

Contextual KP System

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Abstract

In this paper, we consider contextual way of handling string objects in KP Systems. We also considered contextual KP System with Depth First Derivation and Contextual KP System with marked derivation

Keywords: Psystem, KP System, Contextual grammar

1 INTRODUCTION

P System represents a branch of natural computing that brings from cellular biology to computer science a set of concepts, principles and computing mechanisms. P system contains a hierarchical structure of membranes placed inside a main membrane called the skin membrane. Graphically a membrane structure is represented by a venn diagram without intersection and with a unique super set. Starting from an initial configuration and using the evolution rules,

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we get a computation. we consider a computation complete when it halts, no further rule can be applied.

A new class of P System called Kernel P System (KP System) has been introduced in order to generalize various features like the structure of the model, the type of the rules and the execution strategy.

In KP System each rule may have a guard 'g' which is applicable to the multiset and that controls the system. In this paper we define contextual KP Systems, in which the rule consists of attaching contexts to strings depending upon a choice mapping. We consider some variants of contextual grammars and prove that contextual KP System with rules of the type corresponding to these variants are more powerful than ordinary contextual grammars and their variants.

2 PREREQUISITES

Contextual grammars were introduced by S.Marcus as a non- chomskian model to describe natural languages. They provide an important tool in the study of formal language theory. Contextual grammars also play a major role in our understanding of grammars without the use of non terminals, called pure grammars.

In contextual grammars, the string xwy is derived by attaching the context (x,y) to the string w .

Definition 2.1. *A contextual grammar with choice is a construct.*

$$G = (V, A, C, \psi),$$

Where V is an alphabet, A is a finite language over V , C is a finite subset of $V^* \times V^*$ and $\psi: V^* \rightarrow 2^C$. The strings in A are called axioms, the elements (u,v) in C are called contexts and ψ is the choice mapping

Definition 2.2. An external contextual grammar is a contextual grammar

where all derivations are based on the external mode “ ex ” that is, $x \Rightarrow_{ex} y$ if and only if $y = uxv$, for a context (u,v) in $\psi(x)$

Definition 2.3. An internal contextual grammar is a contextual grammar where all derivations are based on the internal mode “ in ” that is, $x \Rightarrow_{in} y$ if and only if $x = x_1x_2x_3$, $y = x_1ux_2vx_3$, for any $x_1, x_2, x_3, \in V^*$, $(u, v) \in \psi(x_2)$.

Definition 2.4. A total contextual grammar is a system

$$G = (V, A, C, \psi),$$

where V is an alphabet, A is a finite language over V , C is a finite subset of $V^* \times V^*$ and $\psi : V^* \times V^* \times V^* \rightarrow 2^c$.

Definition 2.5. For a total contextual grammar $G = (V, B, C, \psi)$ we define the relation \Rightarrow_G on V^* as follows : $x \Rightarrow_G y$ if and only if $x = x_1x_2x_3, y = x_1ux_2vx_3$ for some $x_1, x_2, x_3 \in V^*$, $(u, v) \in C$ such that $(u, v) \in \psi(x_1, x_2, x_3)$

Definition 2.6. A contextual grammar $G = (V, B, C, \psi)$ is said to be without choice if $\psi(x) = C$ for all x in V^*

Five basic families of languages are obtained, they are

1. *TC* = the family of languages generated by total contextual grammars.
2. *ECC* = the family of languages generated by external contextual grammars.
3. *ICC* = the family of languages generated by internal contextual grammars.
4. *EC* = the family of languages generated by external contextual grammars without choice.
5. *IC* = the family of languages generated by internal contextual grammars without choice.

Definition 2.7. A language $L \subseteq V^*$ has the external bounded step (EBS) property if there is a constant p such that for each $x \in L, |x| > p$, there is $y \in L$ such that $x = uyv$ and $0 < |uv| \leq p$

Definition 2.8. A language $L \subseteq V^*$ has the internal bounded step (IBS) property if there is a constant p such that for each $x \in L, |x| > p$, there is $y \in L$ such that $x = x_1ux_2vx_3, y = x_1x_2x_3$ and $0 < |uv| \leq p$

Definition 2.9. A language $L \subseteq V^*$ has the bounded length increase (BLI) property if there is a constant p such that for each $x \in L$, there is $y \in L$ such that $-p \leq |x| - |y| \leq p$

- A language is in the family ECC if and only if it has the EBS property
- A language is in the family TC if and only if it has the IBS property.
- All contextual languages have the BLI property.

Definition 2.10. A contextual grammar

$$G = (V, A, (S_1, C_1), (S_2, C_2), \dots, (S_n, C_n))$$

We define a depth - first derivation in G as a derivation

$$w_1 \Rightarrow_{df} w_2 \Rightarrow_{df} \dots \Rightarrow_{df} w_m, m \geq 1$$

where

i) $w_1 \in A, w_1 \Rightarrow_{in} w_2$ in the usual sense

ii) for each $i = 1, 2, 3, \dots, m$ if $w_{i-1} = x_1x_2x_3,$

$$w_i = x_1ux_2vx_3, (u, v) \text{ is the context}$$

adjoined to w_{i-1} in order to get w_i then, $w_i = y_1y_2y_3, w_{i+1} = y_1u'y_2v'y_3$ such that $y_2 \in s_j, (u', v') \in c_j$ for some $j, 1 \leq j \leq n$ and y_2 contains atleast one of the words u, v as a substring.

Definition 2.11. A contextual grammar with marked derivation is a construct

$$G = (V, A, C_1, C_2, \psi, \Phi)$$

Where

$$(V, A, C_1, \psi)$$

is a usual contextual grammar, C_2 is a finite set of contexts over V and $\psi : V^* \rightarrow 2^{C_2}$. The contexts in C_2 are the terminal ones and ϕ is a mapping selecting the terminal context.

For a grammar G as above, let us denote by G^1 the underlying usual grammar (V, A, C_1, ψ) and by $\Longrightarrow_{\alpha}^{\phi}$, for $\alpha \in \{ex, in\}$, the derivation step where a context in C_2 is adjoint (in the external or internal mode, respectively) as selected by the mapping ϕ . Then the language generated by G in the mode $\alpha \in \{ex, in\}$ is $L_{\alpha} = \{x \in V^* / \omega \Longrightarrow_{\alpha}^* y \Longrightarrow_{\alpha}^{\phi} x, \text{ for } \omega \in A, y \in L_{\alpha}(G^1)\}$ we denote by $ECC_{m_k}(F)$, $ICC_{m_k}(F)$ the families of languages generated in the external and internal mode respectively, by grammars with F choice.

3 KERNEL P SYSTEMS (KP SYSTEMS)

A KP System is a formal model that uses some well known features of existing P System and includes some new elements and more importantly, it offers a coherent view on integrating them in to the same formalism. The system was introduced by M Gheorghe et al. Here a broad range of strategies to use the rule against the multiset of objects available in each compartments is provided. Now will see the definition of compartments and KP System

Definition 3.1. Given a finite set A called alphabet of elements, called objects and a finite set L , of elements called labels, a compartment is a tuple $C = (l, \omega_0, R^{\sigma})$ where $l \in L$ is the label of the compartments ω_0 is the initial multiset over A and R^{σ} denotes the DNA code of C , which comprises the set of rules, denoted R , applied in this compartments and a regular expression

σ , over $Lab(R)$ the labels of the rule of R

Definition 3.2. A kernel P System of degree n is a tuple

$$K\Pi = (A, L, I_0, \mu, C_1, C_2, \dots, C_n, i_0)$$

where A and L are as in definition 3.1, the alphabet and the set of labels respectively; I_0 is a multiset of objects from A , called environment; μ defines the membrane structure which is a graph (V, E) , where V are vertices, $V \subseteq L$ (the nodes are labels of these compartments), and E edges, C_1, C_2, \dots, C_n are n compartments of the system - the inner part of each compartment is called region, which is delimited by a membrane, the labels of the compartments are from L and initial multiset are over A . i_0 is the output compartments where the result is obtained.

4 CONTEXTUAL KP SYSTEMS

Definition 4.1. A contextual KP system is a construct

$$K\Pi = (A, \mu, C_1, C_2, C_3, \dots, C_n, i_0)$$

where A is a finite set of elements called objects, μ is a membrane structure, C_1, C_2, \dots, C_n are n compartments with

$$C_i = (t_i, w_i)$$

$$t_i = (R_i, \sigma_i)$$

R_i is a contextual rule of the form $(x, (u, v), tar)$ (attaching evolution rules) where $x, u, v \in A$ and $tar \in \{here, in, out\}$ and σ_i is an execution strategy in KP System, i_0 is the output compartment where the result is obtained

The family of all languages generated by contextual KP Systems of degree $n, n \geq 1$ in the mode $X \in \{IC, ICC, EC, ECC\}$ with attaching evolution

rules and by using the target indications of the form $\{here, out, in\}$ is denoted by $KCP(X, n)$

Theorem 4.1. $KCP(EC, 2) - ICC \neq \phi$

Proof. Consider a contextual KP System of degree 2 in the external contextual mode

$$KCP = (A, \mu, c_1, c_2, i_0)$$

$$A = \{a, b\}$$

$$\mu = [1[2]2]_1$$

$$c_1 = (t_1, w_1)$$

$$c_2 = (t_2, w_2)$$

$$t_1 = (R_1, \sigma_1)$$

$$t_2 = (R_2, \sigma_2)$$

$$\omega_1 = \{ab\}$$

$$\omega_2 = \phi$$

$$i_0 = c_1$$

$$R_1 = \{r_1 : (ab, (a, b), in)\}$$

$$R_2 = \{r_1 : (a^n b^n, n \geq 1, (\lambda, \lambda), out)\}$$

$$\sigma_1 = r_1^*$$

$$\sigma_2 = r_1$$

$$L(KCP) = \{a^n b^n, n \geq 1\} \notin ICC$$

due to lemma 5.5 [1]

□

Theorem 4.2. $KCP(IC, 1) - TC \neq \phi$

Proof. consider KP system of degree 1 in the internal contextual mode.

$$\begin{aligned}
 KCP &= (A, \mu, c_1, i_0) \\
 A &= \{a, b\} \\
 \mu &= [1]_1 \\
 C_1 &= (t_1, w_1) \\
 t_1 &= (R_1, \sigma_1) \\
 i_0 &= c_1 \\
 w_1 &= \{aba\} \\
 R_1 &= \{r_1 : (aba, (a, a), here)\} \\
 \sigma_1 &= r_1^* \\
 L(KCP) &= \{a^n b a^n / n \geq 1\} \notin TC
 \end{aligned}$$

due to lemma 10.1[1]. □

5 CONTEXTUAL KP SYSTEMS WITH DEPTH FIRST DERIVATION

The push down automata, known to be equivalent to context free grammars have a memory used in LIFO (Last in first out) manner. Sometimes it is important to use also the context free grammars in this way. Here we are using the Depth first derivation similar to LIFO which means rewriting at each step one of the last introduced non terminals.

Definition 5.1. *A contextual KP System with Depth First Derivation of degree n is a construct $KCP_{df} = (A, \mu, C_1, C_2, \dots, C_n, i_0)$ where A is called alphabet of elements*

μ is a membrane structure

C_1, C_2, \dots, C_n are n compartments with

$$C_i = (t_i, w_i)$$

$$t_i = (R_i, \sigma_i)$$

$R_i, 1 \leq i \leq n$ is a contextual rule of the form $(x, (u, v), tar)$ (attaching evolution rules) where $x, u, v \in A$ and $tar \in \{here, in, out\}$ and

σ_i is an execution strategy in KP System and we are using the contextual rewriting rules in the last generated string

- The union of all words w_m , $m \geq 1$ generated by Depth First Derivation in contextual KP System constitute the language generated by the contextual KP System in the Depth First Derivation Manner and is denoted by $L(KCP_{d_f}(F))$
- The families of languages KCP_{d_f} for contextual gramars with F choice and possibly using the empty context (λ, λ) is denoted by $L(KCP_{d_f}^\lambda(F))$.

Theorem 5.1. $KCP_{d_f}(FIN) - ICC \neq \phi$

Proof.

$$KCP_{d_f} = (A, \mu, c_1, c_2, c_3, i_0)$$

$$A = \{a, b, c\}$$

$$\mu = [1[2[3]3]2]1$$

$$c_1 = (t_1, w_1)$$

$$c_2 = (t_2, w_2)$$

$$c_3 = (t_3, w_3)$$

$$w_1 = \{a\}$$

$$w_2 = \{c^2\}$$

$$w_3 = \{\}$$

$$t_1 = (R_1, \sigma_1)$$

$$t_2 = (R_2, \sigma_2)$$

$$t_3 = (R_3, \sigma_3)$$

$$i_0 = 3$$

$$R_1 = \{r_1 : (a, (\lambda, a), \text{here})$$

$$r_2 : (a^n, (\lambda, \lambda), \text{in})\}$$

$$\sigma_1 = r_1^* r_2$$

$$R_2 = \{r_1 : (c, (a^2 c^2, c^2 b^2), \text{in})$$

$$r_2 : (c, (\lambda, \lambda), \text{in})$$

$$r_3 : (a^n, (\lambda, \lambda), \text{in})\}$$

$$\sigma_2 = r_1 r_2 r_3$$

$$R_3 = \{r_1 : (a^2 c^5 b^2, (a^2, b^2), \text{here})$$

$$r_2 : (c, (\lambda, \lambda), \text{here})$$

$$r_3 : (a^n, (\lambda, \lambda), \text{here})\}$$

$$\sigma_3 = r^* r_2 r_3$$

$$L(KCP_{d_f}) = \{c\} \cup \{a^n/n \geq 1\} \cup \{a^{2n}c^5b^{2n}/n \geq 1\}$$

this language is not in the *ICC* family due to lemma 5.5 [1]

□

Theorem 5.2. $KCP_{d_f}(F) \subseteq KCP_{d_f}^\lambda(F)$ where $F \in \{FIN, REG\}$

6 CONTEXTUAL KP SYSTEMS WITH MARKED DERIVATION

In contextual grammars the marked derivation proceeds as usual and it end by adjusting a component from a special set. This special set can be seen as symmetric to the axiom set the contextual KP System with marked derivation is denoted by KCP_{m_k}

Definition 6.1. *A contextual KP System with marked derivation of degree n is a construct $KCP_{m_k} = (A, \mu, C_1, C_2, \dots, C_n, i_0)$ where A is called alphabet of elements*

μ is a membrane structure

C_1, C_2, \dots, C_n are n compartments with

$C_i = (t_i, w_i)$

$t_i = (R_i, \sigma_i)$

$R_i, 1 \leq i \leq n$ is a contextual rule of the form $(x, (u, v), tar)$ (attaching evolution rules) where $x, u, v \in A$ and $tar \in \{ here, in, out \}$ and

σ_i is an execution strategy in KP System and we are using the contextual rewriting rules as usual and finally in the output compartment we use an execution strategy to generalize the powers of the components of the generated string.

Theorem 6.1. $KCP_{m_k}(CF) - TC \neq \phi$

Proof.

$$KCP_{m_k}(P) = \{A, \mu, C_1, C_2, C_3, i_0\}$$

$$A = \{a, b\}$$

$$\mu = [{}_1[{}_2[{}_3]_3]_2]_1$$

$$C_1 = (t_1, \omega_1)$$

$$C_2 = (t_2, \omega_2)$$

$$t_1 = (R_1, \sigma_1)$$

$$t_2 = (R_2, \sigma_2)$$

$$\omega_1 = \{b\}$$

$$\omega_2 = \{\}$$

$$C_3 = (t_3, \omega_3)$$

$$t_3 = (R_3, \sigma_3)$$

$$\omega_3 = \phi$$

$$i_0 = 3$$

$$R_1 = \{r_1 : (b, (a, a), \text{here})\}$$

$$r_2 : (a^n b a^n, (\lambda, \lambda), \text{in})\}$$

$$\sigma_1 = r_1^* r_2$$

$$R_2 = \{r_1 : (a^n b a^n, (\lambda, b), \text{in})\}$$

$$\sigma_2 = r_1$$

$$R_3 = \{r_1 : (a^n b a^n b, (\lambda, a), \text{here})\}$$

$$r_2 = (a^n b a^n b a^k, (\lambda, a), \text{here}) \}$$

$$\sigma_3 = r_1^k r_2^{n-k}$$

$$L(KCP_{m_k}) = \{a^n b a^n b a^n, n \geq 1\}$$

The above language does not have the IBS property. So it is not in the TC family.

□

7 CONCLUSION

In this paper we used the contextual way of processing string objects in KP Systems. Also considered some variants of contextual KP systems with Depth-First Derivation and contextual KP System with marked derivation. With these variants we can generate powerful languages

References

- [1] Gh. Paun, Marcus Contextual Grammars ,Kluwer Academic publishers ,1997
- [2] Gheorghe Păun, *Membrane Computing An Introduction*, Springer-Verlag, Berlin, 2002.
- [3] Madhu Mutyam, Kamala Krithivasan, Contextual P Systems, Fundamenta Informaticae, 2002
- [4] Marian Gheorghe, Florentin Ipate, Ciprian Dragomir, Laurentiu Mierla, LuisValencia-Cabrera, Manuel Garcia-Quismondo, and Mario J, Kernel P System - Version 1, International Conference On Membrane Computing, 2013