.3:

A **Triangular Snake** T_n is obtained from a Path $u_1u_2,\ldots,u_n$ by joining u_i and u_{i+1} to a new vertex v_i, $1 \le i \le n-1$. That is every edge of a Path is replaced by a triangle C_3.

Definition 1.4:

An **Alternate Triangular snake** $A(T_n)$ is obtained from a Path $u_1u_2\ldots u_n$ by joining u_i and u_{i+1} alternatively to a new vertex v_i. That is every alternate edge of a Path is replaced by C_3.

Definition 1.5:

A **Double Triangular snake** $D(T_n)$ consists of two Triangular snakes that have a common Path.

Definition 1.6:

An **Alternate Double Triangular snake** $A(D(T_n))$ consists of two Alternate Triangular snakes that have a common Path.

Now we shall use frequent reference to the following theorems.

Theorem 1.7 [5]: Triangular snakes and Alternate Triangular snakes are Geometric mean graphs.

Theorem 1.8: Double Triangular and Alternate Double Triangular snakes are Geometric mean graphs.

2. Main Results

Theorem 2.1:

A Double Triangular snake $D(T_n)$ is a Super Geometric mean graph.

Proof

Let D (T_n) be the Double Triangular snake.

Consider a Path u_1 $u_2,\ldots,u_n$.

Join u_i u_{i+1} with two new vertices v_i and w_i, $1 \le i \le n-1$. The graph is displaced below.

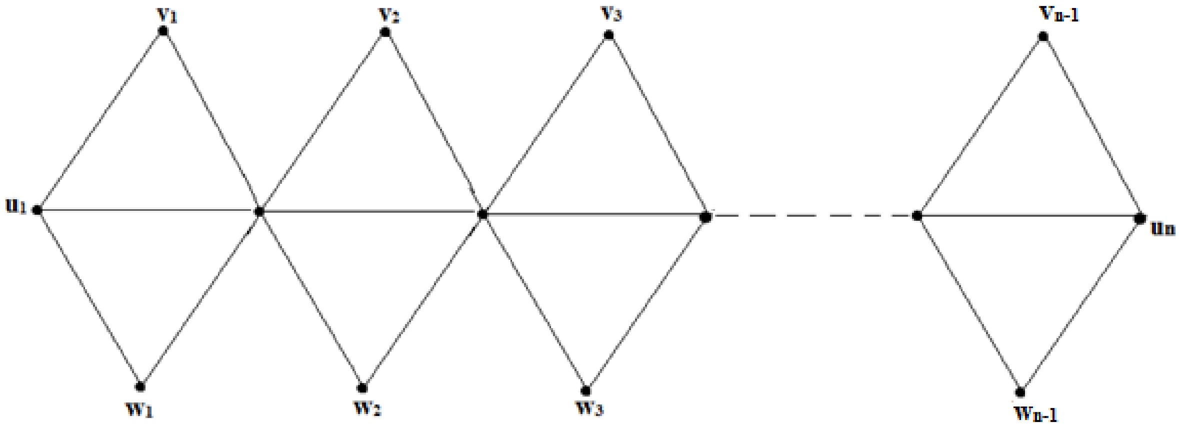

Figure: 1

Define a function f: $V(D(T_n)) \rightarrow \{1,2,\ldots,p+q\}$by,

$f(u_1) = 4$

$f(u_i) = 8i-7$, $2 \le i \le n$

$f(v_1) = 1$

$f(v_i) = 8i-4$, $2 \le i \le n-1$

$f(w_i) = 8i-1$ $1 \le i \le n-1$

Edges are labeled with,

$f(u_1u_2) = 6$

$f(u_iu_{i+1}) = 8i-3$, $2 \le i \le n-1$

$f(u_iv_i) = 8i-6$, $1 \le i \le n-1$

$f(v_1u_2) = 3$

$f(v_iu_{i+1}) = 8i-2$, $2 \le i \le n-1$

$f(u_1w_1) = 5$

$f(u_i w_i) = 8i-5$, $2 \le i \le n-1$

$f(w_iu_{i+1}) = 8i$, $1 \le i \le n-1$

Thus we get distinct edge labels

Hence $D(T_n)$ is a Super Geometric mean graph.

Example 2.2

A Super Geometric mean labeling of $D(T_5)$ is given below.

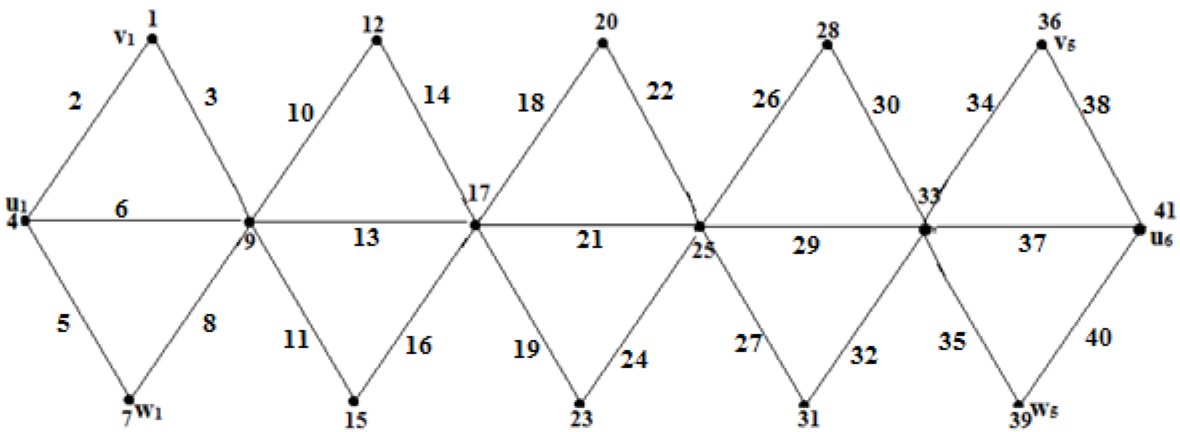

Figure: 2

Theorem 2.3

Alternate Double Triangular snake A ($D(T_n)$) is a Super Geometric mean graph.

Proof:

Let G be the graph $A(D(T_n))$

Consider the Path $u_1u_2\ldots u_n$

To construct G, join u_i and u_{i+1} (alternatively) with two new vertices v_i and w_i

There are two different cases to be considered.

Case 1: If $A(D(T_n))$ starts from u_1, we need two subcases.

Subcase 1(a) If n is odd, then

Define a function f: $V(G) \to \{1,2,\ldots,p+q\}$ by,

$f(u_1) = 4$

$f(u_{2i-1}) = 10i-9,\ 2\leq i\leq\left(\frac{n-1}{2}\right)+1$

$f(u_{2i}) = 10i-1,\ 1\leq i\leq \frac{n-1}{2}$

$f(v_1) = 1$

$f(v_i) = 10i-6,\ 2 \leq i \leq \frac{n-1}{2}$

$f(w_i) = 10i-3,\ 1\leq i \leq \frac{n-1}{2}$

Edges are labeled with,

$f(u_1 u_2) = 6$

$f(u_iu_{i+1}) = 5i,\ 2\leq i\leq n-1$

$f(u_{2i-1}\, v_i) = 10i-8,\ 1 \le i \le \frac{n-1}{2}$

$f(v_1 u_2) = 3$

$f(v_i u_{2i}) = 10i-4,\ 2 \le i \le \frac{n-1}{2}$

$f(u_1 w_1) = 5$

$f(u_{2i-1}\, w_i) = 10i-7,\ 2 \le i \le \frac{n-1}{2}$

$f(w_i u_{2i}) = 10i-2,\ 1 \le i \le \frac{n-1}{2}$

Thus we ge get distinct edge labels.

The labeling pattern of A $(D(T_4))$ is shown below

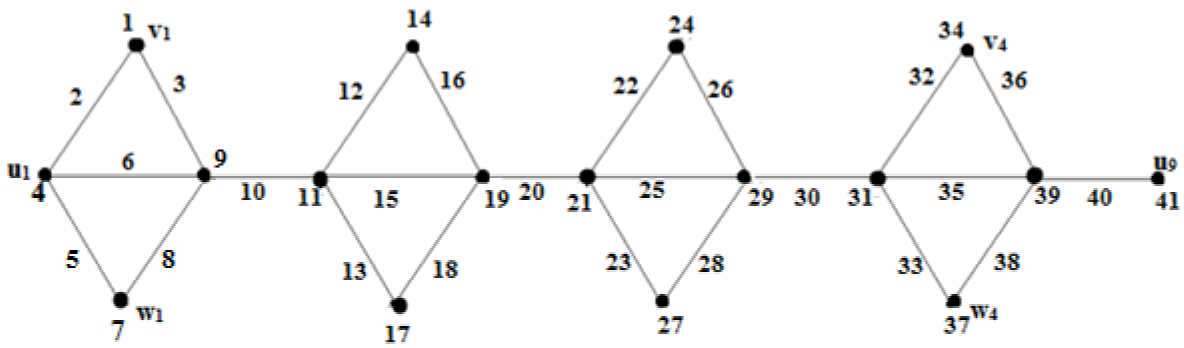

Figure: 3

Subcase 1(b): If n is even, then

Define a function $f: V(G) \rightarrow \{1,2,...,p+q\}$ by,

$f(u_1) = 4$

$f(u_{2i-1}) = 10i-9,\ 2 \le i \le \frac{n}{2}$

$f(u_{2i}) = 10i-1,\ 1 \le i \le \frac{n}{2}$

$f(v_1) = 1$

$f(v_i) = 10i-6,\quad 2 \le i \le \frac{n}{2}$

$f(w_i) = 10i-3\quad ,\ 1 \le i \le \frac{n}{2}$

Edges are labeled with

$f(u_1 u_2) = 6$

$f(u_i\, u_{i+1}) = 5i,\ 2 \le i \le n-1$

$f(u_{2i-1}\, v_i) = 10i-8,\ 1 \le i \le \frac{n}{2}$

$f(v_1 u_2) = 3$

$f(v_i\ u_{2i}) = 10i\text{-}4,\ 2 \le i \le \frac{n}{2}$

$f(u_1\ w_1) = 5$

$f(u_{2i-1}\ w_i) = 10i\text{-}7,\ 2 \le i \le \frac{n}{2}$

$f(w_i\ u_{2i}) = 10i\text{-}2,\quad 1 \le i \le \frac{n}{2}$

Therefore the edge labels are distinct.

The labeling pattern of $A(D(T_4))$ is displaced below.

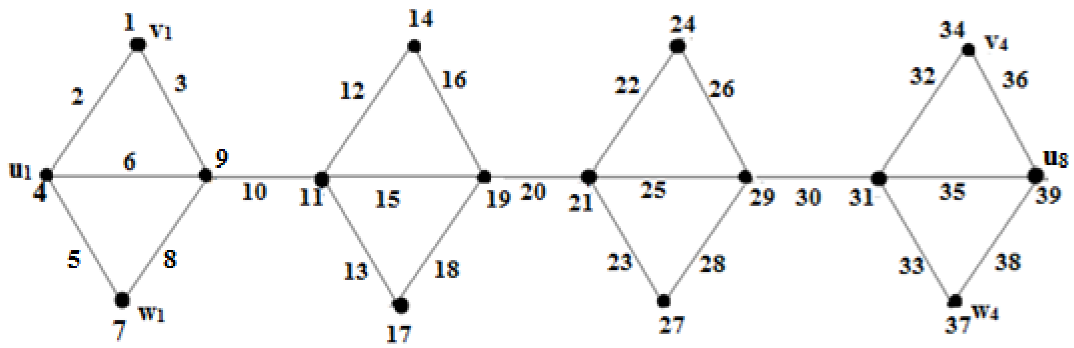

Figure: 4

In this case f provides a Super Geometric mean labeling of G.

Case 2: If $A(D(T_n))$ Starts from u_2, we have to consider two subcases.

Subcase 2(a): If n is odd then,

Define a function $f: V(G) \rightarrow \{1,2,\ldots,p+q\}$ by,

$f(u_{2i-1}) = 10i\text{-}9,\quad 1 \le i \le \left(\frac{n-1}{2}\right)+1$

$f(u_{2i}) = 10i\text{-}7,\quad 1 \le i \le \frac{n-1}{2}$

$f(v_i) = 10i\text{-}3,\quad 1 \le i \le \frac{n-1}{2}$

$f(w_i) = 10i\text{-}1,\quad 1 \le i \le \frac{n-1}{2}$

Edges are labeled with,

$f(u_{2i-1}\ u_{2i}) = 10i\text{-}8,\quad 1 \le i \le \frac{n-1}{2}$

$f(u_{2i}\ u_{2i+1}) = 10i\text{-}4,\quad 1 \le i \le \frac{n-1}{2}$

$f(v_i\ u_{2i}) = 10i\text{-}6,\quad 1 \le i \le \frac{n-1}{2}$

$f(v_i\, u_{2i+1}) = 10i\text{-}2, \quad 1 \le i \le \frac{n-1}{2}$

$f(u_{2i}\, w_i) = 10i\text{-}5, \quad 1 \le i \le \frac{n-1}{2}$

$f(w_i\, u_{2i+1}) = 10i, \quad 1 \le i \le \frac{n-1}{2}$

Thus we get distinct edge labels.

The labeling pattern of $A(D(T_4))$ is given below.

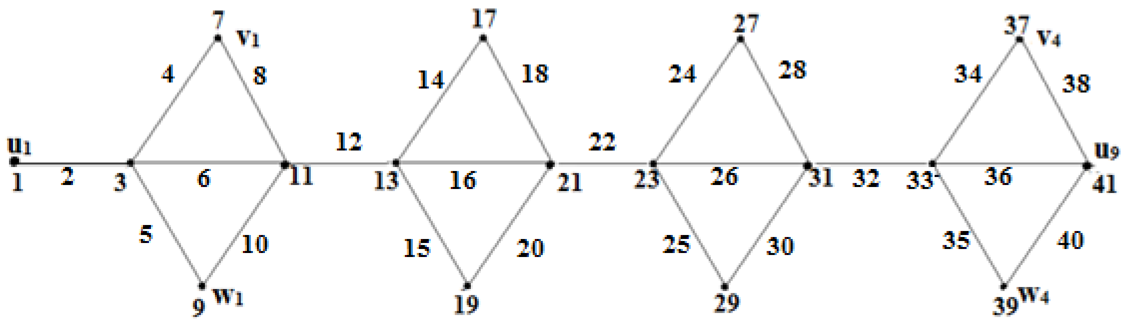

Figure: 5

Subcase 2(b): If n is even, then

Define a function, $f: V(G) \rightarrow \{1,2,\ldots,p+q\}$ by,

$f(u_{2i-1}) = 10i\text{-}9, \; 1 \le i \le \frac{n}{2}$

$f(u_{2i}) = 10i\text{-}7, \; 1 \le i \le \frac{n}{2}$

$f(v_i) = 10i\text{-}3, \quad 1 \le i \le \frac{n-2}{2}$

$f(w_i) = 10i\text{-}1, \; 1 \le i \le \frac{n-2}{2}$

Edges are labeled with

$f(u_{2i-1}\, u_{2i}) = 10i\text{-}8, \quad 1 \le i \le \frac{n}{2}$

$f(u_{2i}\, u_{2i+1}) = 10i\text{-}4, \quad 1 \le i \le \frac{n-2}{2}$

$f(v_i\, u_{2i}) = 10i\text{-}6, \quad 1 \le i \le \frac{n-2}{2}$

$f(v_i\, u_{2i+1}) = 10i\text{-}2, \quad 1 \le i \le \frac{n-2}{2}$

$f(u_{2i}\, w_i) = 10i\text{-}5, \quad 1 \le i \le \frac{n-2}{2}$

$f(w_i\, u_{2i+1}) = 10i \quad 1 \le i \le \frac{n-2}{2}$

$\therefore$ The edge labels are distinct

$\therefore$ The labeling pattern of $A(D(T_3))$ is shown below.

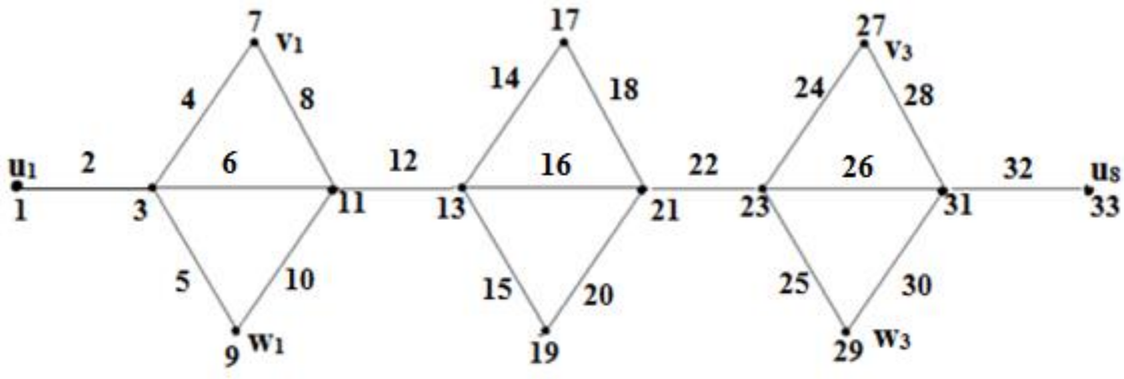

Figure: 6

In this case, f provides a Super Geometric mean labeling of G.

∴ From all the above cases, we conclude that Alternate Double Triangular snake $A(D(T_n))$ is a Super Geometric mean graph.

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