A Study of Inventory System with Ramp Type Demand Rate and Shortage in The Light Of Inflation – I

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Abstract: In this paper, we develop an order-level inventory system for deteriorating items under inflation with ramp type demand function and partial exponential type-backlogging function of time. Three costs are considered under inflation as significant: deterioration, holding, shortage. The backlogging rate is an exponentially decreasing, time-dependent function specified by a parameter. For this model we derive results, which ensure the existence of a unique optimal policy and provide the solution procedure for the problem. The method is illustrated by numerical example, and sensitivity analysis of the optimal solution with respect to the parameters of the system is carried out.

Key-words: Inventory, Deteriorating items, Inflation, Ramp type demand.

Introduction: Most of the literature available during 20th century in the field of inventory management has not taken into account the effect of inflation. Perhaps this has happened mostly because of the perception and belief that inflation would not influence the policy variables to any significant degree. But during 21st century, the monetary situation of most of the countries, affluent or otherwise has changed to such an extent due to large scale inflation and consequent sharp decline the purchasing power of money, that it has not been possible to ignore the effects of inflation and so several efforts have been made by researchers to reformulate the optimal inventory management polices taking into account inflation.

The first attempt in this direction was done by Buzacott [2], where he dealt with an economic order quantity model with inflation subject to different types of pricing policies. After Buzacott [2], several other researchers have extended his approach to various interesting situations taking into consideration the inflation rate. In this connection the works of Misra [5,6], Aggarwal [1], Jeya Chandra and Bahner [3] etc. are worth mentioning. But in all these studies, the marked demand rate has been assumed to be constant and unsatisfied demand is completely backlogged.

However, for fashionable commodities and high-tech products with short product life cycle, the willingness for a customer to wait for backlogging during a shortage period is diminishing with the length of the waiting time. Hence the longer the waiting time, smaller the backlogging rate would be. To reflect this phenomenon, Papachristos and Skouri [7] established a partially backlogged inventory model in which the backlogging rate decreases exponentially as the waiting time increases.

In a recent communication Mandal and pal [4] attempted to study on order-level inventory model for deteriorating items, where the demand rate is a ramp-type function of time. Kun – shan Wu and Liang – Yuh –Ouyang [9] extended their work where inventory starts with shortages. This type of demand pattern is generally seen in the case of any new brand of consumer goods coming to the market. The demand rate for such items increases with time up to a certain time period and then ultimately stabilizes and becomes constant. It is believed that this type of demand rate is quite realistic

The above investigation led us to develop an inflationary model for deteriorating items with ramp type demand rate and partial – exponential type backlogging.

We attempt to provide the exact solution for the problem in the light of numerical example followed by sensitivity analysis of the optimal solution with respect to the parameters of the system.

ASSUMPTIONS AND NOTATIONS:

The mathematical model of the deterministic inventory replenishment problem with ramp type demand rate is based on the following assumptions:

i. The replenishment rate is infinite, thus replenishments are instantaneous.
ii. The lead-time is zero.
iii. The on hand inventory deteriorates at a constant rate θ (0< θ <1) per unit time. The deteriorated items are withdrawn immediately from the warehouse and there is no provision for repair or replacement.
iv. The rate of demand R (t) is ramp type demand function of t.

$$R(t) = D_0\left[t-(t-\mu)H(t-\mu)\right] \qquad D_0 > 0$$

Where H(t–μ) is Heaviside's function defined as follows:

$$H(t-\mu) = \begin{cases} 1 & t \geq \mu \\ 0 & t < \mu \end{cases}$$

v. Unsatisfied demand is backlogged at rate $e^{-\alpha x}$, where x is the time up to the next replenishment and 'α' a parameter $0 < \alpha < \frac{1}{T}$

vi. The unit price is subject to the same inflation rate as other related costs.

The following notations are used throughout this investigation:

- T – The fixed length of each ordering cycle.
- S – The maximum inventory level for each ordering.
- r – The inflation rate.
- C_h – The inventory holding cost per unit per unit of time.

- C_s – The shortage cost per unit per unit of time.
- C_d – Deterioration cost per unit of deteriorated item.
- I(t) – The on hand inventory at time t over [0, T).
- CI – The amount of inventory carried during a cycle.
- DI – The total number of items which deteriorate during a cycle
- SI – The amount of shortage during a cycle.

MATHEMATICAL MODELS AND SOLUTIONS:

The objective of the inventory problem here is to determine the optimal order quantity so as to keep the total relevant cost as low as possible under inflation has been subjected to the inventory starts without shortages.

The work of further investigation and we shall discuss the inventory model for deteriorating items under inflation, where the inventory starts without shortages.

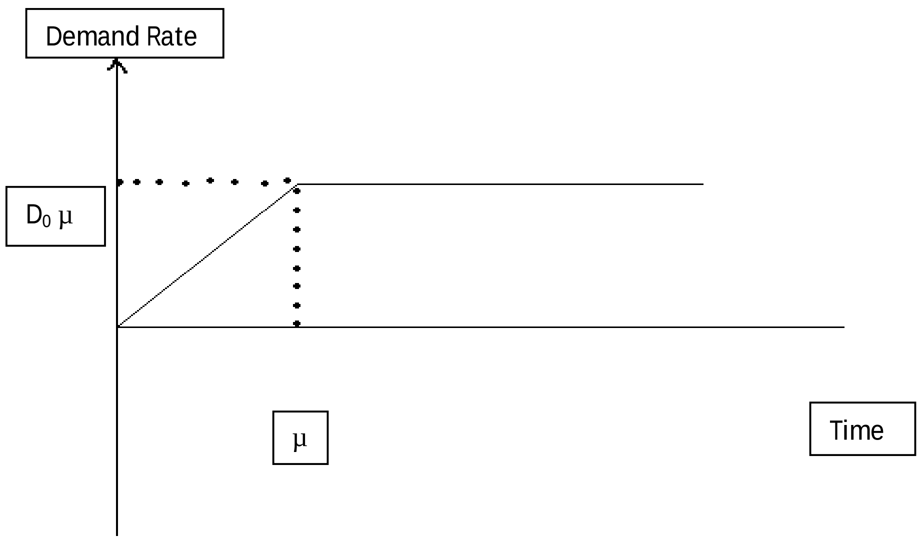

Figure 1. A ramp type function of the demand rate [Adapted from Mandal and Pal [4]]

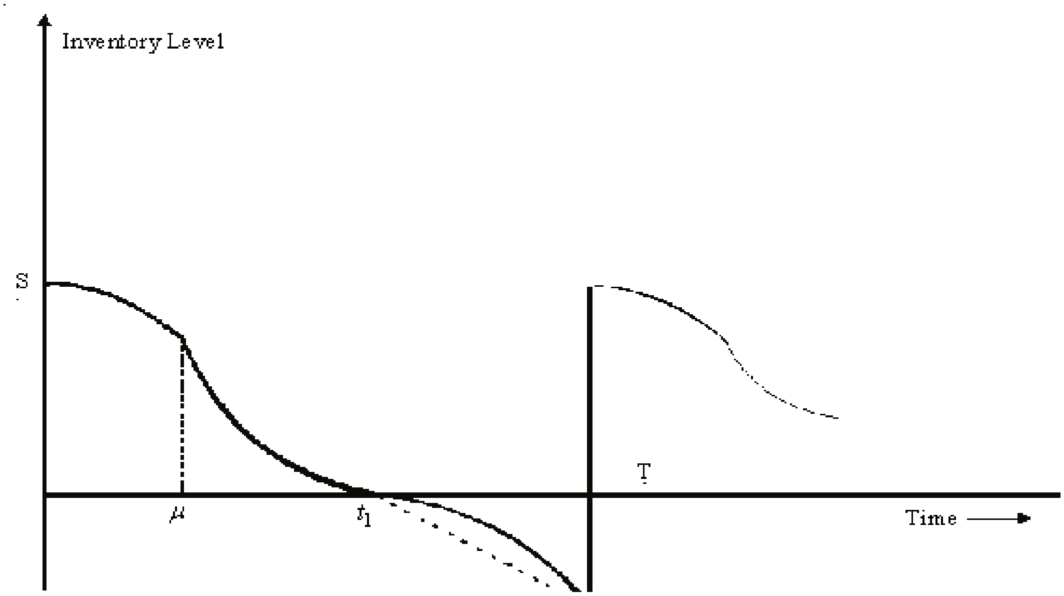

Figure 2. Graphical representation of inventory model

The fluctuation of the inventory level in the system is given in figure2. Replenishment is made at line t=0, when the inventory level is at its maximum S. The inventory at t=0 gradually reduces to zero at t_1 time units. The depiction of inventory level during the interval [0, t1] is due to the joint effect of the demand and the deterioration of items. At t_1, the inventory level reaches zero, thereafter shortage are allowed to occur during the time interval [t_1, T] and the demand during period [t_1, T] is partially backlogged. The total number of backlogged items is replaced by the next replenishment.

The inventory level of the system at time t over period [0, T) can be described by the following equations:

$$\frac{dI(t)}{dt} = -R(t) - \theta I(t) \qquad 0 \le t \le t_1 \qquad (1)$$

and

$$\frac{dI(t)}{dt} = -e^{-\alpha(T-t)} R(t) \qquad t_1 \le t < T \qquad (2)$$

For $\mu<t_1$, the above equation reduce to

$$\frac{d\,I(t)}{d\,t} + \theta\, I(t) = -\,D_0\, t \qquad 0 \le t \le \mu \tag{3}$$

$$\frac{d\,I(t)}{d\,t} + \theta\, I(t) = -\,D_0\, \mu \;\; ; \mu \le t \le t_1 \;\; \text{in view of } I(0)=S \tag{4}$$

and

$$\frac{dI(t)}{dt} = -e^{-\alpha(T-t)}\, D_0\mu \, ; \; t_1 \le t < T \;\; \text{in the light of } I(t_1)=0 \tag{5}$$

Since (3), (4) and (5) are first order liner differential equations it is fairly easy to derive their solutions as

$$I(t) = S\,e^{-\theta t} - D_0\left[\frac{t}{\theta} - \frac{1}{\theta^2}\left(1-e^{-\theta t}\right)\right];0 \le t \le \mu \tag{6}$$

$$I(t) = \frac{D_0\,\mu}{\theta}\left[e^{\theta(t_1-t)} - 1\right];\; \mu \le t \le t_1 \tag{7}$$

and

$$I(t) = -\frac{D_0\,\mu}{\alpha}\left[e^{-\alpha(T-t)} - e^{-\alpha(T-t_1)}\right];\; t_1 \le t < T \tag{8}$$

From equations (6) and (7), the value of I(t) at t=μ should coincide, which implies that

$$S = \frac{D_0\,\mu}{\theta}\, e^{\theta\, t_1} - \frac{D_0}{\theta^2}\left(e^{\theta\mu} - 1\right) \tag{9}$$

The amount of inventory carried during the period [0, t_1] is CI, where

$$CI = \int_0^{t_1} I(t)\,dt = \int_0^{\mu} I(t)\,dt + \int_{\mu}^{t_1} I(t)\,dt \tag{10}$$

The total number of items which deteriorate during [0, t_1] is DI, where

$$DI = S - \int_0^{t_1} R(t)\,dt = \theta \int_0^{t_1} I(t)\,dt \tag{11}$$

And the amount of shortage during the period [t_1, T)

$$SI = \int_{t_1}^{T} I(t)\,dt \tag{12}$$

The present value of the inventory holding cost during the period [0, t_1] is

$$= C_h \int_0^{t_1} e^{-rt} I(t)\,dt = C_h \left[\int_0^{\mu} e^{-rt} I(t)\,dt + \int_{\mu}^{t_1} e^{-rt} I(t)\,dt \right]$$

$$= C_h \left[\frac{S\theta^2 - D_0}{\theta^2(\theta+r)} \left(1 - e^{-(\theta+r)\mu}\right) + \frac{D_0(r-\theta)}{\theta^2 r^2}\left(1 - e^{-r\mu}\right) + \frac{D_0\,\mu\, e^{-rt_1}}{r(\theta+r)} + \frac{D_0\,\mu}{\theta(\theta+r)} e^{\theta t_1 - (\theta+r)\mu} \right] \tag{13}$$

The present value of the deterioration cost during the period [0, t_1] is

$$C_d \int_0^{t_1} e^{-rt}\,\theta\, I(t)\,dt = C_d\,\theta \left[\frac{S\theta^2 - D_0}{\theta^2(\theta+r)} \left(1 - e^{-(\theta+r)\mu}\right) + \frac{D_0(r-\theta)}{\theta^2 r^2}\left(1 - e^{-r\mu}\right) + \frac{D_0\,\mu\, e^{-rt_1}}{r(\theta+r)} + \frac{D_0\,\mu}{\theta(\theta+r)} e^{\theta t_1 - (\theta+r)\mu} \right] \tag{14}$$

The present value of the shortage cost during the period [t_1, T) is

$$C_S \int_{t_1}^{T} e^{-rt} I(t)\,dt = C_S \int_{t_1}^{T} e^{-rt} \frac{D_0\,\mu}{\alpha} \left\{ e^{-\alpha(T-t)} - e^{-\alpha(T-t_1)} \right\} dt$$

$$= C_S \left[\frac{D_0\,\mu}{\alpha(\alpha-r)} e^{-rT} - \frac{D_0\,\mu}{r(\alpha-r)} e^{-\alpha T + (\alpha-r)t_1} + \frac{D_0\,\mu}{\alpha r} e^{-(\alpha+r)T + \alpha t_1} \right] \tag{15}$$

Also the order quantity during the period [0, T) is given by

$$Q = S + I(T) = \frac{D_0\,\mu}{\theta} e^{\theta t_1} - \frac{D_0}{\theta^2}\left(e^{\theta\mu} - 1\right) + \frac{D_0\,\mu}{\alpha}\left(1 - e^{-\alpha(T-t_1)}\right) \tag{16}$$

Hence the total relevant cost of the system during the time interval [0,T), can be put as,

$$X_1 = (C_h + C_d\,\theta)\left[\frac{S\theta^2 - D_0}{\theta^2(\theta+r)} \left(1 - e^{-(\theta+r)\mu}\right) + \frac{D_0(r-\theta)}{\theta^2 r^2}\left(1 - e^{-r\mu}\right) + \frac{D_0\,\mu\, e^{-rt_1}}{r(\theta+r)} + \frac{D_0\,\mu}{\theta(\theta+r)} e^{\theta t_1 - (\theta+r)\mu} \right] + C_S \left[\frac{D_0\,\mu}{\alpha(\alpha-r)} e^{-rT} - \frac{D_0\,\mu}{r(\alpha-r)} e^{-\alpha T + (\alpha-r)t_1} + \frac{D_0\,\mu}{\alpha r} e^{-(\alpha+r)T + \alpha t_1} \right] \tag{17}$$

Thus, the average total cost per unit time is

$$TC_1(t_1) = \frac{X_1}{T} \tag{18}$$

To minimize the average total cost per unit of time, the optimal value of t_1 can be obtained by solving the following equation

$$\frac{d\,TC_1(t_1)}{d\,t_1} = 0 \tag{19}$$

This also satisfies the condition $\left.\dfrac{d^2 T C_1(t_1)}{d\,t_1^2}\right|_{t=t_1} > 0$

Equation (19) is equivalent to

$$\begin{aligned}(C_h + C_d\,\theta)&\left[\frac{D_0\,\mu\,e^{\theta t_1}}{(\theta+r)}\left(1-e^{-(\theta+r)\mu}\right) - \frac{D_0\,\mu}{(\theta+r)}\,e^{-r t_1} + \frac{D_0\,\mu\,e^{\theta t_1 - (\theta+r)\mu}}{(\theta+r)}\right] \\ &+ C_S\left[\frac{D_0\,\mu}{r}\,e^{-(\alpha+r)T+\alpha t_1} - \frac{D_0\,\mu}{r}\,e^{-\alpha T+(\alpha-r)t_1}\right] = 0\end{aligned} \tag{20}$$

This is a non-linear equation. This equation can be easily solved using any iterative method when the value of the parameters is prescribed.

By using the optimal value t_1*, the optimal value of S*, the minimum average total cost per unit of time and the optimal order quantity can be obtained from equation (9), equation (18) and equation (16) respectively.

Discussion and Conclusion:-

In the light of numerical example adopted by Kun-Shan Wu and Liang- Yuh- Ouyang [9], the input parameters in our case are as follows :-

Ch = \$3 per unit year, Cd = \$5 per unit, Cs = \$ 15 per unit per year

D0 = 100 units, μ = 0.12 years θ = 0.0001 T = 1 year

α = 0.08 r = 0.05

t_1* = 0.8276, S* = 9.2153, Q* = 11.2699, TC* = 14.6494

Where * stands for optimal values. Evidently optimal total cost of our inventory model is less than that of Kun-Shan Wu and Liang-Yuh-Ouyang [9]. Hence our model is believed to be better one.

Sensitivity Analysis:-

We will now study the sensitivity of the optimal solution to changes in the values of the different parameters associated with the inventory system in example. The results are shown in Table 1.

Sensitivity Analysis of numerical example:

Parameter	% change	% change in		
		S*	Q*	TC*
r	+50%	-2.46%	-0.03%	-1.02%
	+25%	-0.11%	-0.001%	-0.75%
	-50%	+0.23%	+0.003%	+1.02%
	-25%	+0.117%	+0.002%	+0.045%
α	+50%	-0.13%	-0.065%	-0.253%
	+25%	-0.065%	-0.032%	-0.228%
	-50%	+0.065%	+0.032%	+0.023%
	-25%	+0.13%	+0.064%	+0.045%
C_h	+50%	-8.68%	-0.122%	+37.85%
	+25%	-4.509%	-0.059%	+19.773%
	-50%	+4.89%	+0.052%	-21.64%
	-25%	+10.218%	+0.096%	-45.25%
C_d	+50%	-0.026%	0	-0.141%
	+25%	-0.013%	0	+0.04%
	-50%	0	0	-0.033%
	-25%	+0.013%	0	-0.073%
C_s	+50%	+6.63%	+0.068%	+6.025%
	+25%	+3.884%	+0.043%	+3.503%
	-50%	-5.942%	-0.079%	-5.556%
	-25%	-16.133%	-0.255%	-14.852%
θ	+50%	+0.01%	+0.018%	-0.136%
	+25%	0	+0.01%	-0.128%
	-50%	-0.011%	-0.01%	-0.064%
	-25%	-0.022%	-0.018%	-0.187%
μ	+50%	+44.14%	+45.21%	+48.72%
	+25%	+22.56%	+23.00%	+24.28%
	-50%	-23.54%	-23.80%	-24.93%
	-25%	-48.05%	-48.40%	-49.85%
D_0	+50%	+50.001%	+50%	+49.69%
	+25%	+25%	+25%	+24.74%
	-50%	-50%	-50%	-50.08%
	-25%	-25%	-25%	-25.16%

A careful study of Table 1 reveals the following points.

1. It is evident that S* is insensitive to change in the value of the parameter θ, α, C_d. It is moderately and highly sensitive to change in the value of parameters μ and D_0.
2. It is fairly easy to observe that the optimal order quantity Q* is insensitive to changes in the value of the parameters C_h, C_s, θ, r, α. It is highly sensitive to changes in the value of parameters μ and D_0. It is not affected by the change in the value of the parameter C_d.
3. Finally it is obviously clear that the optimum total cost is insensitive to changes in the values of parameters C_d, θ, α. It is moderately sensitive to change in the value of parameter Cs, r and highly sensitive to change in the value of parameters C_h, μ and D_0.

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